Atmospheric Dynamics

Homework Ⅳ

Deadline: 2006.05.08

(1) Please write problems (8.1) (8.2).

(2) In an conditional unstable atmosphere, Bjerknes (1938) proves that, when the effect of adiabatic warming due to the *subsidence in the cloud environment* ins included, conditional instability prefers the smallest scale.

Let σ be the fraction horizontal area covered by cumulus clouds and $\Gamma_d < \Gamma < \Gamma_m$. (a) Explain the following equations

$$\frac{\partial T}{\partial t} \cong W_{cloud} \, (\Gamma - \Gamma_m)$$

and

$$\frac{\partial T}{\partial t} \cong W_{env} \, (\Gamma - \Gamma_d).$$

(b) Prove the Bjerknes theorem by deriving the following equation ($\sigma << 1$)

$$\sigma < \frac{\Gamma - \Gamma_m}{\Gamma_d - \Gamma}$$

and explain physically the result.

(3) Define absolute zonal momentum M by $M = fy - u$. Now consider two rubes of air (assuming extended infinitely along the x axis) on a θ surface. The M values of the tube of air are $M_1 = fy - u_1$ and $M_2 = f(y + \delta y) - u_2$,

(a) Use the second law of thermodynamics to prove that the mathematical condition for the symmetric instability is $(\partial M/\partial y)_\theta < 0$ or $M_2 < M_1$

(b) Show that the symmetric instability involves not only the adiabatic rearrangement of air tube *without mixing*.

(4)

(a) As compare to symmetric instability, the parcel theory of Kelvin Helmholtz instability requires *momentum mixing* of the two parcels to establish the instability necessary condition $R_i < 1/4$. Consider a parcel of ($\rho + \delta\rho$, $u + \delta u$, $z + \delta z$) on top of another parcel of (ρ, u, z), show that $R_i > 1/4$ is the sufficient condition for stability in this two parcel system.

(b)請簡明整理 Kelvin Helmholtz 不穩度的物理條件與物理特性。