Iteration-1

- Iteration
  - assurance of convergence
  - rate of convergence
  - computational efficiency

- Iteration Criteria

$$|f(x_n)| \le F_{TOL} \quad \text{or} \quad |x_n - x_{n-1}| \le x_{TOL}$$

$$\text{or} \quad \frac{f(x_n)}{F_{SIZE}} \le F_{TOL} \quad \text{or} \quad |x_n - x_{n-1}| \le x_{TOL} \cdot x_n$$

- Newton's method

Iteration-2

Newton's method is old,
and it was already old
when Newton invented it.

- James Gleick, Chaos

Iteration-3

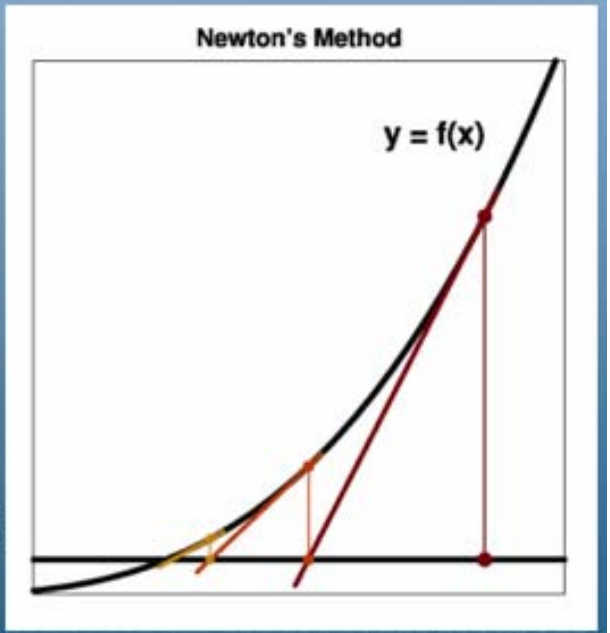
- **Newton's method :**

$$x^{n+1} = x^n - \frac{f(x^n)}{f'(x^n)}$$

- convergence :

$$g'(x) = \frac{f(x)\,f''(x)}{[f'(x)]^2} < 1$$

- for Newton's method to work :
  - a) $f''(x)$ must be small
  - b) $f'(x)$ may be large
  - c) good initial guess



Iteration-4

- Advantage of Newton's method :

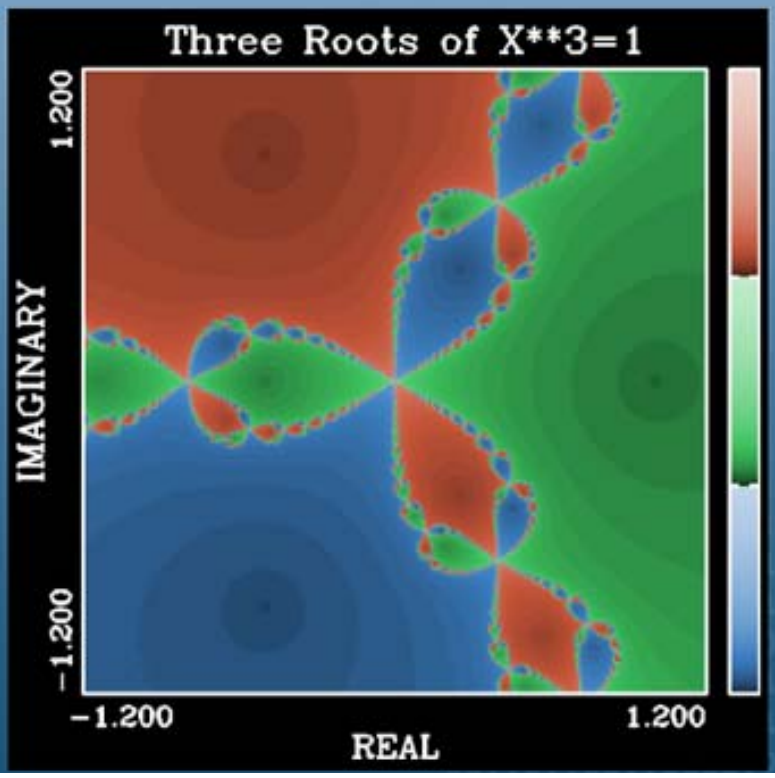
  speed ; converges quadratically

$$e_{n+1} \approx -\frac{1}{2} g''(\zeta) e_n^{\,2}, \quad \text{for large enough n}$$

- Disadvantages of Newton's method :
  - a) initial condition sensitive, good initial guess required
  - b) $f'(x)$ required

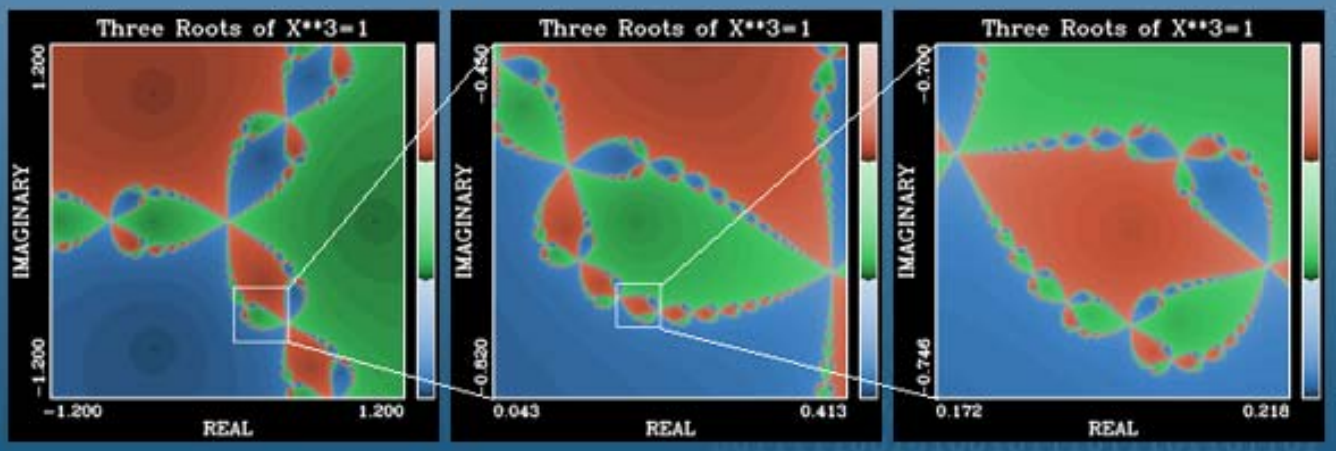
Iteration-5

- Use Newton's method to solve $x^3=1$:



Iteration-6

- Self-similarity



- Calculation of $\sqrt{k}$ by iteration :

$$x^n = \frac{1}{2}\left(x^{n-1} + \frac{k}{x^{n-1}}\right)$$