2nd Homework and Notes
for Lifescience Mathematics

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The problems and the notes are for the "read to learn" and "write to learn" purposes.

————————Part I (**Notes**)————————————-

Below are the summary for some important rules in the differentiation.

The Constant Multiple Rule If c is a constant and f is a differential function, then positive integer, then

$$\frac{d\ cf(x)}{dx} = c\frac{d\ f(x)}{dx}$$

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The Sum Rule If f and g are both differentiable, then

$$\frac{d\ f(x)+g(x)}{dx} = \frac{d\ f(x)}{dx} + \frac{d\ g(x)}{dx}$$

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The Difference Rule If f and g are both differentiable, then

$$\frac{d\ f(x)-g(x)}{dx} = \frac{d\ f(x)}{dx} - \frac{d\ g(x)}{dx}$$

.

The Product Rule If f and g are both differentiable, then

$$\frac{d\ f(x)g(x)}{dx} = g(x)\frac{d\ f(x)}{dx} + f(x)\frac{d\ g(x)}{dx}$$

.

The Quotient Rule If f and g are both differentiable, then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d\,f(x)}{dx} - f(x)\frac{d\,g(x)}{dx}}{[g(x)]^2}$$

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The Chain Rule If f and g are both differentiable and F is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x).$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable function, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

The Power Rule Combined with the Chain Rule If n is any real number and $u = g(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}.$$

The Power Rule If n is a positive integer, then

$$\frac{d\,x^n}{dx} = nx^{n-1}$$

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The Power Rule (General Version) If n is any real number, then

$$\frac{d\,x^n}{dx} = nx^{n-1}$$

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Some useful series for the exponential, logarithmic, and trigononometric functions; and also some useful binomial series are included. Please try to comprehend deeply the **geometrical meaning** (especially the low order components)of the series.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 \cdots, -1 < x < 1$$

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \cdots, -1 < x < 1$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \cdots, -1 < x < 1$$

$$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \cdots, -1 < x < 1$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, -1 < x \leq 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots, -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots, -\infty < x < \infty$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

$$e^{-x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots,$$

$$(x+y)^n = \sum_{m=0}^{n} \binom{n}{m} x^{n-m} y^m,$$

Part II (**HOMEWORK**)

1 Derive the first 7 of the above series from the Taylor expansion formula,

$$f(x_0+x) = f(x_0)+f^{(1)}(x-x_0)+\frac{1}{2!}f^{(2)}(x_0)(x-x_0)^2+\frac{1}{3!}f^{(3)}(x_0)(x-x_0)^3+\cdots,$$

where the $f^{(n)}(x_0)$ is the nth differential of f at x_0. [The origin of the Taylor expansion is another important issue which we have discussed in the class. I may give the derivation in the examination.]

2 The differentiation can be defined as

$$\frac{df(x)}{dx} = \lim_{\triangle x\to 0}\frac{f(x+\triangle x)-f(x)}{\triangle x}.$$

With the help of the above series, basic definition and/or other rules, derive the following 13 differentiations,
(1) If n is a positive integer, then

$$\frac{dx^n}{dx} = nx^{n-1}$$

(2)

$$\frac{d}{dx}\sqrt{x^2+y^2} = \frac{1}{\sqrt{x^2+y^2}}$$

Can you think of any geometric meaning of $df/dx = 0$ in this case.

Derivatives of Logarithmic Functions

(3)

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}.$$

(4)

$$\frac{d}{dx}(\ln x) = \frac{1}{x}.$$

(5)

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}.$$

Derivative of the Exponential Functions

(6)
$$\frac{d}{dx}(e^x) = e^x.$$

(7)
$$\frac{d}{dx}(a^x) = a^x \ln a.$$

Derivatives of Trigonometric Functions

(8)
$$\frac{d}{dx}(\sin(x)) = \cos(x).$$

(9)
$$\frac{d}{dx}(\cos(x)) = -\sin(x).$$

(10)
$$\frac{d}{dx}(\tan(x)) = \sec^2(x).$$

(11)
$$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x).$$

(12)
$$\frac{d}{dx}(\sec(x)) = -\sec(x)\tan(x).$$

(13)
$$\frac{d}{dx}(\cot(x)) = -\csc^2(x).$$