1st Homework
for Lifescience Mathematics

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The problems are for the "read to learn" and "write to learn" purposes. It is not a required turn-in home work. It is a **"Bonus"** homework in which you get extra points for the work you turn in.

1 Struggle for Existence from Charles Darwin

"*A struggle for existence inevitably follows from the high rate at which all organic beings tend to increase. Every being, which during its natural lifetime produces several eggs or seeds, must suffer destruction during some period of its life, and during some season or occasional year; otherwise, on the principle of geometrical increase, its number would quickly become so inordinately great that no country could support the product. Hence, as more individuals are produced than can possibly survive, there must in every case be a struggle for existence, either one individual with another of the same species, or with the individuals of distinct species, or with physical conditions of life. It is the doctrine of Malthus applied with manifold force to the whole animal and vegetable kingdoms; for in this case there can be no artificial increase of food, and no prudential restraint from marriage. Although some species may be now increasing, more or less rapidly, in numbers, all cannot do so, for the world would not hold them....*

The amount of food for each species of course gives the extreme limit to which each can increase: but very frequently it is not the obtaining food, but the serving as prey to other animals, which determines the average numbers of a species."

Darwin was not a mathematician, but his insight as a modeler was profound. Please read the above statement carefully and give a very short summary. Also briefly state what is the doctrine of Malthus? Try to write some equations which describing the Darwin's statement.

2. The meaning of "never".

One of the most important developments in the study of the firng of nerve cells or neurons was the development of a model for this phenomenon in

giant squid in the 1950s by Hodgkin and Huxley. They developed a four-dimensional system of differential equations that described the electrochemical transmission of neuronal signals along the cell membrane, a work for which they later received the Nobel prize. Roughly speaking, this system is similar to systems that arise in electrical circuits. The neuron consists of a cell body,or *soma*, which receives electrical stimuli. This stimulus is then conducts along the *axon*, which can be thought fo as an electrical cable that connects to other neurons via a collection of synapses. Of course, the motion is not really electrical, becasue the current is not really made up of electrons, but rather ions (predominantly sodium and potassium). The four-dimensional **Hodgkin-Huxley** system is difficicult to deal with, primarily because of the highly nonlinear nature of the equations. An important breakthrough from a mathematical point of view was achived by the Fitzhugh and Nagumo in 1961, who produced a simpler model of the **Hodgkin-Huxley** model. Although this system is not as biologically accurate asthe original system, it nevertheless does capture the essential behavior of nerve impulse, including the phenomenon of *excitability*. The **Fitzhugh-Nagumo** equation is discussed in section 5.7 of the Adler book. It will be an important topic in our course.

It has been said by Huxley that *"six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum."* Huxley's statement is nonsense, for it gives a misleading conclusion about very, very large numbers. Please reason with numbers for the validity of Huxley's statement. Could all the monkeys in the world have typed out a single specified book (say Hamlet) in the age of the universe? Could all the human being in the history of mankind have typed out the sentences in this problem or a single specified book simply by arbitrary work?

(a) The age of the universe is $10^{18}s$.

(b) The average lifetime of human being is $10^9 s$ and the total number of lives in the history is 10^{11}. The cumulative number of man-seconds is $10^{20}s$.

(c) The number of monkeys is an arbitrary choice. We can assume the number is about three times greater than the present population of the earth. Then 10^{10} monkeys are assumed to have been seated at typewriters throughout the age of the universe.