Atmospheric Dynamics

Homework Ⅲ

Deadline: 2006.04.10

(1) Please write problems (7.8), (7.9)

(2) One important factor in the success of the atmospheric numerical modeling seems to lie in *distinguishing different scales of motion governing weather systems and appropriate simplification of basic hydrodynamic equation by rational approximation in order to describe motions of a particular scale.*

Consider the following three-dimensional linearized equations:

$$\frac{\partial \mathbf{v}'}{\partial t} = -\frac{1}{\rho_0}\nabla p' - \frac{\rho'}{\rho_0} g\mathbf{k} ,$$

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho_0 \mathbf{v}') = 0 ,$$

$$\frac{\partial}{\partial t}\left(\frac{\theta'}{\theta_0}\right) + w' \frac{\partial \ln \theta_0}{\partial z} = 0 ,$$

$$\frac{\theta'}{\theta} = -\frac{\rho'}{\rho} + \frac{1}{\gamma}\frac{p'}{p_0} .$$

(a) List the conditions that filtered out the buoyancy wave from the above equations. Derive the dispersion relationship for the adiabatic sound waves that are not affected by the buoyancy ("pure sound waves")

(b) List the conditions that filtered out the sound wave from the above equations. Derive the dispersion relationship for the incompressible buoyancy waves.

(c) Derive the energy equation for the incompressible buoyancy waves and the "pure sound waves". What kind of energy exchange are involved in the "pure sound waves" ? What about the incompressible buoyancy waves ? Also name the energy terms! Hint :

$$\rho_0 \frac{\partial}{\partial t}\left[\frac{1}{2}\mathbf{v}'^2 + \frac{1}{2c_s^2}\left(\frac{p'}{\rho_0}\right)^2\right] = -\nabla \cdot (p'\mathbf{v}')$$

$$\rho_0 \frac{\partial}{\partial t}\left[\frac{1}{2}w'^2 + \frac{1}{2N^2}\left(g\frac{\theta'}{\theta_0}\right)^2\right] = 0$$

(d) Under the hydrostatic balance approximation, how do we call the energy term ? Physically interpret these energy in terms of the “work required”, the “pressure gradient force” and the “buoyancy force”.

(3) “抽刀斷水水更流” 還是 “抽刀斷水水不流”？在 f 面水高 H 之淺水系統，若將距離 2L 之柵門抽起，請以重力波速度 c，柯氏參數 f 及 L 定性討論其物理結果。