EAS 422
Atmospheric Dynamics

The 1st one-hour examination Feb. 26, 2003

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Total 110 points

1 (15 pts) Discuss the physical meaning of the following vector operations, also state whether the yield of vector operation is a scalar or a vector.
(a) $\nabla \cdot \mathbf{V}$
(b) $\nabla \times \mathbf{V}$
(c) $\nabla \phi$
(d) $\nabla^2 \phi$
(e) $\mathbf{A} \cdot \nabla \phi$

2 (15 pts) The vector momentum equation as derived with minimum assumptions is

$$\frac{D\mathbf{V}}{Dt} + 2\mathbf{\Omega} \times \mathbf{V} = -\frac{\mathbf{1}}{\rho}\nabla_{\mathbf{z}}\mathbf{p} - \mathbf{g} + \nu\nabla^{\mathbf{2}}\mathbf{V}.$$

(a) Discuss the meaning of each terms.
(b) The $\mathbf{V}$ has three components u, v, and w, what are the physical meanings of these components? Does the definition or meaning of $\mathbf{V}$ changed when we use pressure as an alternate vertical coordinate?
(c) Discuss the Coriolis force from the angular momentum and centrigual force point of views.

3 (10 pts) The approximate u component of equation can be written as

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} = -\frac{1}{\rho}\frac{\partial p}{a\cos\phi\partial\lambda} + 2\Omega v\sin\phi.$$

(a) What are the characteristic magnitudes of each of the five terms for mid-latitude synoptic-scale motion?
(b) What is the Rossby number and what is its significance?

4 (15 pts) Consider the following equation [equation (1.22) in the text book] for transforming gradient quantities from height (z) to s vertical coordinates

$$\nabla_s f = \nabla_z f + \frac{\partial f}{\partial z}\nabla_s z,$$

where f is any arbitrary scalar parameter.
(a) Show that if $f = p$ and $s = p$,

$$g\nabla_p z = \frac{1}{\rho}\nabla_z p.$$

(b) Discuss the meaning of the above equation.
(c) What is the other quantity sometimes used as an alternate vertical coordinate?

5 (10 pts) An atmospheric with a dry adiabatic lapse rate (i.e. constant potential temperature) the geopotential height is given by

$$z = [1 - (\frac{p}{p_0})^{R_d/c_p}]\frac{c_p\theta_0}{g}$$

where p_0 is the pressure at $z = 0$.
(a) What is the depth of this atmosphere?
(b) What is the temperature at the top of this atmosphere?

6 (10 pts) Consider the conservation of angular momentum $(\Omega a\cos\phi + u)a\cos\phi$ in a zonally symmetric, inviscid flow, compute the zonal wind of a parcel which has risen, at the equator, from the ocean surface to a height of 16 km above sea level and move poleward, at 16 km height, to ϕ N latitude. Assume the zonal velocity was zero at sea level. What is its zonal velocity at this latitude? Is the wind induced easterly or westerly?

7 (10 pts) Define and discuss the Brunt-Vaisala frequency N. What is the typical value of N in the atmosphere?

8 (10 pts) Discuss what physical principles govern the dynamics of the atmosphere? [hint: conservation laws and constitutive equation]

9 (15 pts) Consider the continuity equation in the Eulerian form

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial x} + \frac{\partial\rho v}{\partial y} + \frac{\partial\rho w}{\partial z} = 0.$$

(a) From the above expression derive the continuity equation in the Lagrange form

$$\frac{D\rho}{Dt} = -\rho[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}],$$

where

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z}.$$

(b) If

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

has the meaning of rate of volume change, discuss the meaning of continuity equation both in Eulerian and Lagrange forms.
(c) Explain why in the condition of incompressible flow the density is conserved follow the motion.[hint: no volume change if incompressible.]