EAS 422
Atmospheric Dynamics

The Final examination May, 2003

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Total 130 points

1 (10 pts)
The geostrophic vorticity is

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{1}{f_0}\nabla^2\phi$$

Explain why the troughs in the 500 mb isobaric surface are regions rich in positive vorticity. Draw a schematic force balance diagram for the geostrophic vorticity.

2 (40 pts)
The quasi-geostrophic forms of the vorticity and thermodynamic energy equations can be written (in pressure coordinate):

$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V_g}\cdot\nabla\zeta_g - \beta v_g - f_0\nabla\cdot\mathbf{v}, \tag{1}$$

$$\frac{\partial T}{\partial t} = -\mathbf{V_g}\cdot\nabla T - \left(\frac{\sigma p}{R}\right)\omega. \tag{2}$$

(a) Discuss the physical meaning of (1). Show how the planetary vorticity (relative) advection term in (1) tends to move the 500-mb geopotential pattern westward (eastward).

(b) Sketch a schematic diagram for the vorticity advection and horizontal divergence near a trough.

(c) Discuss the physical meaning of (2).

(d) Sketch a schematic diagram for the temperature advection and vertical motion near a surface low pressure system.

3 (80 pts)
The quasi-geostrophic geopotential tendency equation and ω equation can be written (in pressure coordinate):

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial \phi}{\partial t} = -f_0\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\phi + f\right) - \frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\mathbf{V_g}\cdot\nabla\left(-\frac{\partial\phi}{\partial p}\right)\right], \tag{3}$$

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\phi + f\right)\right] + \frac{1}{\sigma}\nabla^2\left[\mathbf{V_g}\cdot\nabla\left(-\frac{\partial\phi}{\partial p}\right)\right]. \tag{4}$$

(a) Discuss the qualitative content of the tendency equation.

(b) Discuss the qualitative content of the omega equation.

(c) What are the driving mechanisms for the evolution of mid-latitude synoptic disturbances.

(d) Explain what is meant by the statement, "The secondary circulation associated with the vertical motion tends to oppose horizontal advection." In your explanation, describe physically how this occurs for both vorticity and temperature.

(e) What do the Laplace type of operator in the left hand side of (3) and (4) tell about the spatial distribution of $\partial\phi/\partial t$ and ω versus the spatial distribution of advection.

(f) How do the $\partial/\partial p$ advection height variation term in the right hand side of (3) (and in (4)) affect the geopotential tendency (and the vertical motion)?

(g) Fill the diagram with the vertical motion fields well as the physical processes that give rise to the vertical circulation in various region.

(h) Fill the table for the characteristics of a developing baroclinic disturbance. [hint: state either positive or negative]

4 (70 pts) The vector Ekman layer equation is

$$K\frac{\partial^2\vec{V}}{\partial z^2} = f\hat{k}\times\vec{V} - f\hat{k}\times\vec{V}_g. \tag{5}$$

(a) What is K called? What is the typical magnitude for K in the atmosphere?

(b) What forces are represented in (5)?

(c) Assume the isobars are oriented in the x-direction and the wind vanishes at the surface. Draw the Ekamn spiral solution for the wind field and the frictional force.

(d) To what dynamical field is the Ekman pumping proportional? Explain physically.

(e) What does the terms "secondary circulations" and "spin-down" mean?

(f) Use the equation (5) to prove that the total ageostrophic mass transport in the ocean is directed 90 degree to the right of the surface wind stress in the Northern Hemisphere. Mathematically this is stated as

$$\vec{M}_a = \int_{-\infty}^{0} \rho(\vec{V} - \vec{V}_g)dz, \tag{6}$$

$$\vec{M}_a = -\frac{\rho K}{f}\hat{k}\times(\frac{\partial\vec{V}}{partial z})_{z=0}. \tag{7}$$

(g) Give three phenomena that can be explained by (7).