數學模式與科學研究

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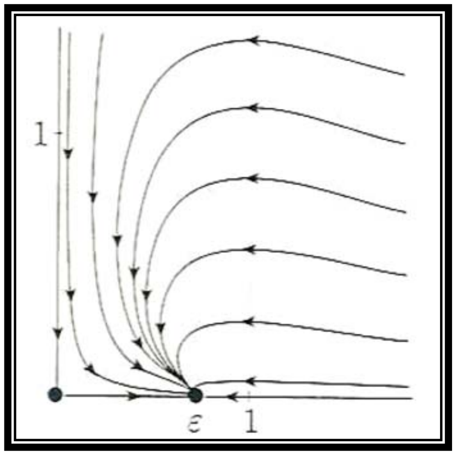
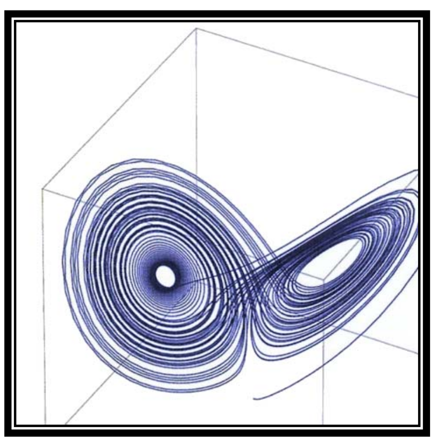


（掠食者的滅絕）

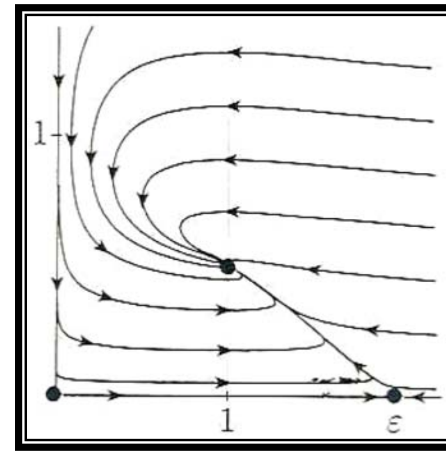


（Lorenz 吸子）

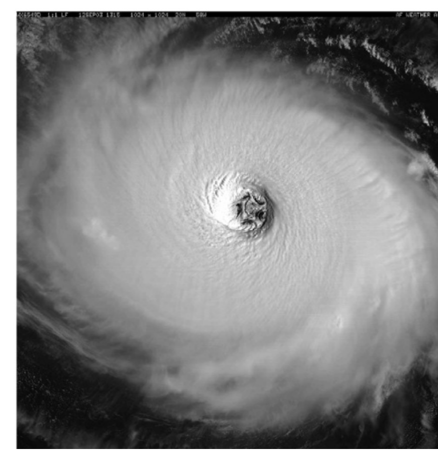
內容：

科學研究是探索未知知識的建構過程，而數學是科學的語言。隨著電腦的進步，資料大幅度的數位化，科學計算更成為打開非線性科學研究的敲門磚。數學建模、科學計算、分析詮釋與驗證等過程，更是現今數學科學的典範。演講將簡介大氣科學數學建模的過程，並以生命科學例子說明數學建模、科學計算、分析詮釋與驗證等過程。

2008/5/14　　輔仁大學



（掠食者與被掠食者共存）



To derive the equations ..... or not to derive the equations that is a question !!

Derivation of (4.11)

$$\frac{\partial u_g}{\partial t} + u\frac{\partial u_g}{\partial x} + v\frac{\partial u_g}{\partial y} + w\frac{\partial u_g}{\partial z} - fv + \frac{\partial \phi}{\partial x} = 0 \qquad -(a)$$

$$\frac{\partial v_g}{\partial t} + u\frac{\partial v_g}{\partial x} + v\frac{\partial v_g}{\partial y} + w\frac{\partial v_g}{\partial z} + fu + \frac{\partial \phi}{\partial y} = 0 \qquad -(b)$$

(i) x component

$$-\frac{\partial(b)}{\partial z} \Rightarrow \frac{D}{Dt}\left(-\frac{\partial v_g}{\partial z}\right) - \frac{\partial u}{\partial z}\frac{\partial v_g}{\partial x} - \frac{\partial v}{\partial z}\frac{\partial v_g}{\partial y} - \frac{\partial w}{\partial z}\frac{\partial v_g}{\partial z} - f\frac{\partial u}{\partial z} - \frac{\partial}{\partial y}\frac{\partial \phi}{\partial z} = 0 \qquad (1)$$

$$\frac{1}{f}\frac{\partial u_g}{\partial y}\frac{\partial(b)}{\partial z} \Rightarrow \frac{1}{f}\frac{\partial u_g}{\partial y}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial z}\right) + \left(\frac{\partial u}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial y} + \frac{\partial w}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial z}\right)\frac{1}{f} + \frac{\partial u}{\partial z}\frac{\partial u_g}{\partial y} + \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial y}\right)\cdot\frac{1}{f}\frac{\partial u_g}{\partial y} = 0 \qquad (2)$$

$$-\frac{1}{f}\frac{\partial u_g}{\partial z}\frac{\partial(b)}{\partial y} \Rightarrow -\frac{1}{f}\frac{\partial u_g}{\partial z}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial y}\right) - \left(\frac{\partial u}{\partial y}\frac{\partial v_g}{\partial x}\frac{\partial u_g}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v_g}{\partial y}\frac{\partial u_g}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial z}\right)\frac{1}{f} - \frac{\partial u}{\partial y}\frac{\partial u_g}{\partial z} - \frac{\partial}{\partial y}\frac{\partial \phi}{\partial y}\cdot\frac{1}{f}\frac{\partial u_g}{\partial z} = 0 \qquad (3)$$

and $\frac{1}{f}\frac{\partial v_g}{\partial z}\frac{\partial(a)}{\partial y}$

and $-\frac{1}{f}\frac{\partial v_g}{\partial y}\frac{\partial(a)}{\partial z}$

$$\frac{\partial \phi}{\partial z} = \frac{g\theta}{\theta_0} \quad ; \quad \frac{1}{f}\frac{\partial \phi}{\partial y} = -u_g \qquad \frac{1}{f}\frac{\partial \phi}{\partial x} = v_g$$

$$\frac{D}{Dt}\left(-\frac{\partial v_g}{\partial z}\right) - \frac{\partial u}{\partial z}\frac{\partial v_g}{\partial x} - \frac{\partial v}{\partial z}\frac{\partial v_g}{\partial y} - \frac{\partial w}{\partial z}\frac{\partial v_g}{\partial z} - f\frac{\partial u}{\partial z} - \frac{g}{\theta_0}\frac{\partial \theta}{\partial y} = 0$$

$$\frac{1}{f}\frac{\partial u_g}{\partial y}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial z}\right) + \frac{1}{f}\left(\frac{\partial u}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial y} + \frac{\partial w}{\partial z}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial z}\right) + \frac{\partial u}{\partial z}\frac{\partial u_g}{\partial y} - \frac{\partial u_g}{\partial z}\frac{\partial u_g}{\partial y} = 0$$

$$-\frac{1}{f}\frac{\partial u_g}{\partial z}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial y}\right) - \frac{1}{f}\left(\frac{\partial u}{\partial y}\frac{\partial v_g}{\partial x}\frac{\partial u_g}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v_g}{\partial y}\frac{\partial u_g}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial z}\right) - \frac{\partial u}{\partial y}\frac{\partial u_g}{\partial z} + \frac{\partial u_g}{\partial y}\frac{\partial u_g}{\partial z} = 0$$

$$\frac{1}{f}\frac{\partial v_g}{\partial z}\frac{D}{Dt}\left(\frac{\partial u_g}{\partial y}\right) + \frac{1}{f}\left(\frac{\partial v_g}{\partial z}\frac{\partial u}{\partial y}\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial z}\frac{\partial v}{\partial y}\frac{\partial u_g}{\partial y} + \frac{\partial v_g}{\partial z}\frac{\partial w}{\partial y}\frac{\partial u_g}{\partial z}\right) - \frac{\partial v}{\partial y}\frac{\partial v_g}{\partial z} + \frac{1}{f}\frac{\partial}{\partial y}\frac{\partial \phi}{\partial x}\frac{\partial v_g}{\partial z} = 0$$

$$-\frac{1}{f}\frac{\partial v_g}{\partial y}\frac{D}{Dt}\left(\frac{\partial u_g}{\partial z}\right) - \frac{1}{f}\left(\frac{\partial v_g}{\partial y}\frac{\partial u}{\partial z}\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial v}{\partial z}\frac{\partial u_g}{\partial y} + \frac{\partial v_g}{\partial y}\frac{\partial w}{\partial z}\frac{\partial u_g}{\partial z}\right) + \frac{\partial v}{\partial z}\frac{\partial v_g}{\partial y} - \frac{1}{f}\frac{\partial}{\partial z}\frac{\partial \phi}{\partial x}\frac{\partial v_g}{\partial y} = 0$$

$$\frac{D}{Dt}\left(-\frac{\partial v_g}{\partial z}\right) + \frac{1}{f}\left(\frac{\partial u_g}{\partial y}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial z}\right) + \frac{\partial v_g}{\partial z}\frac{D}{Dt}\left(\frac{\partial u_g}{\partial y}\right) - \frac{\partial u_g}{\partial z}\frac{D}{Dt}\left(\frac{\partial v_g}{\partial y}\right) - \frac{\partial v_g}{\partial y}\frac{D}{Dt}\left(\frac{\partial u_g}{\partial z}\right)\right) - \frac{g}{\theta_0}\frac{\partial \theta}{\partial y}$$

$$- \frac{\partial u}{\partial z}\left(f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} - \frac{1}{f}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial x} + \frac{1}{f}\frac{\partial v_g}{\partial y}\frac{\partial u_g}{\partial x}\right)$$

$$- \frac{\partial u}{\partial y}\left(\frac{\partial u_g}{\partial z} + \frac{1}{f}\frac{\partial v_g}{\partial x}\frac{\partial u_g}{\partial z} - \frac{1}{f}\frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial x}\right)$$

$$- \frac{\partial u}{\partial x}\left(-\frac{\partial v_g}{\partial x} + \frac{1}{f}\frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial y} - \frac{1}{f}\frac{\partial u_g}{\partial z}\frac{\partial v_g}{\partial y}\right) = 0$$

$$\frac{D}{Dt}\left(-\frac{\partial v_g}{\partial z} + \frac{1}{f}\left(\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial z} - \frac{\partial v_g}{\partial y}\frac{\partial u_g}{\partial z}\right)\right) - \frac{g}{\theta_0}\frac{\partial \theta}{\partial y} \qquad \frac{D}{Dt}\zeta_{gx} - \frac{g}{\theta_0}\frac{\partial \theta}{\partial y} = \zeta \cdot \nabla u$$

$$= \left(f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} - \frac{1}{f}\frac{\partial u_g}{\partial y}\frac{\partial v_g}{\partial x} + \frac{1}{f}\frac{\partial v_g}{\partial y}\frac{\partial u_g}{\partial x}\right)\frac{\partial u}{\partial z}$$

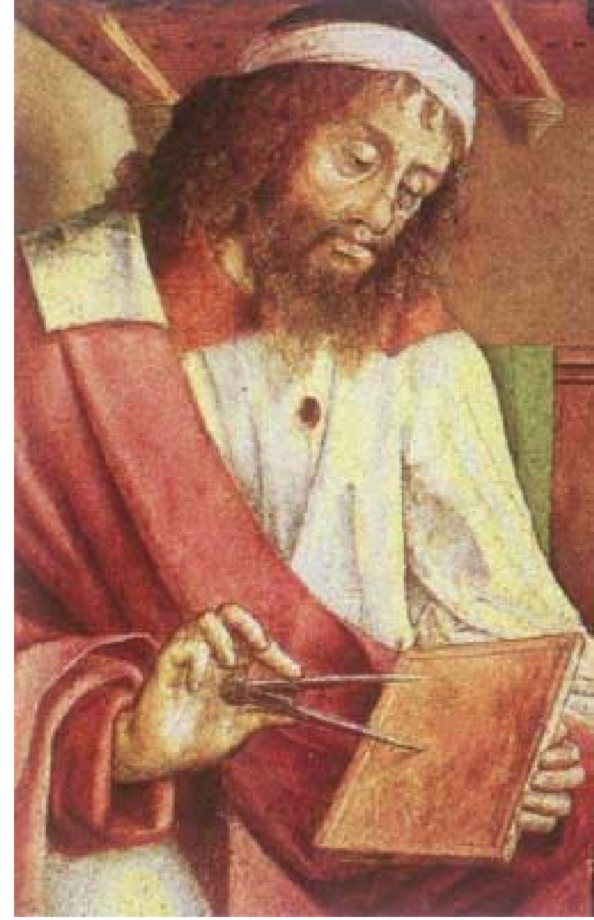
$$+ \left(\frac{\partial u_g}{\partial z} + \frac{1}{f}\left(\frac{\partial v_g}{\partial x}\frac{\partial u_g}{\partial z} - \frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial x}\right)\right)\frac{\partial u}{\partial y}$$

$$+ \left(-\frac{\partial v_g}{\partial x} + \frac{1}{f}\left(\frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial y} - \frac{\partial v_g}{\partial z}\frac{\partial u_g}{\partial y}\right)\right)\frac{\partial u}{\partial x}$$

⇒ similar derivation for y, z component

讀 算 寫

幾何
代數
微積分
電腦計算繪圖



+ -	x /	
加、減	乘、除	
線性	**非線性**	數量化、數位化
大題大作	小題大作	數學化--模式--動力系統

數學科學

Formulation　　微分、差分方程式

Solution / Analysis　　分析、解

Interpretation　科學詮釋

中階課程：微分方程(ODE,PDE)
統計、線性代數
程式、計算與繪圖

數量化、數位化
數學化--模式--動力系統

數學模式

理論、觀點 Theory

False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness.

Darwin, The Origin of Man, chapter 6

解釋資料

Never trust an observation without a supporting interpretation

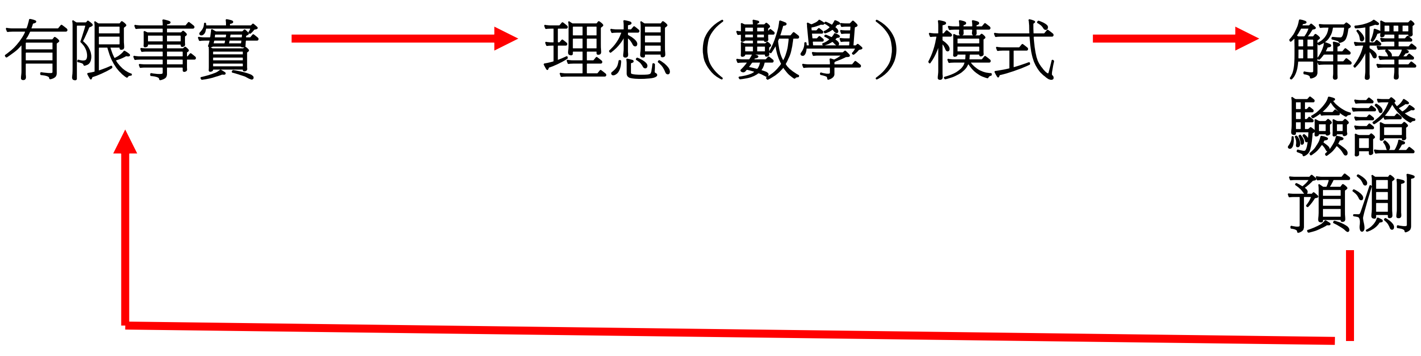
預測 準確性 預測能力

資料同化 利用科學數學模式整合有限的觀測，建構出較完整的資料

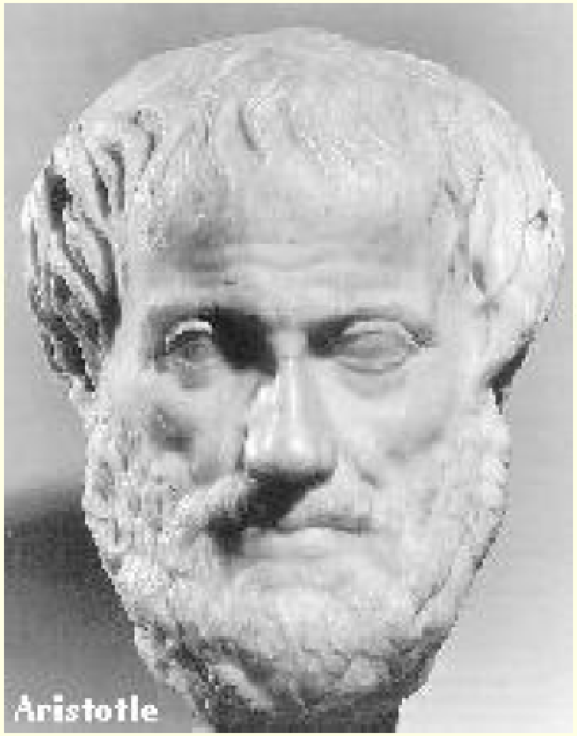
以特殊事實為憑藉，逐漸推廣引伸，成立概念式定律的系統，
以便籠罩更複雜耿廣泛的對象，
科學家依據事實為前提來證明普遍的結論。　　方東美

吾生也有涯，吾知也無涯，已有涯逐無涯，殆矣。
有限時空之觀察或有限資料去推導無限時空的科學定律。

問蒼茫大地誰主浮沈？
有物有則 因果律　　為什麼？　　形而上學



Aristotle's *Meteorologia*



Aristotle (384-322 BC) was a past master at asking questions.

He wrote the first book on Meteorology, the *Μετεωρολογια* (*μετεωρον*: Something in the air)

This work dealt with the causes of various weather phenomena and with the origin of comets.

While a masterly speculator, Aristotle was a poor observer: for example, he believed that the lightning followed the thunder!

形而上學：憑藉第一原因，一切事物方能知曉，但其本身是自明的。


南方有倚人焉
曰黃繚問天地
所以不墜不陷
風雨雷霆之故
惠施不辭而應
不慮而對偏為
萬物說
莊子 天下
郭鴻基
謝建輝

Sir Isaac Newton
(1642-1727)

Isaac Newton

Principia 1687

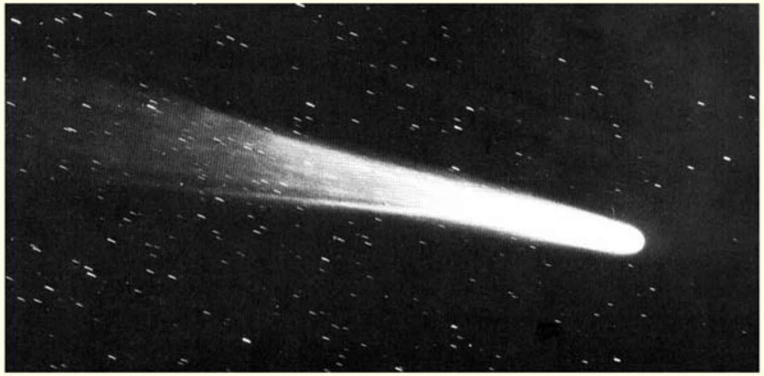
Nature and nature's law
lay hid in night,
God said,
Let Newton be,
and all was light.   A. Pope

Edmund Halley (1656–1742)

Edmund Halley was a contemporary and friend of Isaac Newton.

He was largely responsible for persuading Newton to publish his *Principia Mathematica*.

Halley and his Comet



Halley's analysis of what is now called Halley's comet is an excellent example of the scientific method in action.

A Tricky Question

If the Astronomers can make accurate 76-year forecasts, why can't the Meteorologists do the same?

- Size of the Problem　大氣海洋自由度無限 + 熱力學

  The solar system is discrete, with relatively few degrees of freedom; Dynamics is enough.

  The atmosphere is a continuum with (effectively) infinitely many variables; Thermodynamics is essential.

- Order versus Chaos　大氣海洋的混沌、蝴蝶效應

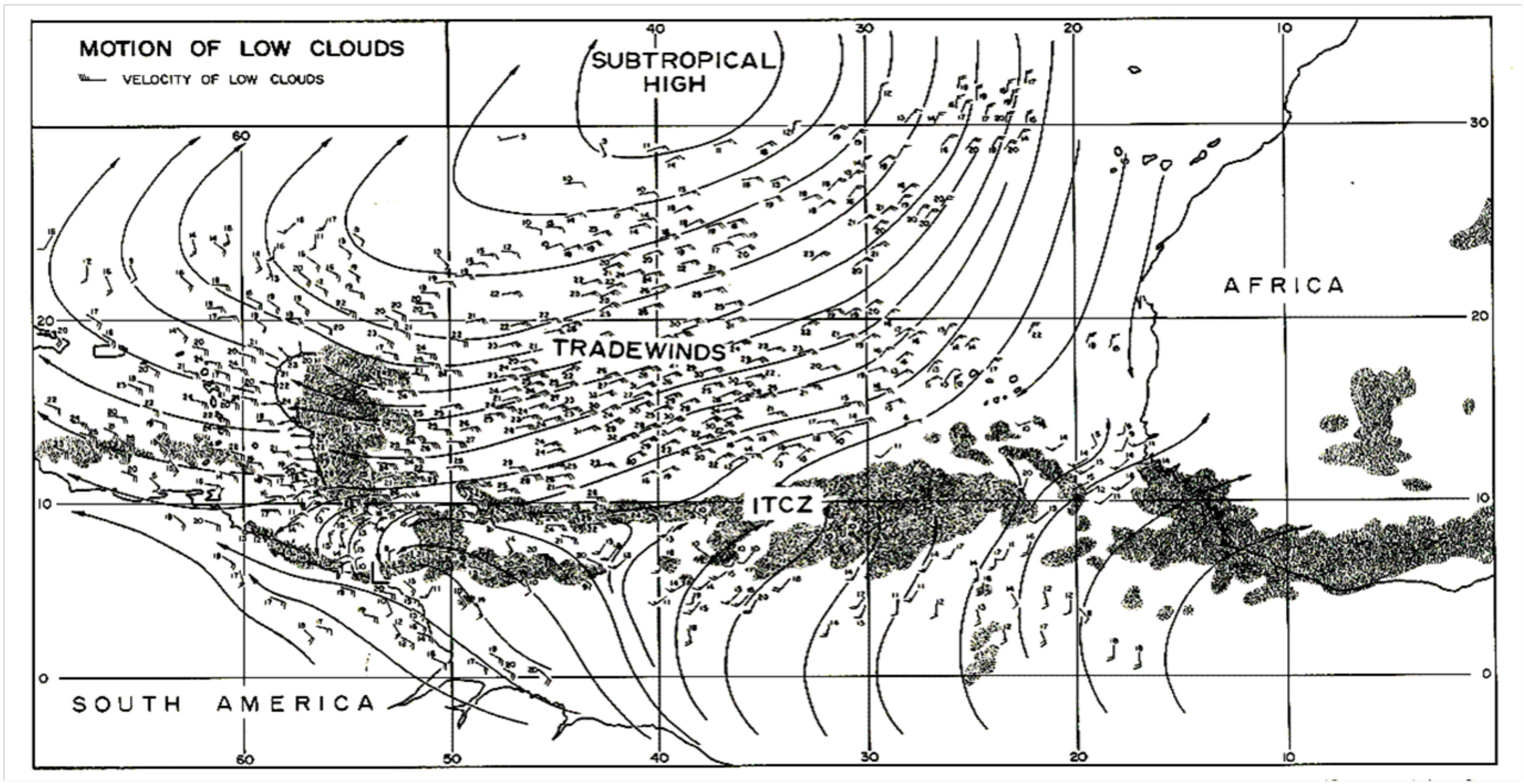
  The equations of the solar system are quasi-integrable and the motion is regular.

  The equations of the atmosphere are essentially nonlinear and the motion is chaotic.

Halley 1686

First proposed the atmospheric motion is connected with the distribution of sun heat (follow the sun in the daily scale; thus wind is westward.)

Fujita, 1971



Euler's Equations for Fluid Flow

流體力學之父

Leonhard Euler, born on 15 April, 1707 in Basel. Died on 18 September, 1783 in St Petersburg.

Euler formulated the equations for incompressible, inviscid fluid flow:

$$\frac{\partial \mathbf{V}}{\partial t} + \boxed{\mathbf{V} \cdot \nabla \mathbf{V}} + \frac{1}{\rho} \nabla p = \mathbf{g} .$$

$$\nabla \cdot \mathbf{V} = 0$$

Partial Differential Equations

偏微分方程式 **PDE**

非線性

Euler 18 century 流體動量、質量守恒

Fluid Dynamics

- Pressure gradient force 壓力梯度力

$$-\frac{1}{\rho}\nabla P$$

- Eulerian-Lagrangian transformation 座標轉換

$$\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$$

- Mass conservation (continuity equation) 質量守恒

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z}=0$$

Isothermal sound speed (same mistake as Newton)

$$\left(\frac{\partial P}{\partial\rho}\right)_T \quad v.s. \quad \left(\frac{\partial P}{\partial\rho}\right)_\theta$$

Euler 1755

$$\frac{d}{dt}\int_{v_m} \rho\vec{v}\,dv = -\int_{\partial v_m} p\,d\vec{s}$$

$$\int_{v_m} \rho\frac{d\vec{v}}{dt}\,dv = -\int_{v_m} \nabla p\,dv$$

$$\rho\,\frac{d\vec{v}}{dt} = -\nabla p$$

Lagrange 1781

$$\frac{\partial\vec{u}}{\partial t} + \vec{\zeta}\times\vec{u} = -\frac{1}{\rho}\nabla p - \nabla K - \nabla\Phi$$

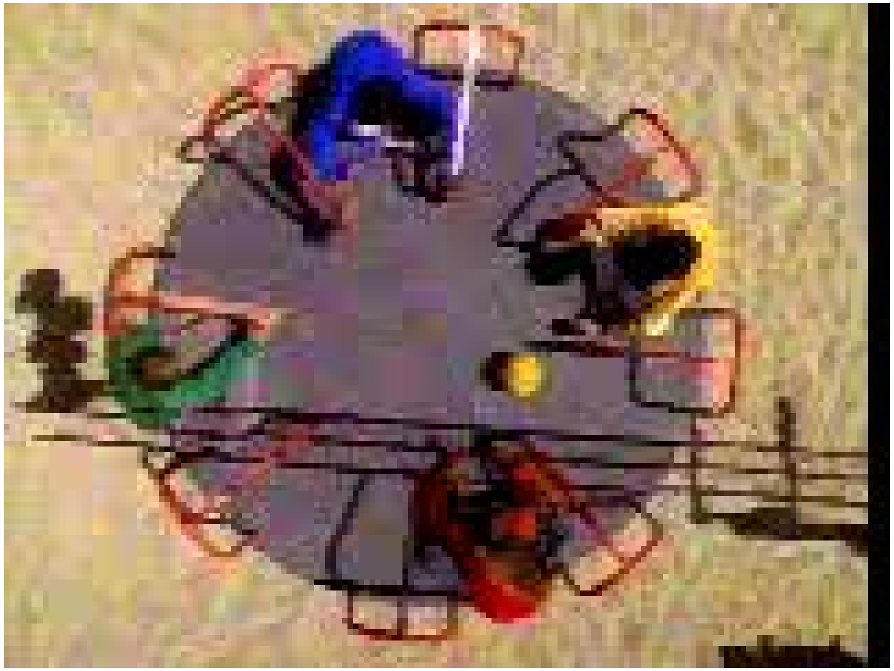
Rotation Vortex

Lorentz Force Law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\mathbf{F} = q(-\nabla V + \mathbf{v}\times\mathbf{B})$$

Coriolis Force = Conservation of Angular Momentum
Centrifugal Force



fv: E-W Coriolis force

Conservation of angular momentum, 角動量守恒

Hadley 1735

旋轉力學

■ fu: N-S Coriolis force

Centrifugal force, thermal wind balance 向心力

Ferrel, 1859

■ Coriolis force, Coriolis, 1835 科氏力

▲ **Falkland ship battle in WW I**

■ Laplace, (1740-1827)

Atmospheric Observational net work (1800-1815)

Hydrostatic balance approximation

Tidal wave equation,

Laplacian

Adiabatic sound speed

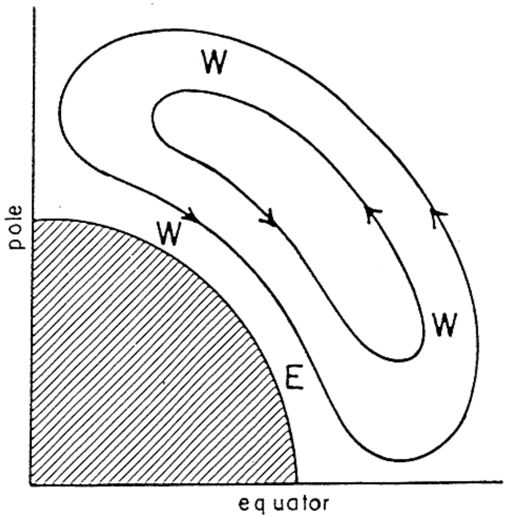


Fig. 2. A schematic view of a meridional cross-section of the general circulation as visualized by Hadley (1735). Streamlines indicate the meridional and vertical flow, while letters E and W indicate regions of easterly and westerly flow.

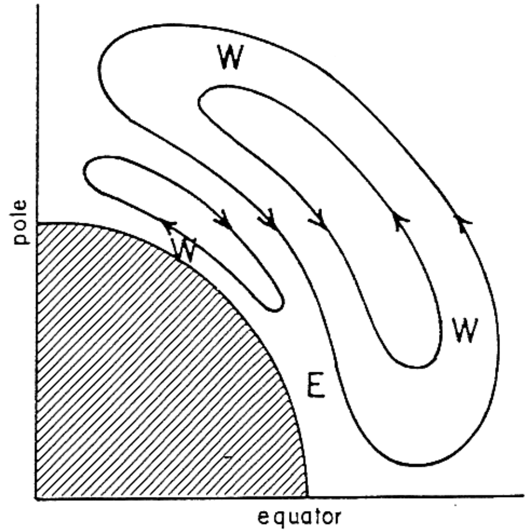


Fig. 3. The same as Fig. 2, but for the general circulation as visualized by Thomson (1857) and Ferrel (1859).

Hadley (1685-1758)

Distribution of sun heating (north and south; seasonal scale)

Earth rotation (conservation of angular momentum)

Thomson (1857)
Ferrel (1859)

Centrifugal force

Coriolis 1835

D'Alembert 1746

Math. Model for Atmospheric Motion in aqua-planet
(Won the 1746 Berlin Academy's Award; Euler's endorsement)

Solar and Lunar Force

Fourier 1768-1830

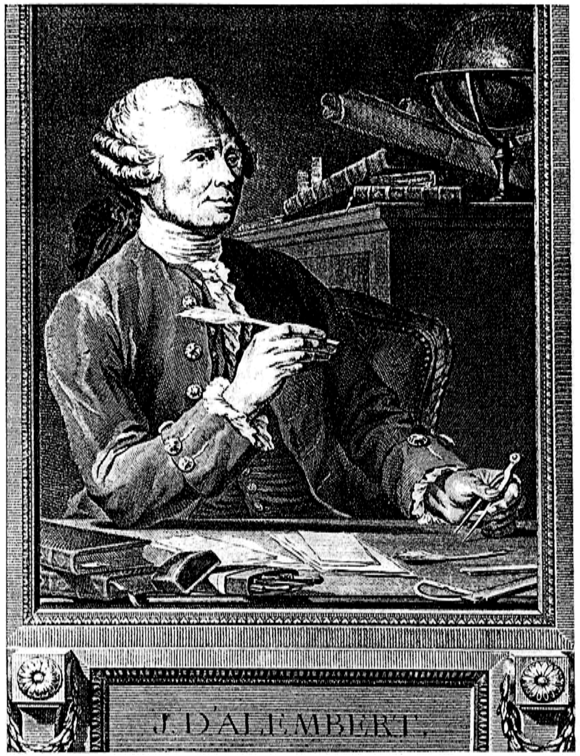
Why the earth not heating up when receive sun energy continuously?

Heat emission or diffusion (by IR)

His calculations showed a very cold surface (No green house effect)

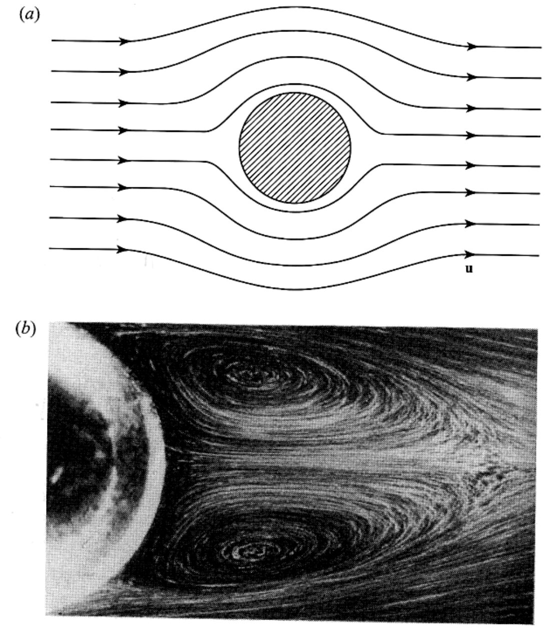
Arrhenius 1896

CO_2 green house effect, but were dismissed by scientists [WHY??]



1717-1783

D'Alembert Paradox



$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathbf{Re} << 1.$$

D'Alembert Solution of the Wave Equation
[f(x + ct) and f(x-ct)]

Re small viscosity important
Re large viscosity unimportant

Atmospheric Motion first expressed mathematically
(Won the 1746 Berlin Academy's Award; aqua-planet
Endorsement of Euler)

Solar and Lunar Force Cause the Atmospheric Motion

Development of Thermodynamics 熱力學

19 century

雲微物理

Precipitation

第一定律　能量作功，能量守恆

First law: Energy is what makes it go and energy is conserved.

$$\triangle Q = \triangle U + WORK$$

Enthalpy

Entropy

Gibbs Free energy

Second law: Entropy tells it where to go!

第二定律　時間之矢，自然單向

Joule, Rudolf Clausius, Lord Kelvin and others

宏觀　微觀

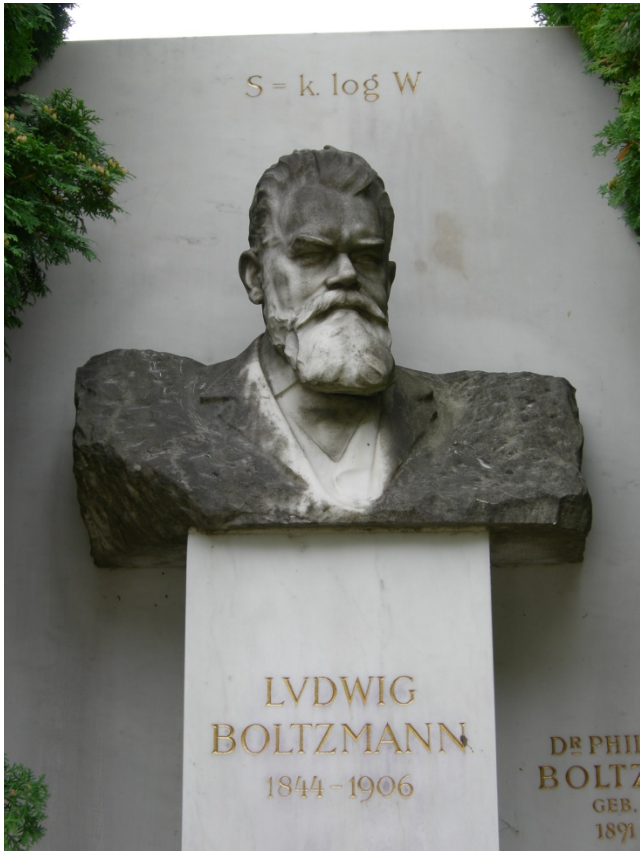
Macro --- Micro

Classical and Statistical Thermodynamics

統計熱力學

Ludwig Boltzmann, 1844-1906, whose work led to an understanding of the macroscopic world on the basis of molecular dynamics.

S = k Log W



Ideal Gas Law **Equation of State 理想氣體方程**

- 1662, Boyle law, PV = c when T = c.
- 1787, Charles law, V/T = c when P = c.
- 1803, Gay-Lussac law, P/T = c when V = c.
- 1811, Avagardro, 1 mole gas is 22.4 l in volume.

Universal Gas Constant

$R^* = 8314.3$ J /(deg$_{-/1}$ kmol)

$PV = n R^* T$

$PV = m/M R^* T$ $P = m/V R^*/M T$

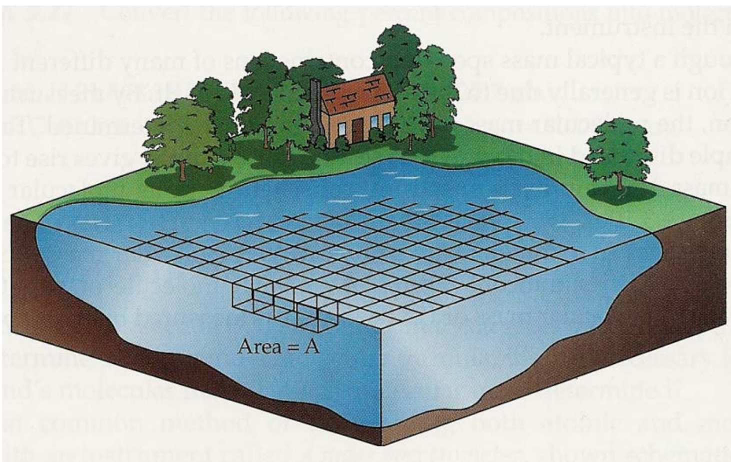
$P = \rho R T$, $R = R^*/M$

$R_d = 287$ J/deg.kg (R^*/M_d)

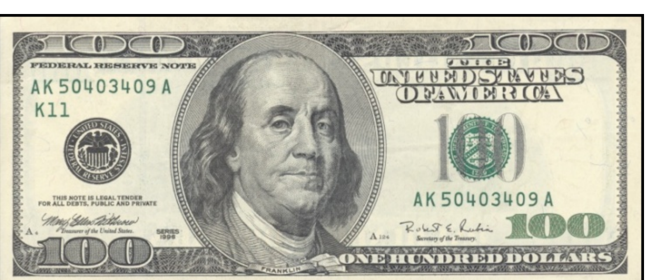
$R_v = 461$ J/deg.kg (R^*/M_v)

Estimate Avogadro's Number

Benjamin Franklin (1773)



Oil spreads on water
→ molecular size
→ Avogadro's number



(1) Molecular size

$$l = \frac{V}{A} = \frac{4.9\ \text{cm}^3}{2.0 \times 10^7\ \text{cm}^2} = 2.4 \times 10^{-7}\ \text{cm}$$

(2) Number of molecules

$$N = \frac{A}{l^2} = \frac{2.0 \times 10^7\ \text{cm}^2}{(2.4 \times 10^{-7}\ \text{cm})^2} = 3.5 \times 10^{20}\ \text{molecules}$$

(3) Mass of the oil

$$m = V \times D = 4.9\ \text{cm}^3 \times 0.95\ \frac{\text{g}}{\text{cm}^3} = 4.7\ \text{g}$$

(4) Number of moles of oil

$$\text{Moles of oil} = \frac{4.7\ \text{g}}{200\ \text{g/mol}} = 0.024\ \text{mol}$$

(5) Avogadro's number

$$\text{Avogadro's number} = \frac{3.5 \times 10^{20}\ \text{molecules}}{0.024\ \text{mol}} = \boxed{1.5 \times 10^{22}}$$

Now we know: $N_A = 6.022142 \times 10^{23}\ /\ mol$

Planck, Unwilling Revolutionary: the idea of quantization

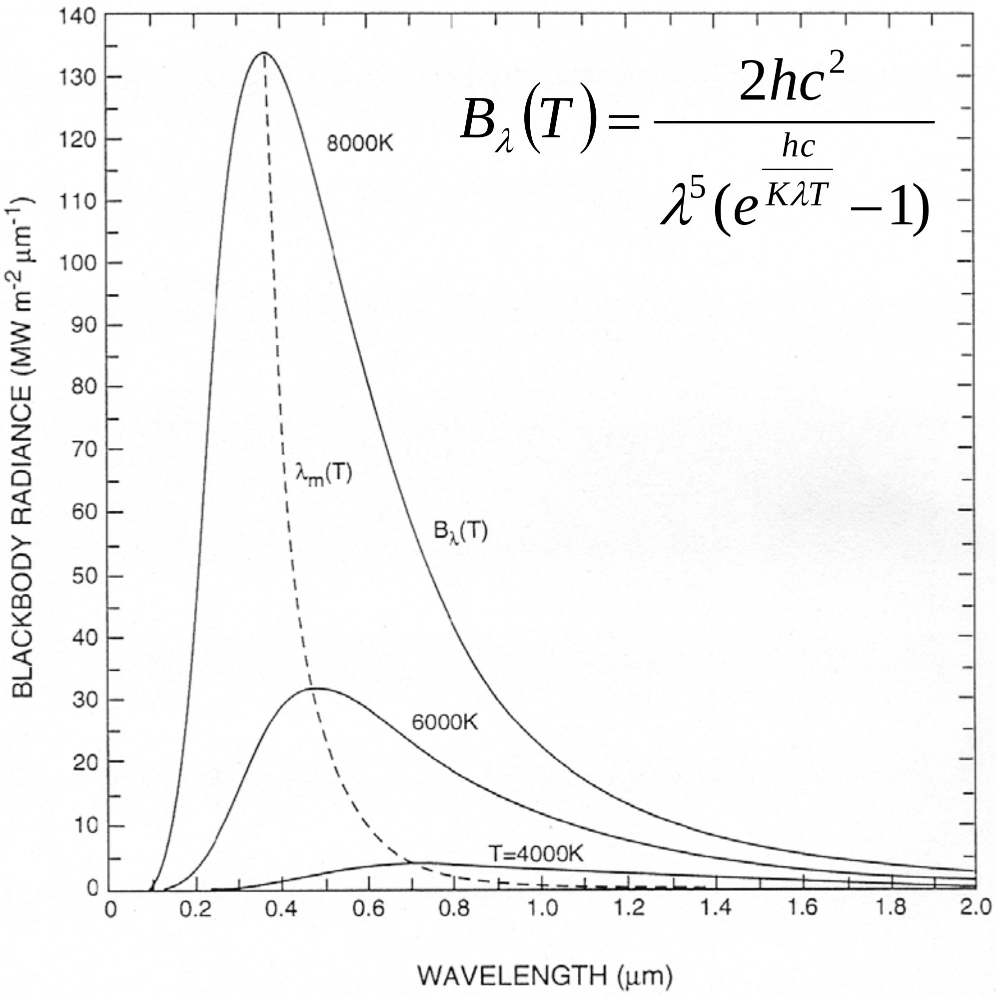


Figure 8.7 Spectra of emitted intensity $B_\lambda(T)$ for blackbodies at several temperatures, with wavelength of maximum emission $\lambda_m(T)$ indicated.

1900

Hall of Fame in Science

Gravitational Law

Blackbody Radiation

E= MC^2

Vilhelm Bjerknes (1862–1951)

科氏力(18 19)

Momentum Conservation (18)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \nu\nabla^2 w$$

Mass conservation (18)

$$\frac{\partial \rho}{\partial t} + \frac{\partial u\rho}{\partial x} + \frac{\partial v\rho}{\partial y} + \frac{\partial w\rho}{\partial z} = 0$$

Energy conservation (19)

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = Q$$

Radiation 大氣輻射 (19,20)

Moisture Latent heat

Equation of State(17,18,19)

$$p = \rho R_a T, \quad \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_a}{C_P}}$$

(19)

雲物理 (19,20)

問蒼茫大氣，誰主浮沈？

質量、動量、能量與大氣狀態方程式

The Ultimate Problem in Meteorology   Bjerknes 1911
氣象的終極問題

I The Present state of the atmosphere must be characterized as accurately as possible. **正確的觀測大氣現狀 [多重時空尺度]**

II The intrinsic laws, according to which the subsequent states develop out of the preceding ones, must be known.
正確的大氣運作規律

Numerical Weather Prediction 數值天氣預報
[第一部電腦ENIAC, EBV model, 1950]
The Observation component 觀測
The diagnostic or analysis component 診斷分析
The prognostic component 預報

Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed by hand the pressure change at a single point.

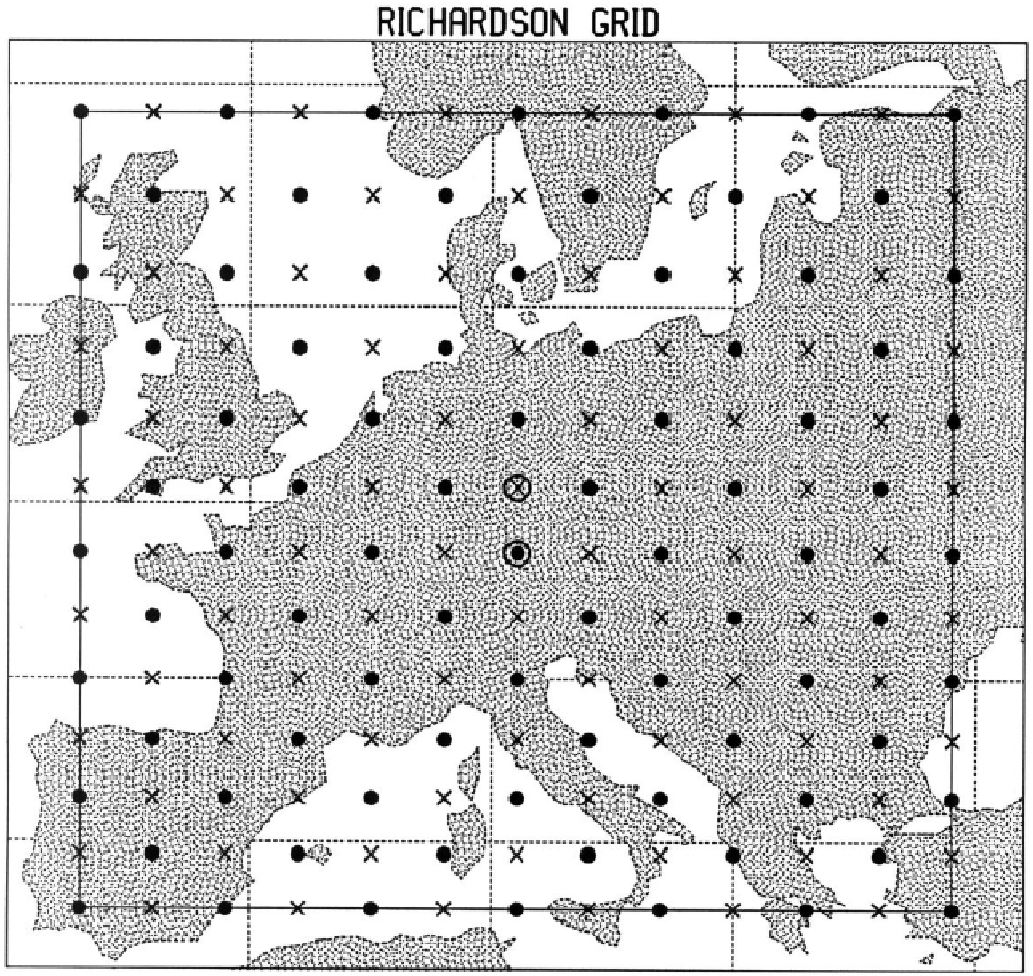
It took him two years !

His ‘forecast’ was a catastrophic failure:

$$\Delta p = 145 \text{ hPa in 6 hours}$$

His method was unimpeachable.

So, *what went wrong?*



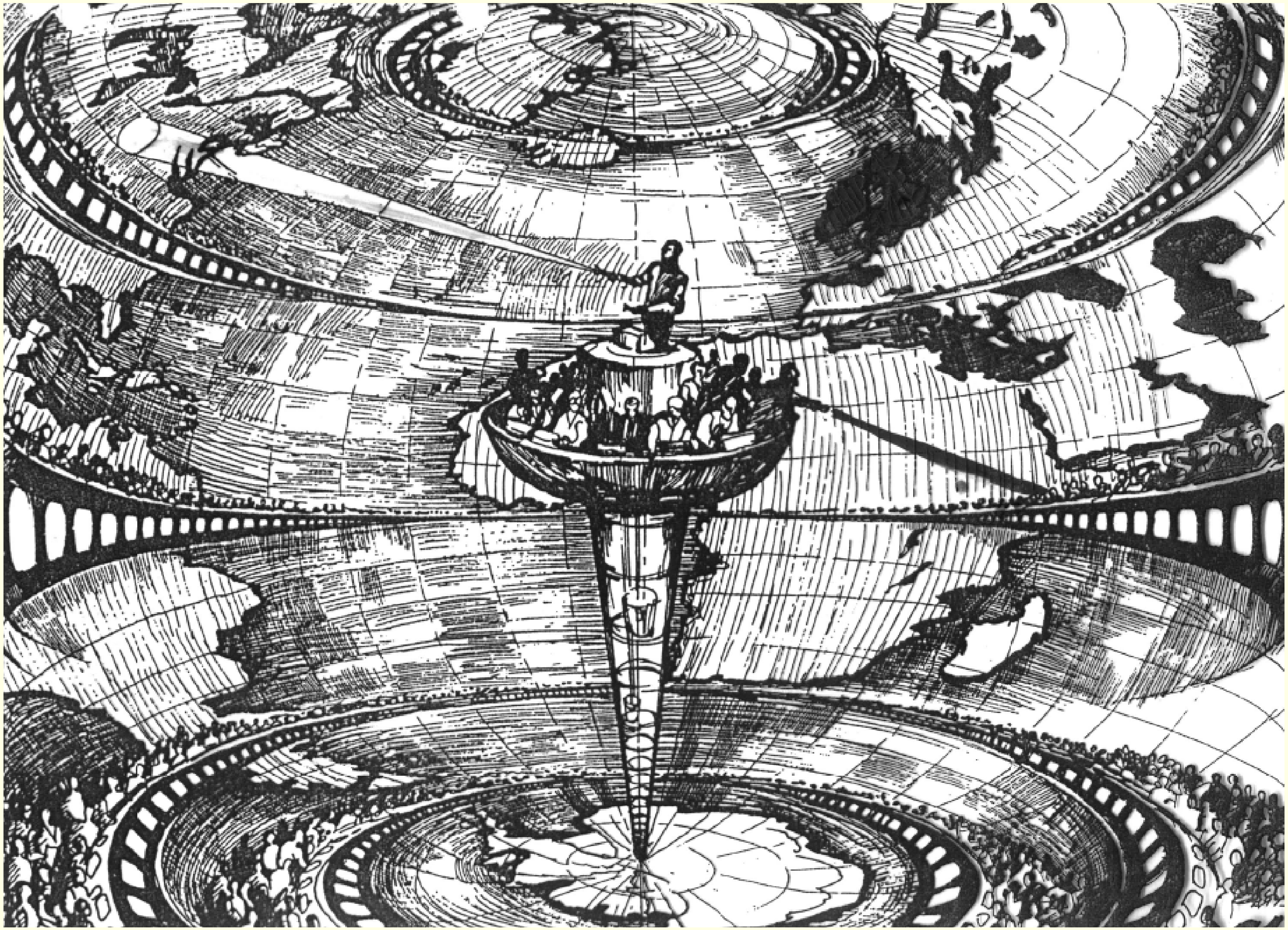
$$\frac{df}{dx} \rightarrow \frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x}$$

$$\frac{dQ}{dt} \rightarrow \frac{Q^{n+1}-Q^{n-1}}{2\Delta t} = F^n$$

13×13=169個ODE

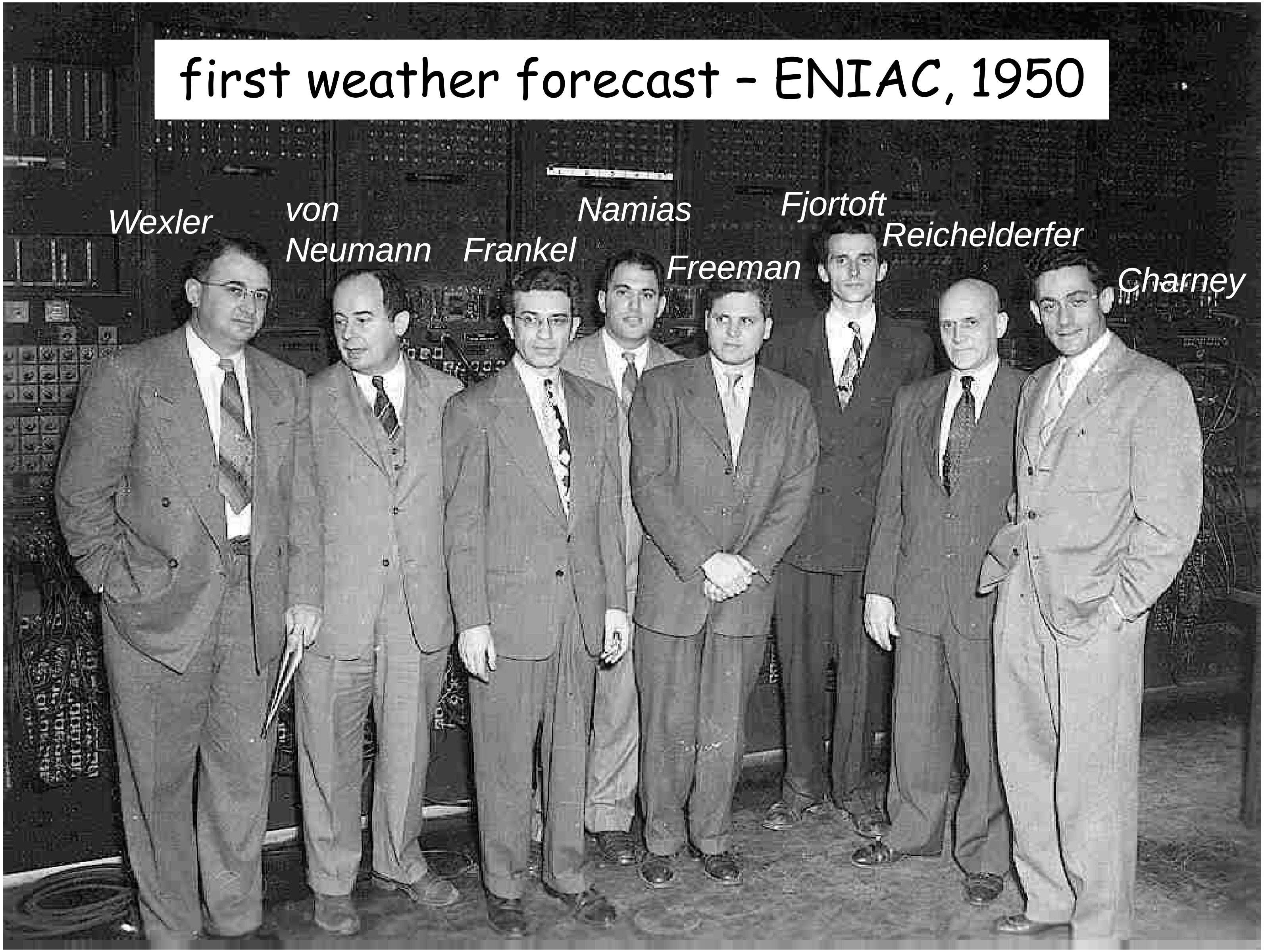
169 自由度

Richardson’s Dream



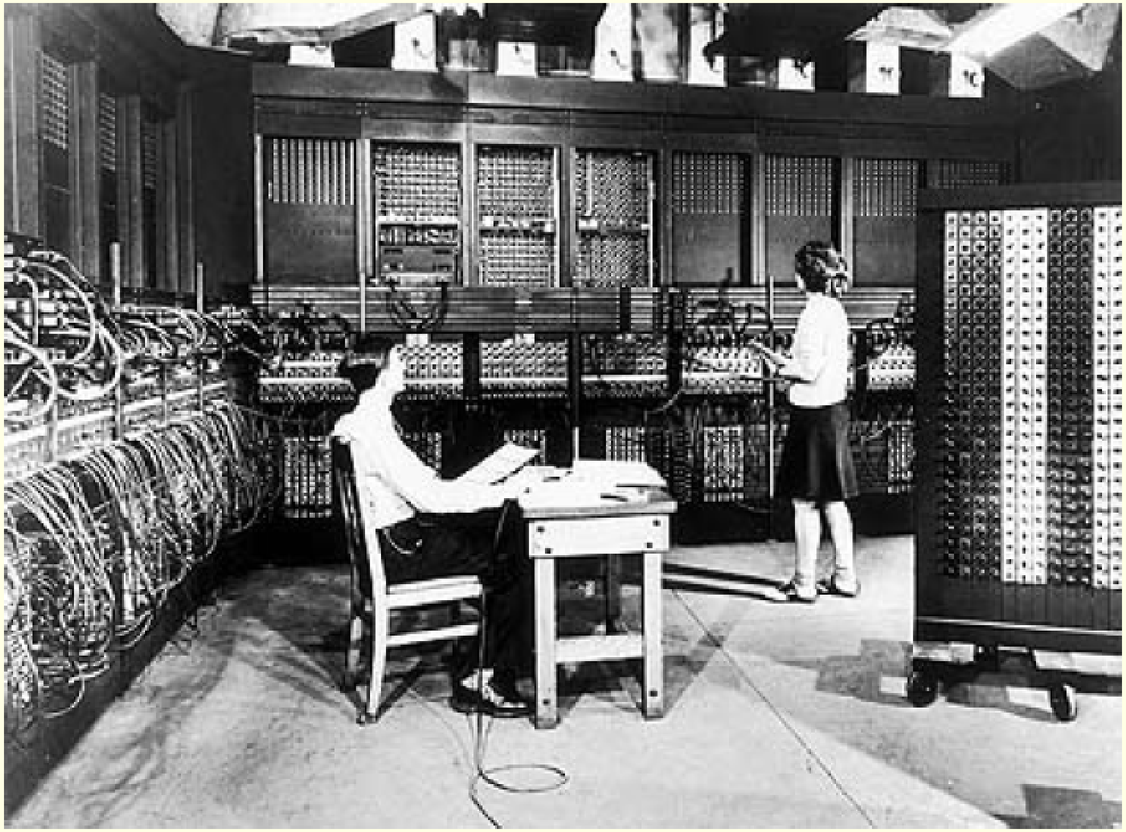
Richardson’s Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor



In front of the Eniac, Aberdeen Proving Ground, April 4, 1950, on the occasion of the first numerical weather computations carried out with the aid of a high-speed computer.

The ENIAC Electronic Numerical Integrator and Computer



18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

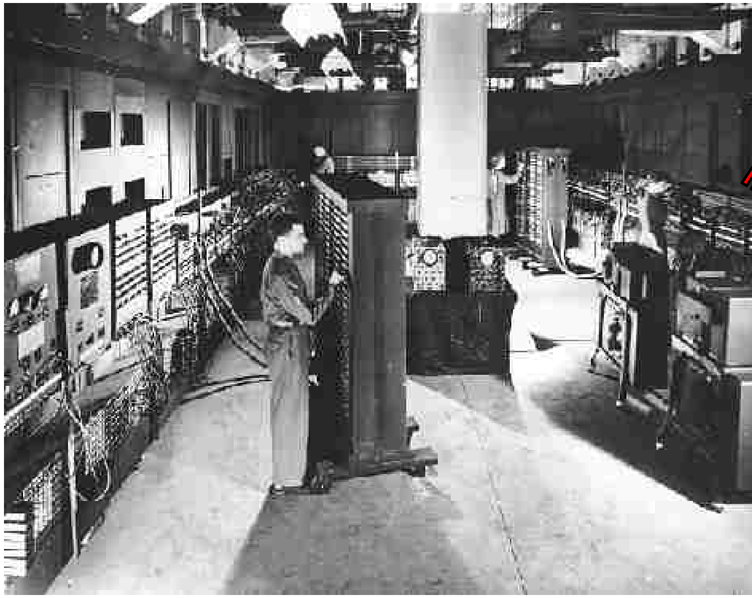
No high-level language
Assembly language

500 Flops
Function Table 0.001 s

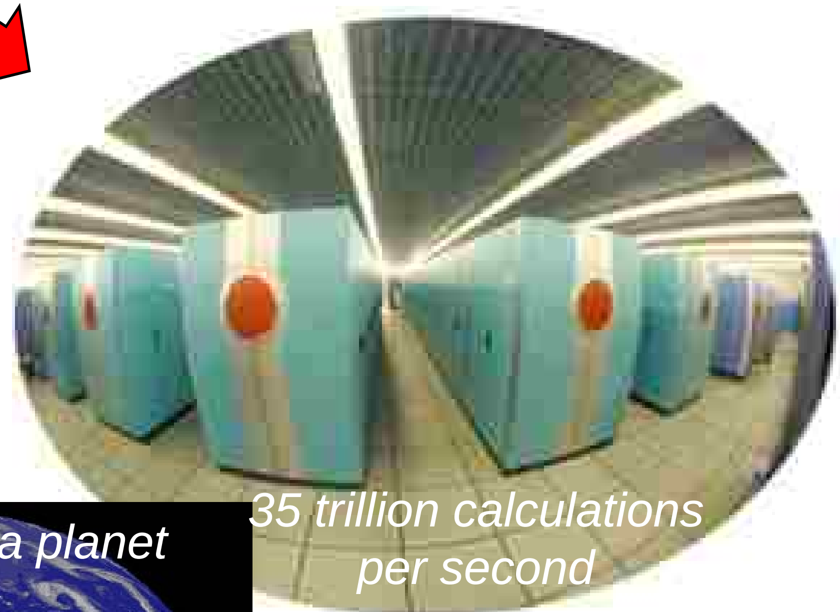
3,700,000,000 times slower than current day large computer

第一部電腦　　氣象預報

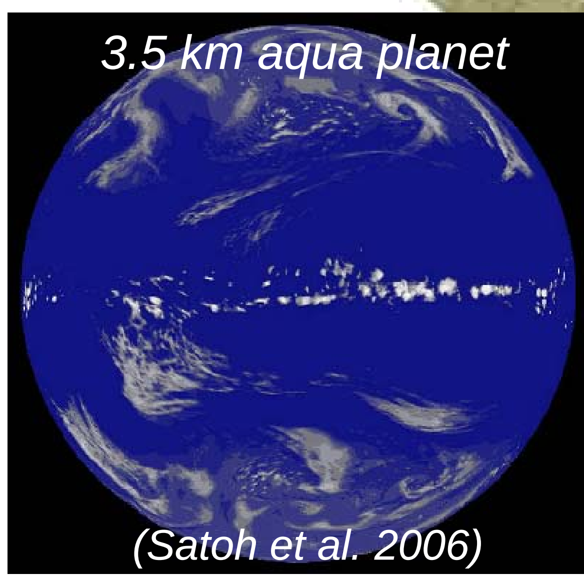
ENIAC – late 40s

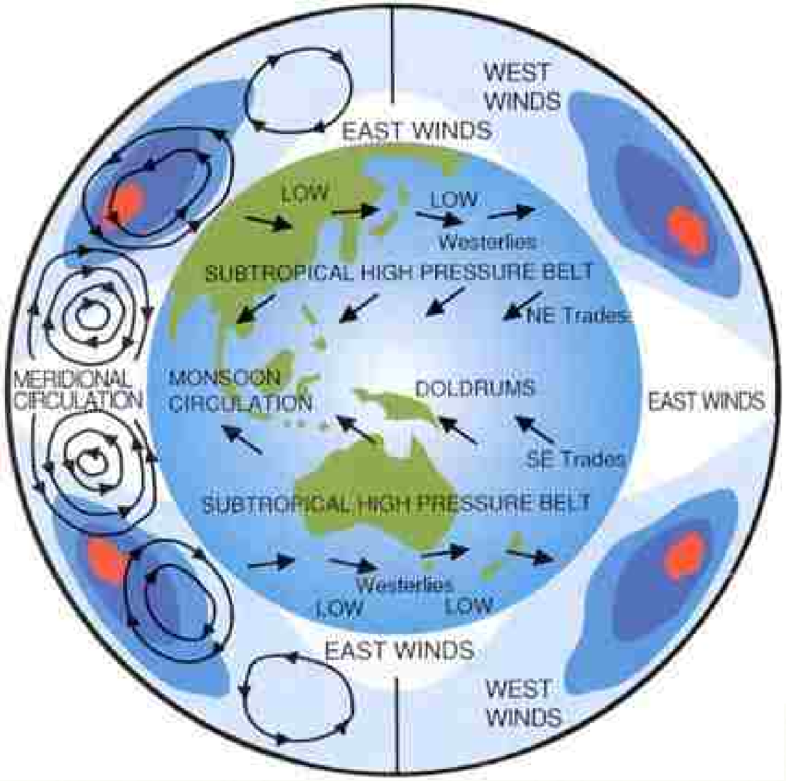


Earth Simulator -- 2002



NASDA, JAERI, JAMSTEC





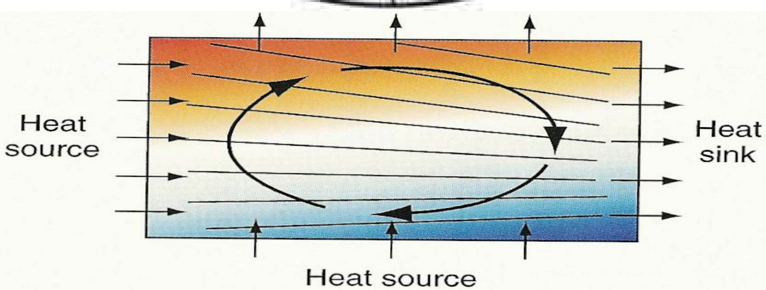


Fig. 7.23 Vertical cross section through a steady-state circulation in the laboratory, driven by the distribution of heat sources and heat sinks as indicated. The colored shading indicates the distribution of temperature and density, with cooler, denser fluid represented by blue. The sloping black lines represent pressure surfaces. Note that the flow is directed down the horizontal pressure gradient at both upper and lower levels.

Heat Driven Atmosphere

3 Cells symmetric dynamics

Solar radiation is symmetric and yet asymmetric phenomena exist

20th century: Wave Mean flow interaction

- Solar + IR + Green house effect
- Earth Rotation
- Deep convection
- Instabilities
- Land-sea contrast
- Seasonal migration of radiation
- Ocean
- Biosphere
- Hydrological cycle

Earth Simulator (Japan)

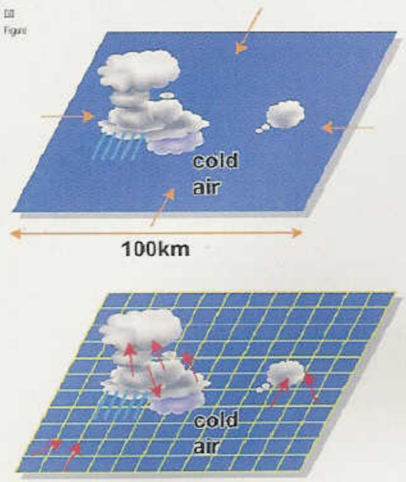
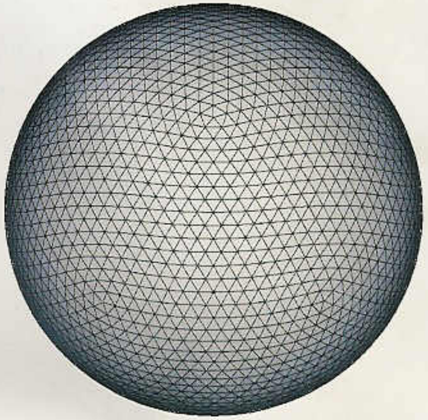


Fig1 : (upper) The cumulus are not resolved directly due to coarse mesh. (lower) Those can be resolved by the fine mesh



~10^8 Grids

Fig2 : Icosahedral grid. The mesh is distributed homogeneously.

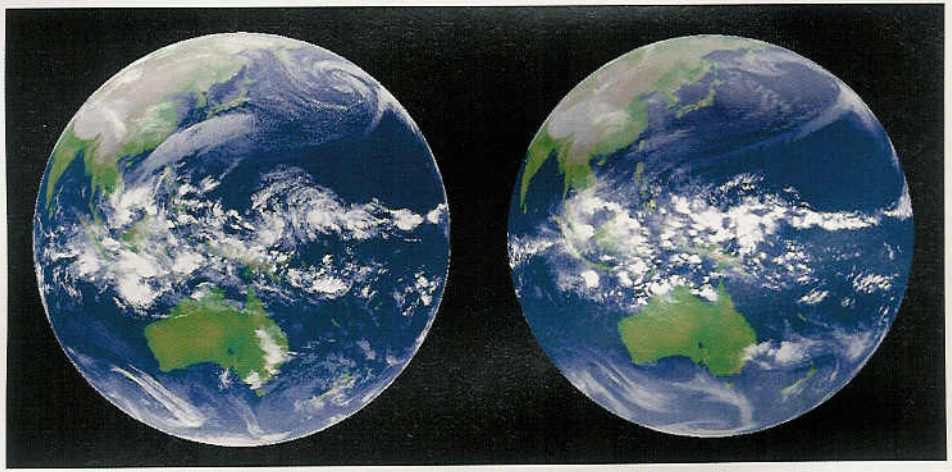


Fig.3 : (Left) Image of MTSAT－1R (Right) the results from NICAM 3.5km mesh.

Satellite observation Numerical simulated

20th Century

Geophysical Fluid Dynamics (GFD)

Atmospheric Oceanic Fluid Dynamics (AOFD)

is for those interested in doing research in the physics, chemistry, and/or biology of Earth fluid environment.

(b)

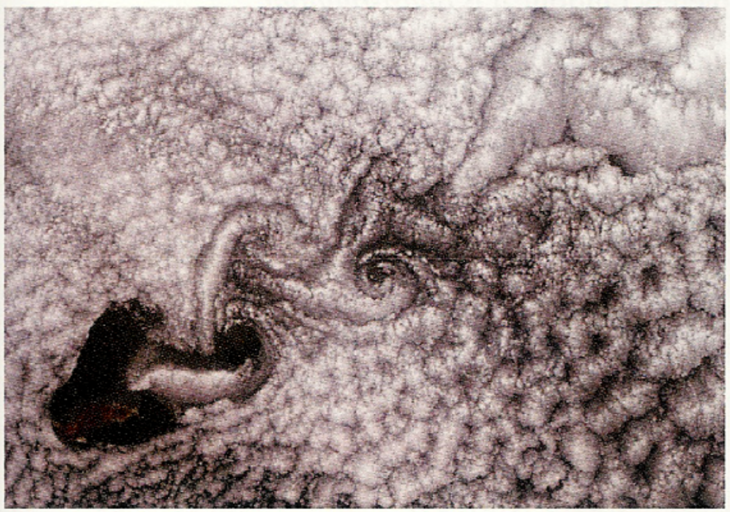
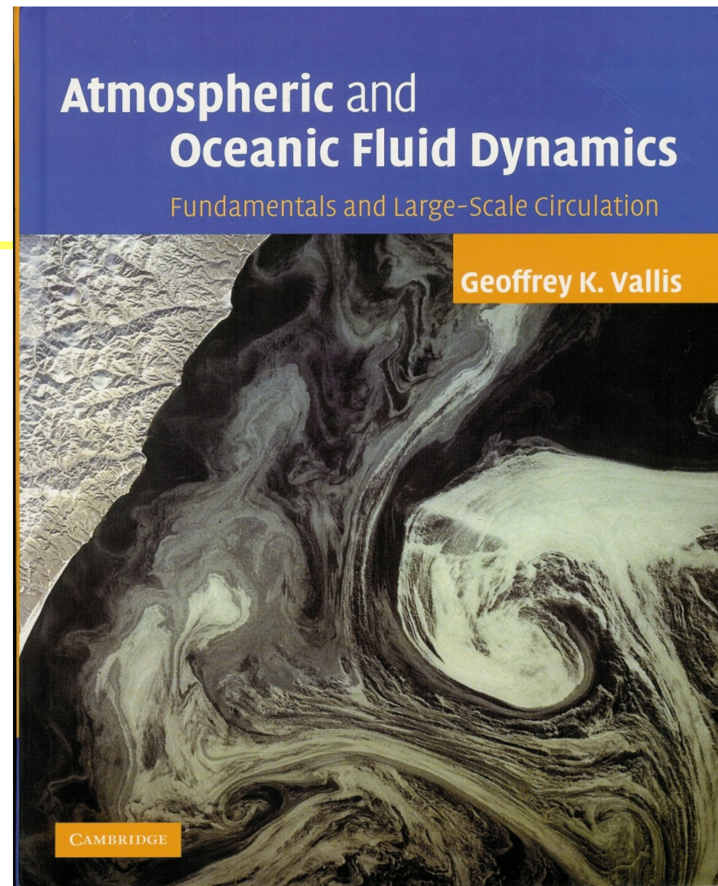


Fig. 9.2 Karman vortex streets in (a) the laboratory, for water flowing past a cylinder [From M. Van Dyke, *An Album of Fluid Motion*, Parabolic Press, Stanford, Calif. (1982) p. 56.], and (b) in the atmosphere, for a cumulus-topped boundary layer flowing past an island [NASA MODIS imagery].



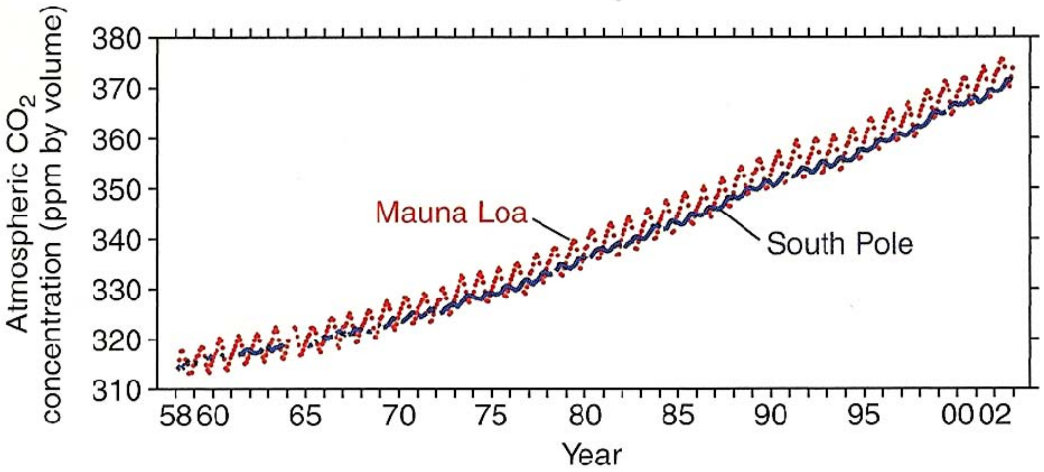
凝結尾　人為

Fig. 10.43 Condensation trails. [Photograph courtesy of Art Rangno.]

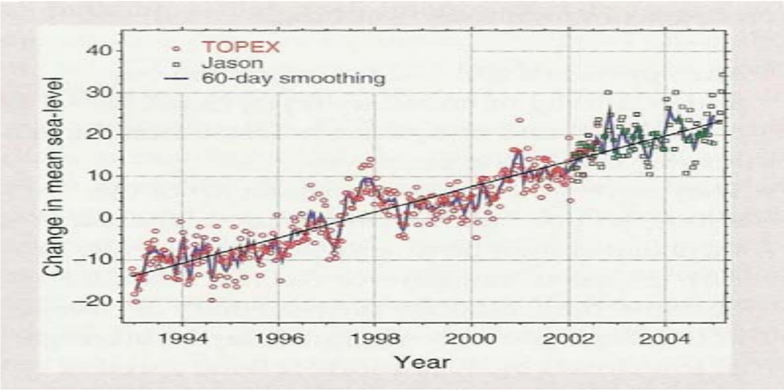
All commercial aircraft flights in U.S. were stopped for
3 days after the 911 attack.

Ground diurnal Temperature
is larger by 1.1K in these 3 days
as compared to the climate
mean.

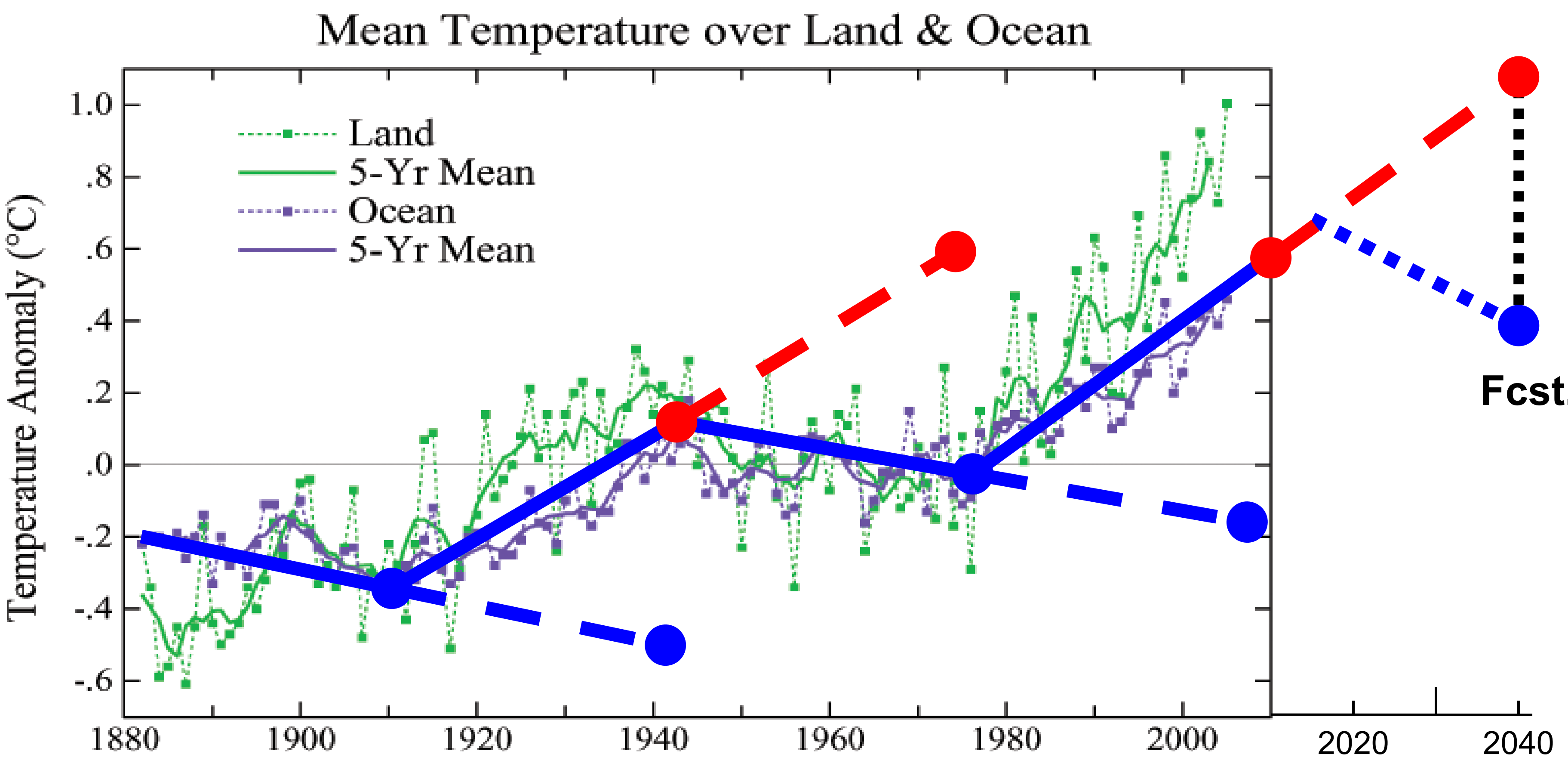
Keeling Curve (1958-)



Charles Keeling (left) at the dedication of the Keeling Building at the Mauna Loa Observatory, Hawaii (1997)



3 mm/year mm


Mean Temperature over Land & Ocean
Land
5-Yr Mean
Ocean
5-Yr Mean
Temperature Anomaly (°C)
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
1880
1900
1920
1940
1960
1980
2000
2020
2040
Fcst.

SEPTEMBER 10

1945

JANUARY 31

1977

Science Digest

February, 1973

Reports that the world's
climatologists are agreed
that "we must prepare for
the next ice age."

Time magazine's June 24, 1974, story showed how Arctic snow and ice had grown from 1968 to 1974.

Newsweek

April 28, 1975

In an article titled “The Cooling World” said that meteorologists are almost **unanimous** that catastrophic famines might result from global cooling.

APRIL 3


APRIL 3, 2006
www.time.com AOL Keyword: TIME
SPECIAL REPORT GLOBAL WARMING
TIME
BE WORRIED. BE VERY WORRIED.
Climate change isn't some vague future problem—it's already damaging the planet at an alarming pace. Here's how it affects you, your kids and their kids as well
EARTH AT THE TIPPING POINT
HOW IT THREATENS YOUR HEALTH
HOW CHINA & INDIA CAN HELP SAVE THE WORLD—OR DESTROY IT
THE CLIMATE CRUSADERS

2006

2005—2055 科技探索 Institute For The Future

生物模擬與計算

數學模型與生物結合

健康醫療規劃

生物資訊分析(Bioinformatics)

透過人腦、電腦介面，強化人體功能

生物檢測

複雜系統：生態、經濟、氣象等數學模式應用議題

....

數量化、數位化
數學化--模式--動力系統

"Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum."　　Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。通典

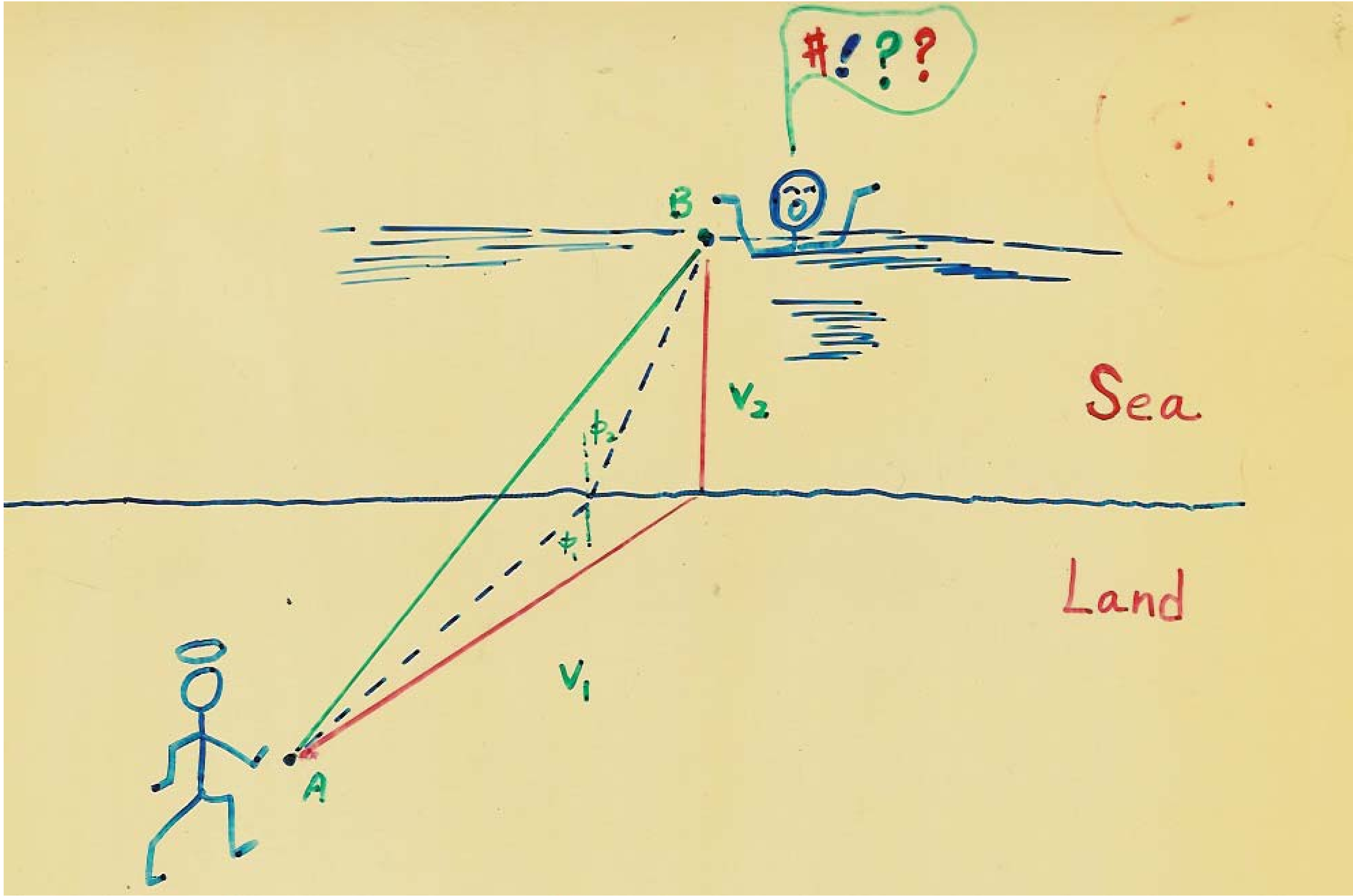
共**49**個字，假設中文常用字爲**1000**字，共有**10^147**個選擇

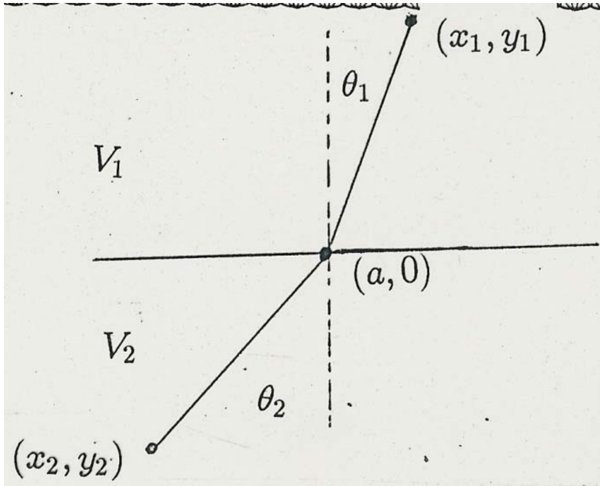
地球歷史 **10^18** sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 **10^4**，

10^10*10^18*10^4 = 10^32

10^32/10^147=10^(-115) ~ 0　機率爲零　，不可能的巧合！

研究學問是苦心孤詣的事業！　　不要人云亦云！


#!??
B
V_2
ϕ_2
ϕ_1
Sea
Land
V_1
A



$$T(a) = \frac{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}{V_1} + \frac{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}{V_2}$$

$$\frac{\partial T}{\partial a} = 0$$

$$\frac{-(x_1-a)}{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}\frac{1}{V_1} + \frac{(a-x_2)}{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}\frac{1}{V_2} = 0$$

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2}}$$

This is the **Snell's Law.**

Hamilton's principle

Fermat's principle

Minimization

一樣觀魚多樣情！

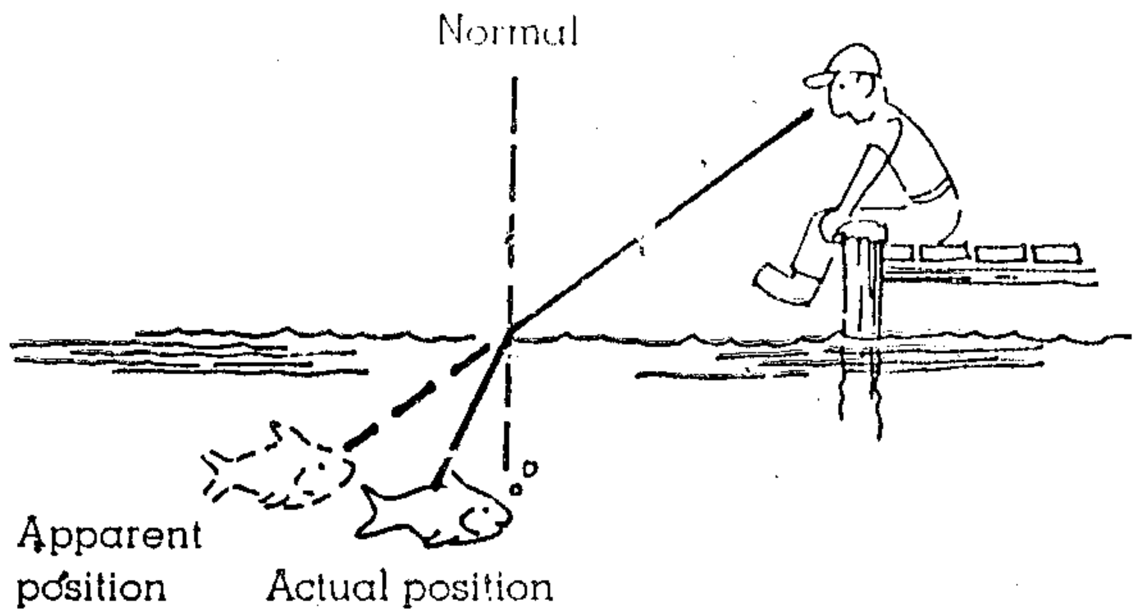
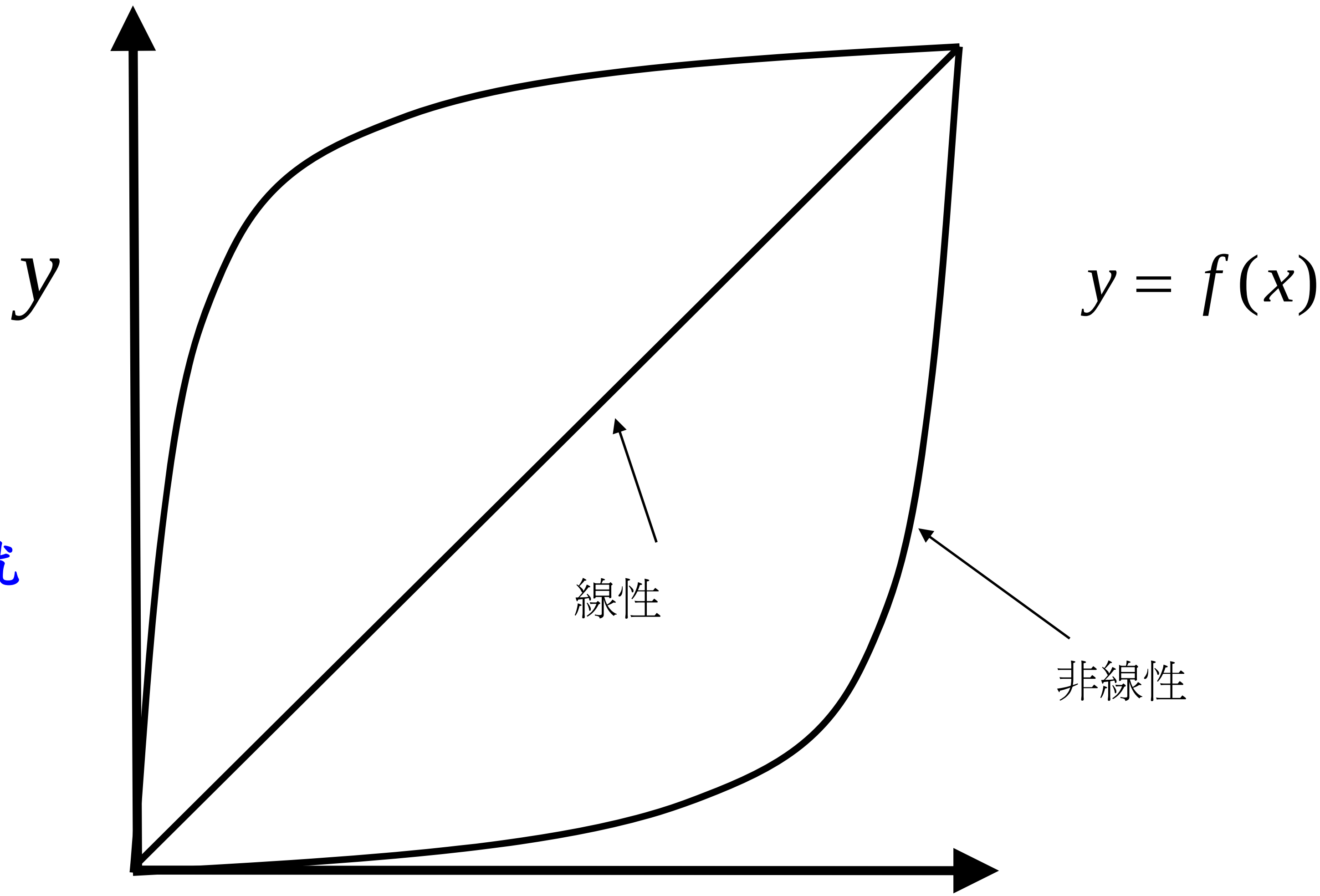


FIGURE 5.13 The refraction of light as it passes from the water into the less-dense air causes a fish to appear closer to the surface than it actually is.

（1）魚快樂嗎？
（2）熱血沸騰，立志革命！
（3）折射定律，最小原理。

一分耕耘 ， 一分收穫？？



y：成就

$$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ?$$

x：努力

Buckingham's Pi Theorem

n variables can always be combined to form
Exactly (n- r) independent nondimensional variables, where
R is the rank of the dimensional matrix.

Eg. T, M, L, g are the 4 variable in the simple pendulum problem

kg, m, s are the rank 3 dimensional unit

4-3 =1 nondimensional variable $T \sim \sqrt{\frac{l}{g}}$

Similar result can be derived from Newton's mechanics or
from the fact that the difference of potential energy and kinetic energy over
a period will be minimized; the Hamilton principle.

$$\Pi = \frac{r_f}{E^{1/5} t^{2/5} \rho_0^{-1/5}}$$

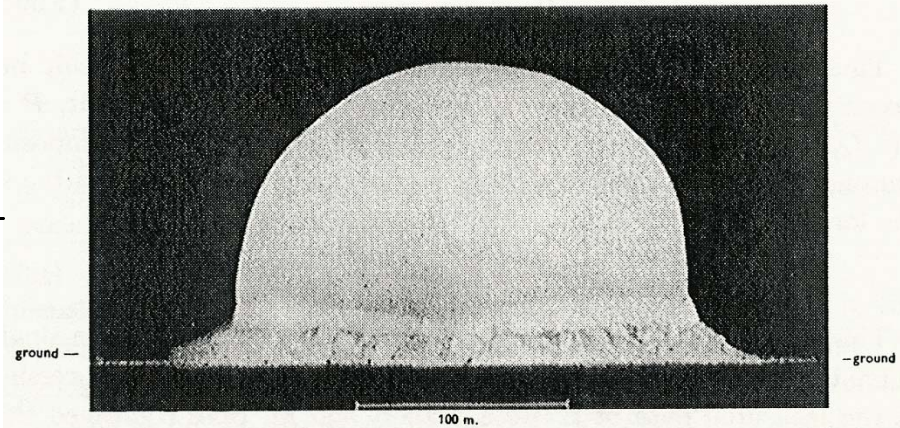


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963).

G.I. Taylor 1950

Symbol	*Definition*	*Representative value or first guess*
R	radius of wavefront	10^2 m
t	time	10^{-2} s
p_0	ambient pressure	10^5 Pa
ρ_0	ambient density	1 kg m^{-3}
E	energy released	10^{14} J

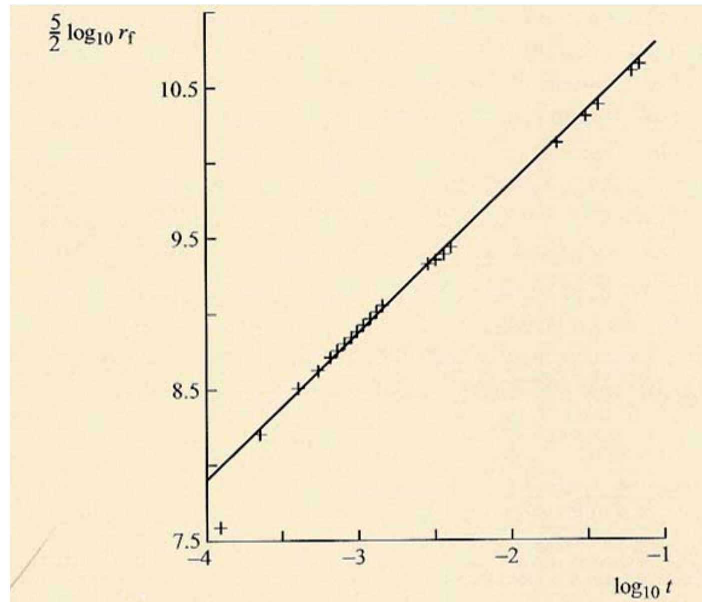
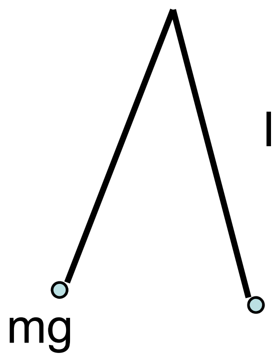


Figure 0.3. Logarithmic plot of the fireball radius, showing that $r_f^{5/2}$ is proportional to the time t (Taylor 1950b, 1963).

原子彈能量～ 10^{14} J



$$T \sim \sqrt{\frac{l}{g}}$$

$$V \sim \frac{l}{T} \sim \sqrt{l}$$

Walking speed

$$T \sim \sqrt{\frac{I}{mgr}} \sim \sqrt{\frac{ml^2}{mgl}} \sim \sqrt{l}$$

$$T \sim \sqrt{\frac{I}{L_{max}}}$$

$$T \sim \sqrt{\frac{l^3 l^2}{l^2 l}} \sim l$$

Runner speed is independent of leg length

$I = l^2 m/3$ the moment of inertia

Force is proportional to muscle area
Mass m is proportional to volume

L_{max} is the torque (force * length)

$$L_{max} \sim F_m l \sim l^2 l,$$

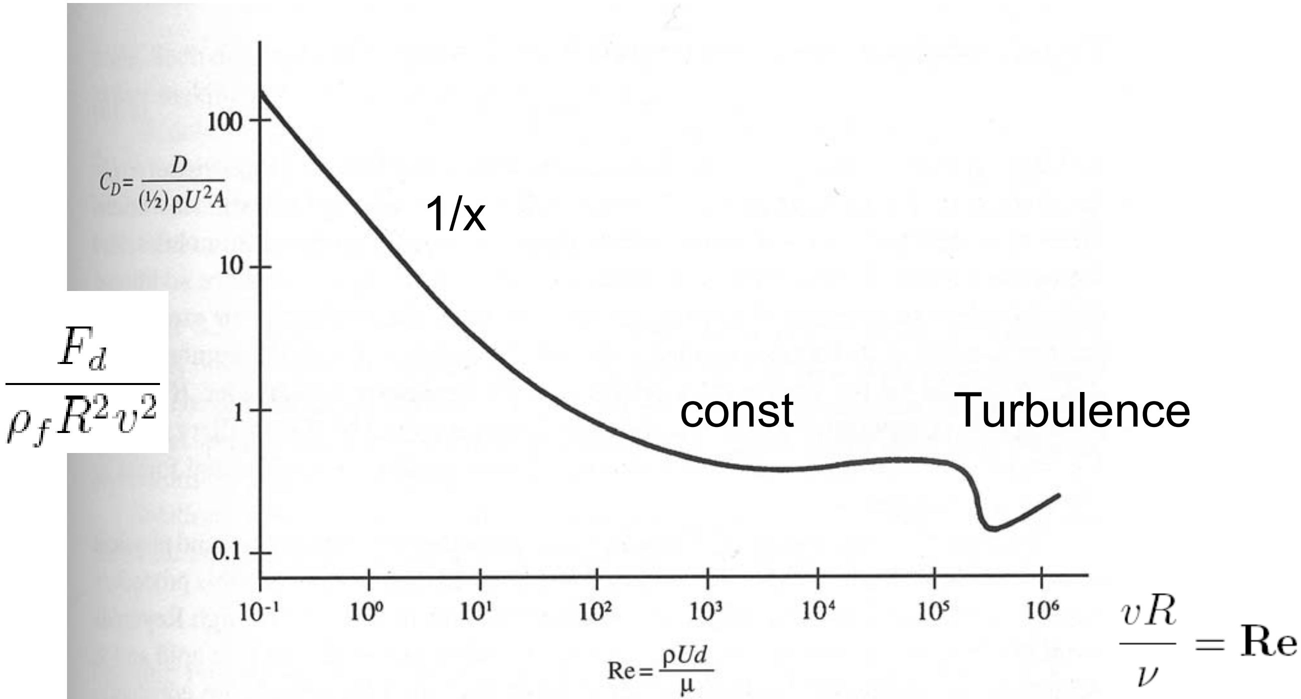
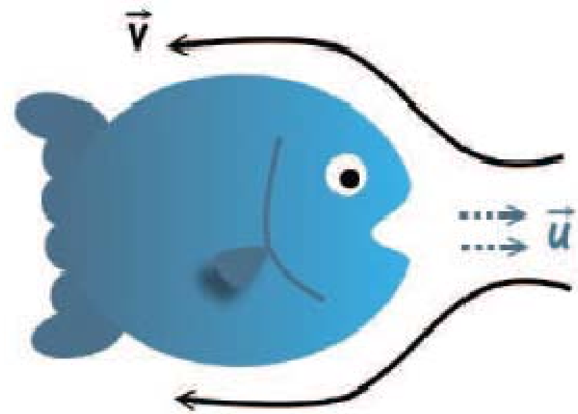


Figure 8.2 Drag coefficient for a sphere. The characteristic area is taken as $A = \pi d^2/4$. The reason for the sudden drop of C_D at Re $\sim 5 \times 10^5$ is the transition of the laminar boundary layer to a turbulent one, as explained in Chapter 10.

F_d $[M][L][T]^{-2}$ drag force;
ρ_f $[M][L]^{-3}$ density of fluid
R $[L]$ radius of sphere
v $[L][T]^{-1}$ speed
ν $[L]^2[T]^{-1}$ kinematic viscosity.

$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathbf{Re} << 1.$$



Power = Force x velocity

Force = Density x velocity^2 x Area

據觀察，魚游向前的速度約爲背景速度的1.5倍，爲什麼？

$$cu^3 \frac{l}{u-v} = E \text{ (power } \times \text{ time = energy)}$$

$$\frac{dE}{dt} = 0 \text{ 魚最省力}$$

$$\Rightarrow \frac{3u^2}{u-v} + \frac{u^3}{(u-v)^2} = 0 \Rightarrow u = \frac{3}{2}v$$

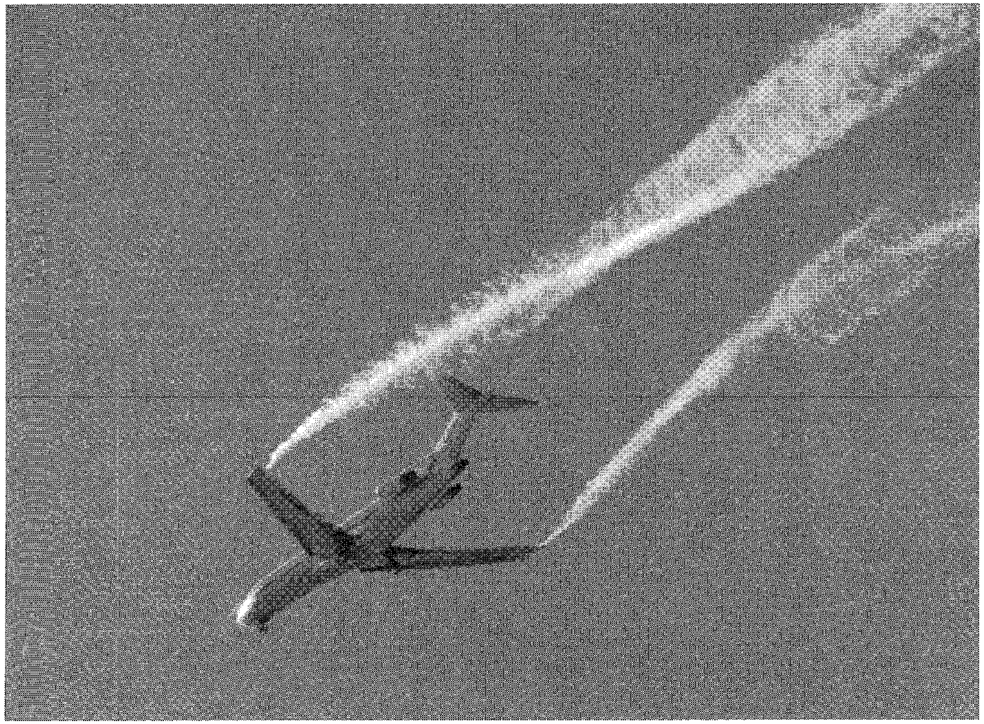
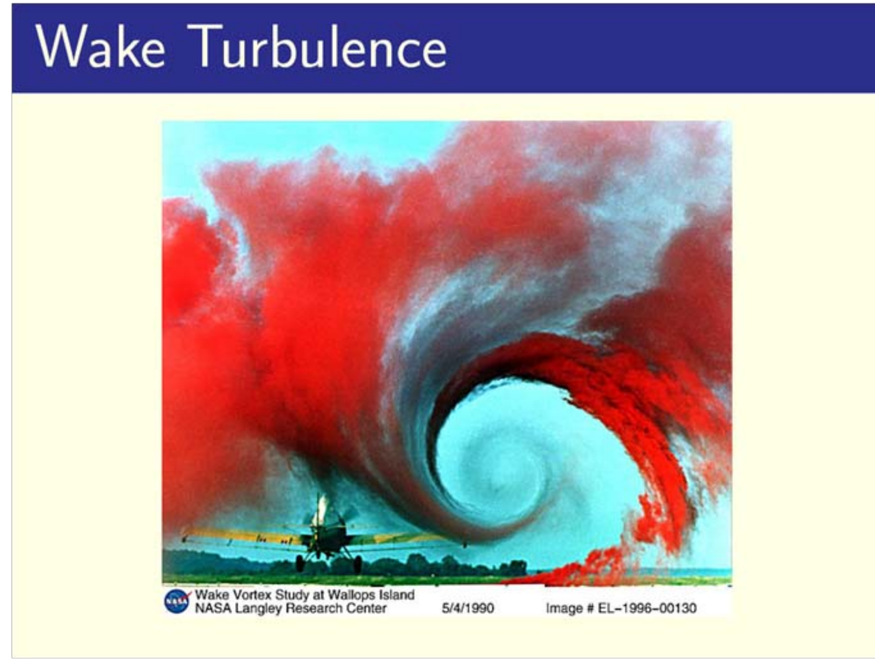


Fig. 8.9. Vortices trailing form the wingtips of a Boeing 727. Figure courtesy of NASA.



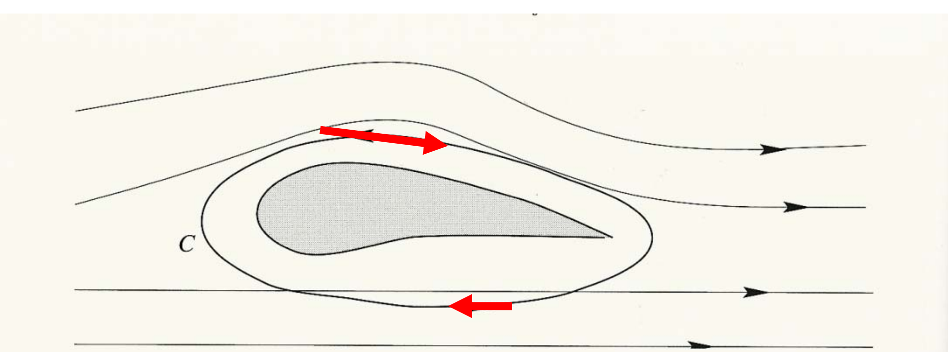


Fig. 8.10. Sketch of the flow along an airfoil. The wing is shown in grey, the contour C is shown by the thick solid line.

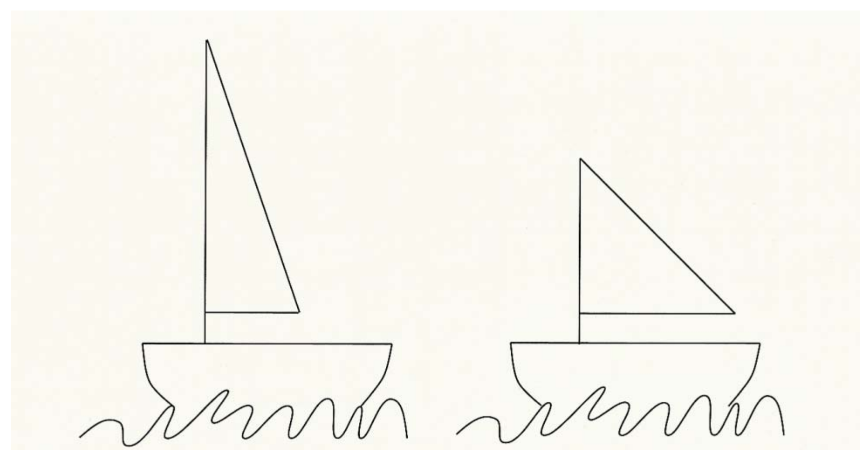
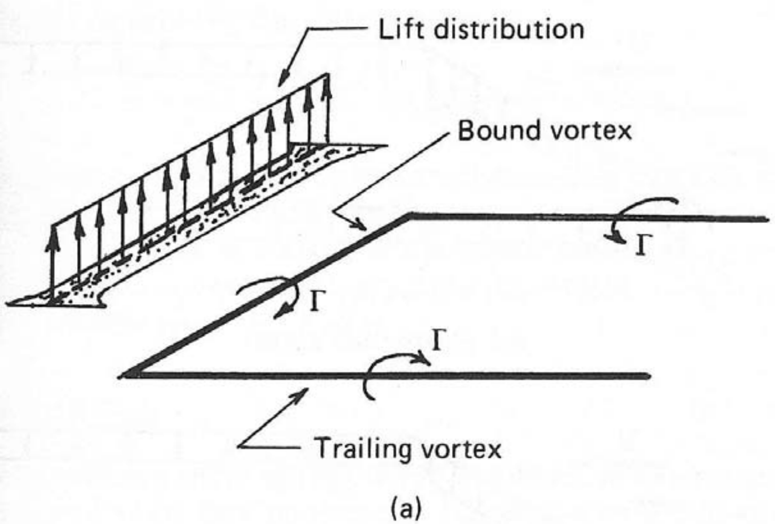


Fig. 8.12. Two boats carrying sails with very different aspect ratios.

Biomath



Lanchester

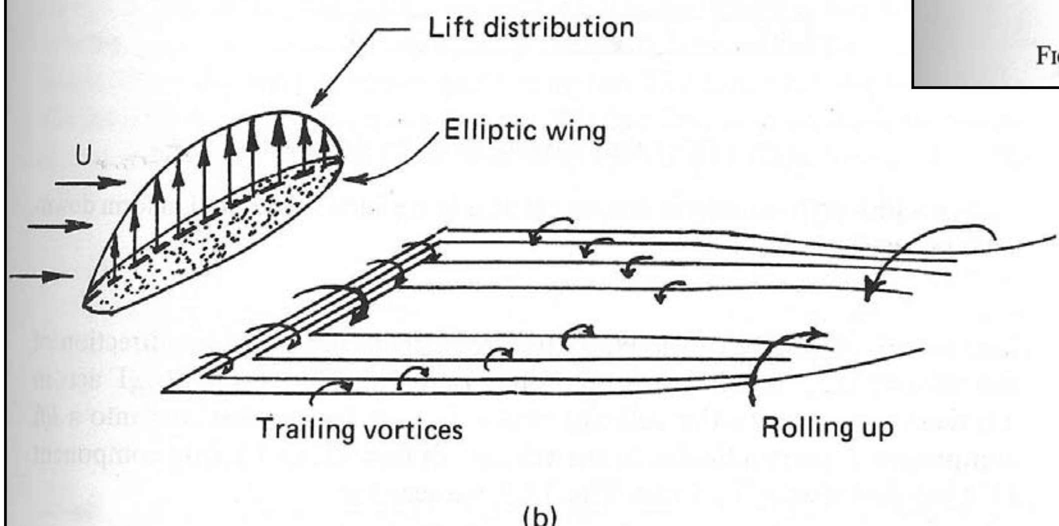


FIGURE 3.10:2 (a) A horse-shoe vortex representing a wing with a uniform lift distribution. (b) Lift distribution on an elliptic wing.

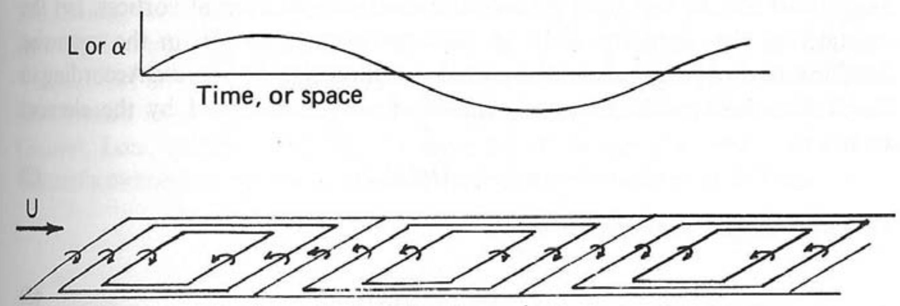


FIGURE 3.10:4 The vortices in the wake of an oscillating wing, idealized under the assumption that the lift fluctuation is very small so that the distortion of the wake due to the vortices in it is also very small. L is the lift, α is the angle of attack.

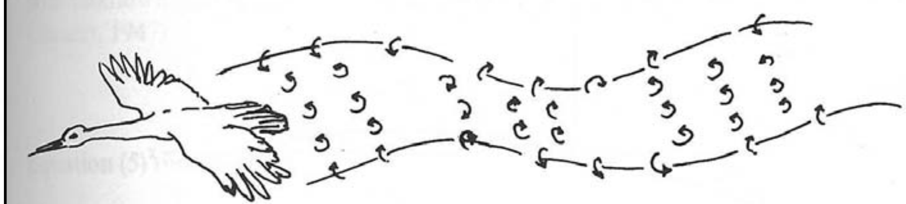


FIGURE 3.10:5 The vortex wake behind a stork in level flight.

Y. C. Fung

$$F = C_L \ \rho_{air} \ v^2 \ S$$

v: speed

V: volumn

S: area

$$W = \rho_f \ V \ g = \rho_f \ S^{1.5} \ g$$

$$F = W$$

$$S^{0.5} = \frac{C_L \ \rho_{air} \ v^2}{\rho_f \ g}$$

$$W = \rho_f \ g \ S^{1.5} = \frac{{C_L}^3 \ \rho_{air}^3 \ v^6}{\rho_f^2 \ g^2}$$

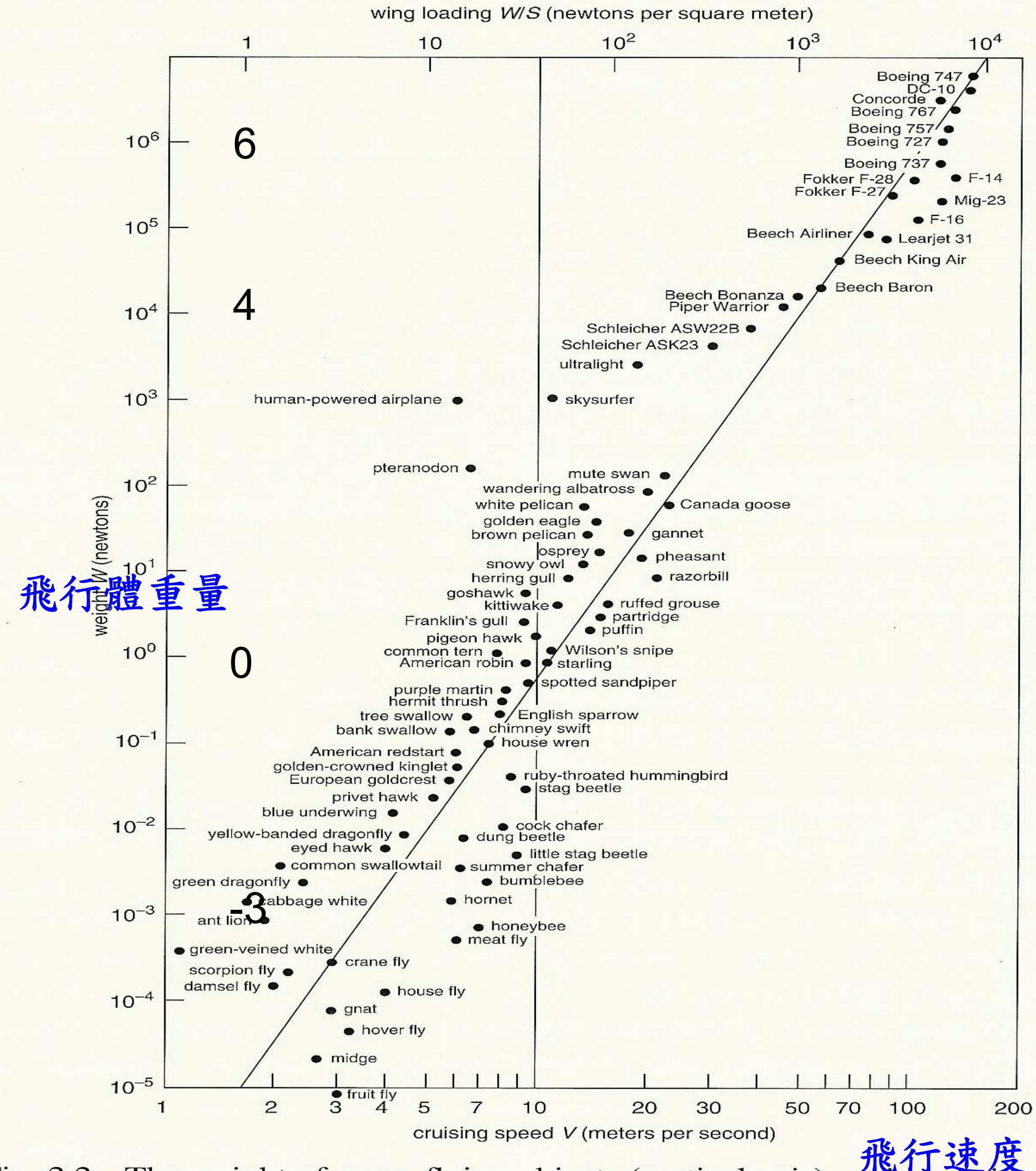
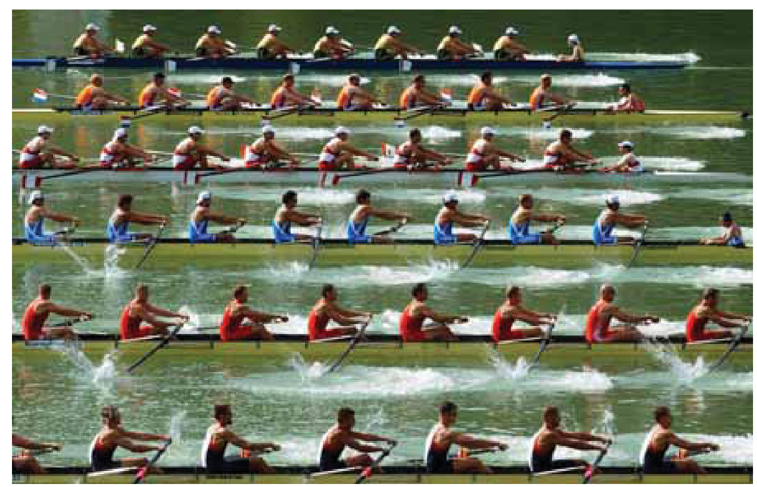


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.




CHN


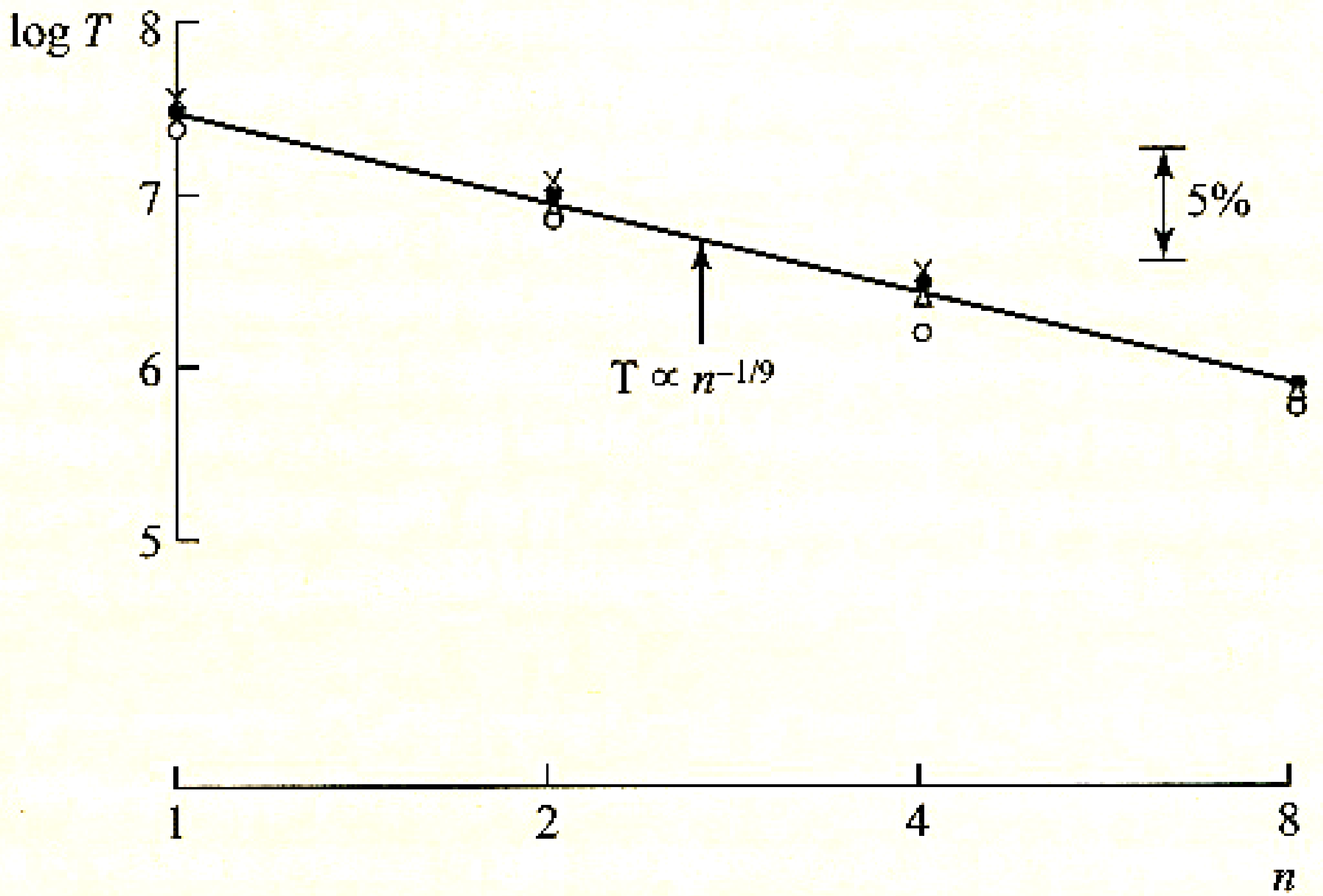


Figure 1.13. The $-1/9$ power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; $\times$, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).

$$F \sim \rho v^2 l^2$$

假設： G, A 為常數

A:strength of oarsperson

$$P \sim \rho v^3 l^2$$

$$\boxed{G \sim \frac{l^3}{n}}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$\boxed{v \sim n^{1/9}}$$

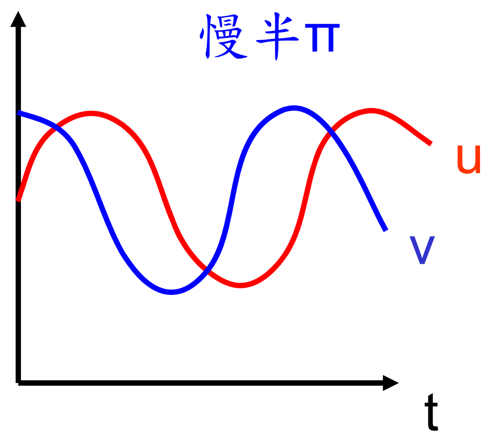
動力系統

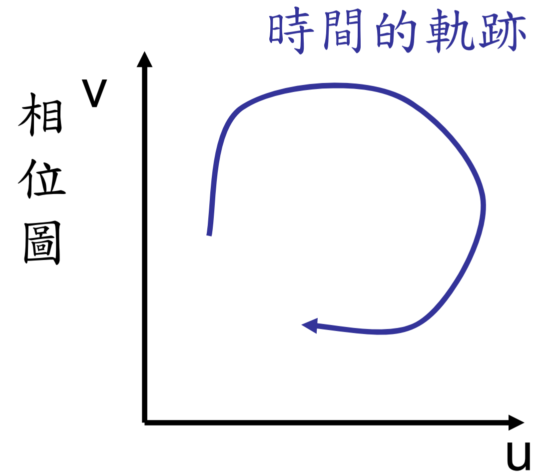
$$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}, \gamma_i)$$

時間變化謂之動力　　變數　　許多外在及內在控制參數

$$\int_0^{2\pi} \underset{u}{\cos t}\,\underset{v}{\sin t}\,dt = 0$$

$$\overline{uv} = 0$$





Cos 和 Sin 零相關、不來電！

相位圖

推背圖：前知三百年，後知三百年

可以解釋〝已知〞，可以預測〝未來〞

微積分數學

$$u = u(x, y)$$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

偏微分

例如: $\frac{\partial x^2 y}{\partial y} = x^2$

只對y變數微分，不改變x變數

你快樂嗎？一個簡單的生涯規劃動力系統

u：快樂指數

x:考試作業量

y:玩魔獸的時間

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

天縱英明的資優生 <0 >0 <0 <0 >0

考試越少越不快樂，
玩魔獸的時間越多越不快樂

$\frac{\partial u}{\partial x}$>0 考試越多越快樂

$\frac{\partial u}{\partial y}$<0 電動越玩越不快樂

→ 人的個性

$\frac{dx}{dt}$

$\frac{dy}{dt}$

→ 人的境遇

個性+境遇=人生

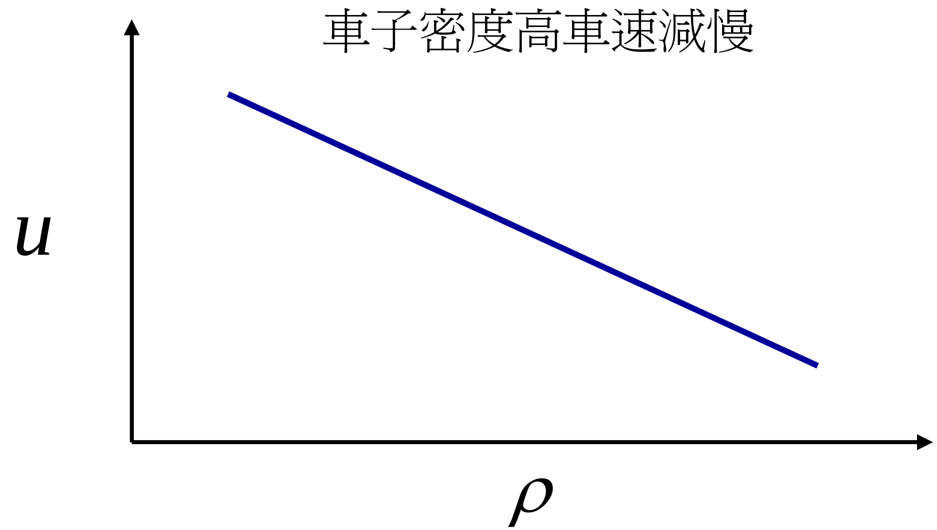
相形不如論心

論心不如則術

形不勝心

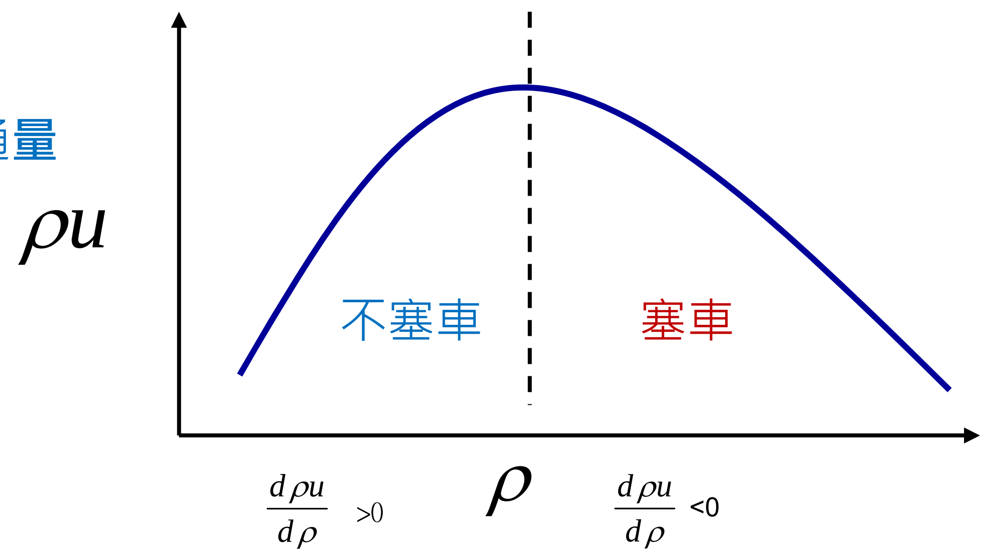
心不勝術 荀子非相

汽車密度



$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial \rho u}{\partial x} = 0 \\ u = u(\rho) \end{cases}$$

汽車通量



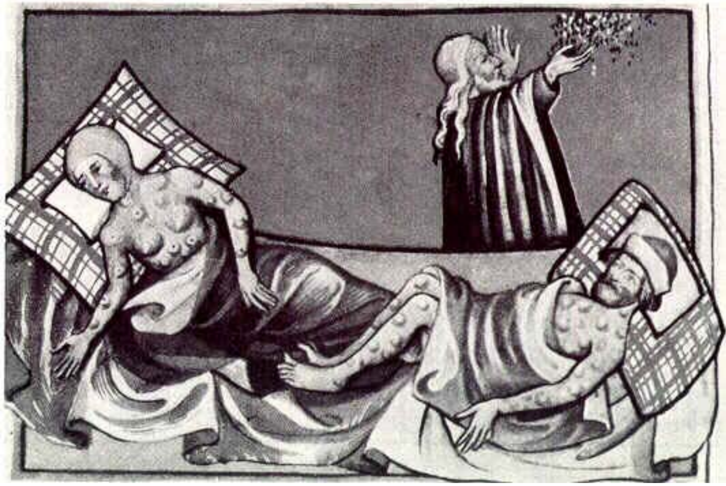
$$\frac{\partial \rho}{\partial t} + \frac{d \rho u}{d \rho} \frac{\partial \rho}{\partial x} = 0 \qquad c = \frac{d \rho u}{d \rho}$$

$$\frac{\partial \rho}{\partial t} + c \frac{\partial \rho}{\partial x} = 0 \qquad \rho = f\left(x - ct\right)$$

密度向下游傳送　密度向上游傳送

Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)



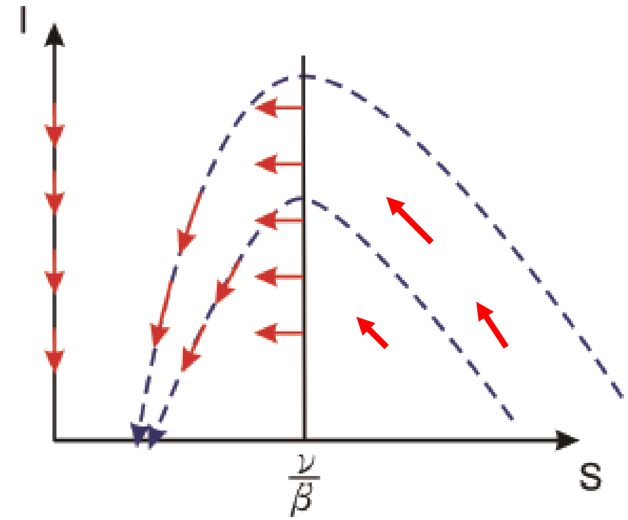
The "***Black Death***" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu)I$$

$$\frac{dR}{dt} = +\nu I$$



ν Recovery Rate

β Infection Rate

No Death in the model

$S = \frac{\nu}{\beta}$有 null cline

Forecast and control of epidemics in a globalized world **PNAS vol.101 no.42**

Hufnagel†, Brockmann, and Geisel

演講者: 陳怡文　日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

HIV Modeling

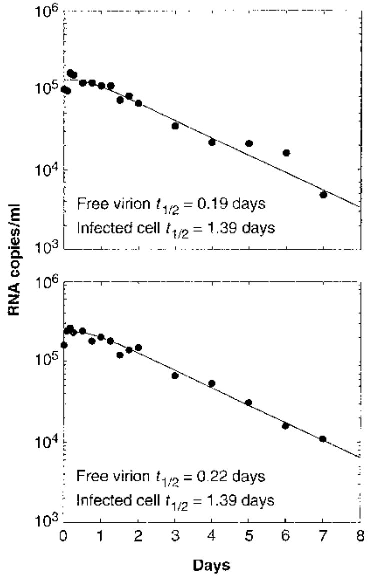
Perelson and Nelson (1999)

$$\frac{dV}{dt} = P \boxed{- cV,}$$

藥物治療

$$\frac{dT}{dt} = kT_0V - \alpha T,$$

$$P = N\alpha T.$$



觀察病人服藥後反應決定C

Figure 4.5. Log of plasma concentrations (copies per mL) of HIV-1 RNA (circles) for two representative patients (upper panel, patient 104; lower panel, patient 107) after *ritonavir* treatment was begun on day 0. The solid line is a nonlinear least square fit to the data. HIV-1 RNA level is an easier measure of HIV virions since each HIV virion contains two RNA molecules. (See exercise 5 for more details.) (From Perelson et al. [1996], used by permission of Alan S. Perelson.)

$$P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \quad (1/(day \cdot ml))$$

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
  : non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

何大一雞尾酒療法

Thomas Robert Malthus

Wikipedia

（1766～1834）

English demographer and political economist

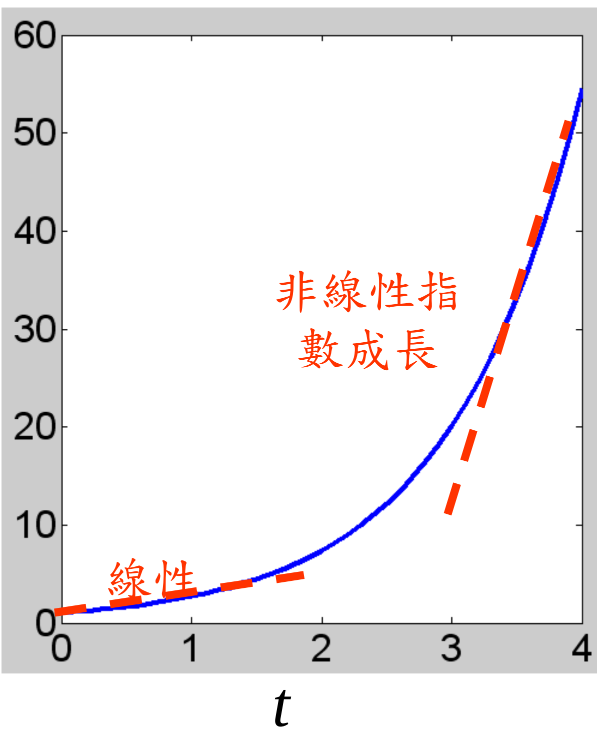
人口學家
政治經濟學家

Malthusian Model

Population Growth

$$\lim_{\Delta \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\frac{dp}{dt} = \alpha p \quad \Rightarrow \quad p = p_0 e^{\alpha t}$$



線性　　　非線性

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

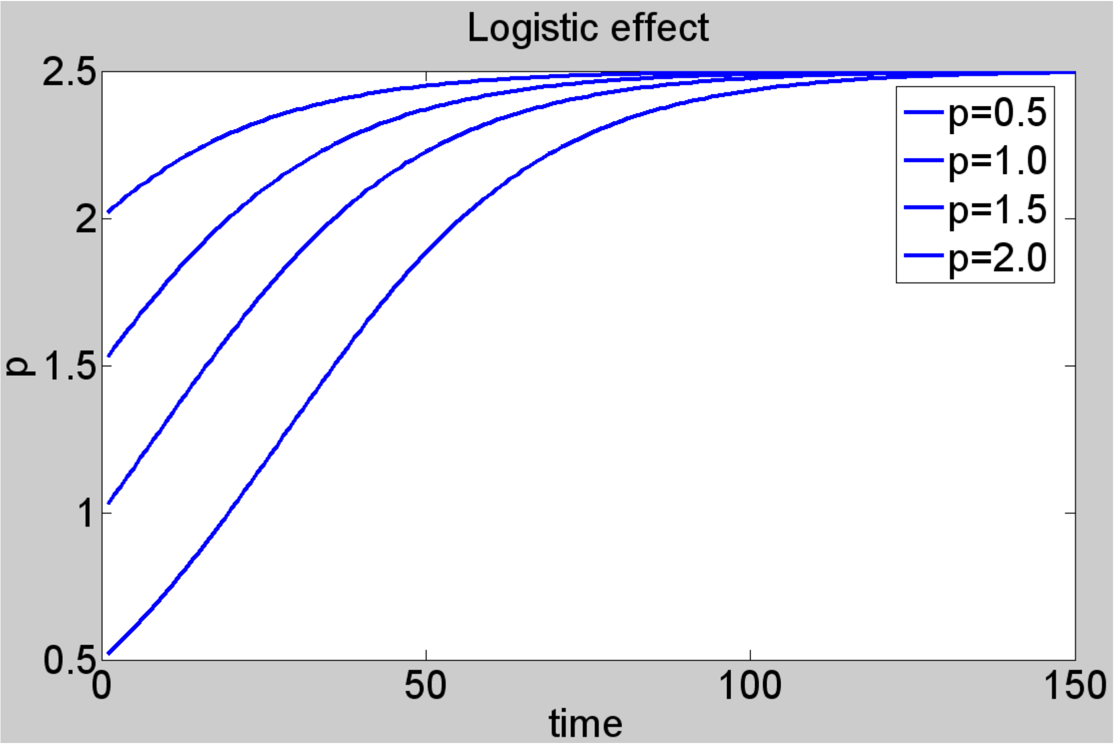
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Logistic Effect

Time Series (時間序列)

$$\frac{dp}{dt} = \alpha p - \beta p^2 = \alpha p\left(1 - \frac{p}{\frac{\alpha}{\beta}}\right)$$

其中 $\frac{\alpha}{\beta}$ → 環境之承載效應



Lotka-Volterra Model

A Predator-Prey Model

Humberto D'Ancona
1926

World War One 掠食者比例變大 (1915–1918)

	Port	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
Predator	**Fiume**	12%	21%	22%	21%	36%	27%	16%	16%	15%	11%
Prey	**Trieste**	14%	7%	16%	15%	-	18%	15%	13%	11%	10%

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

Vito Volterra

Wikipedia

(1860～1940)

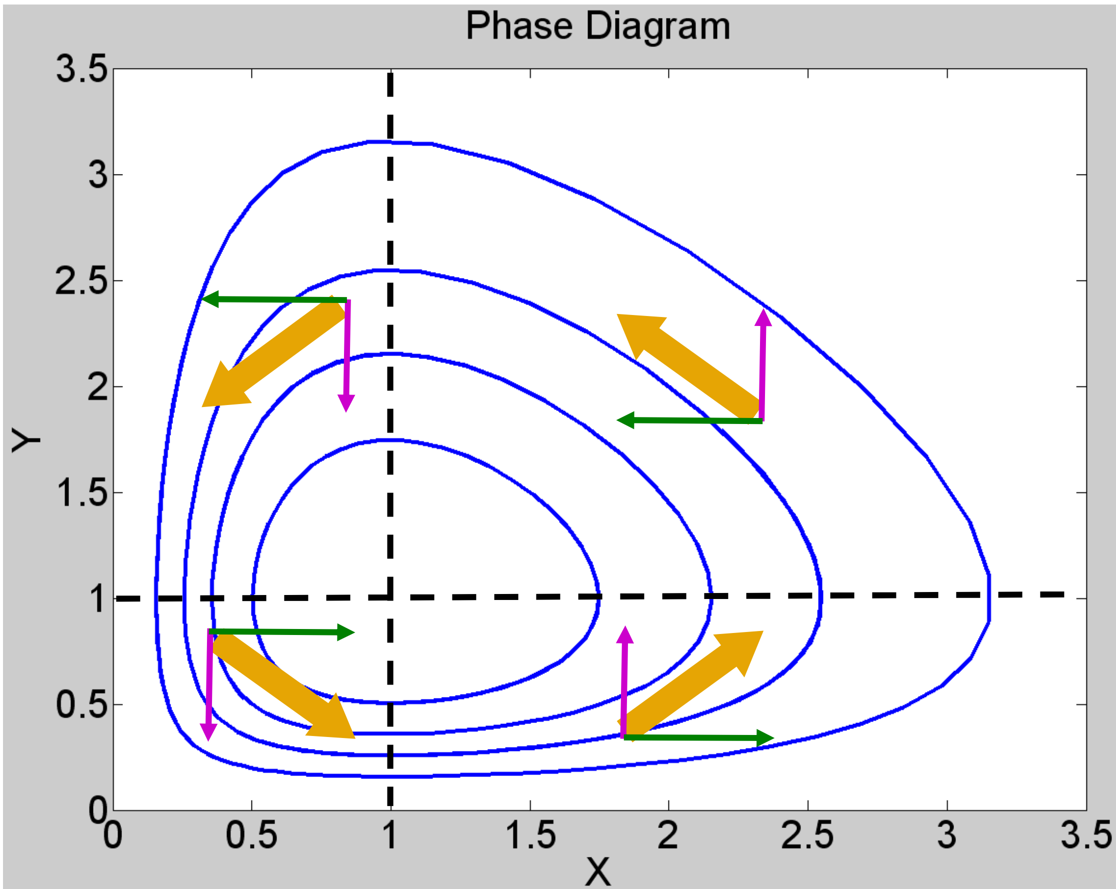
Italian
Mathematician & Physicist

Contribution:

1) Mathematical biology

2) Volterra-Lotka equations

動力系統



dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

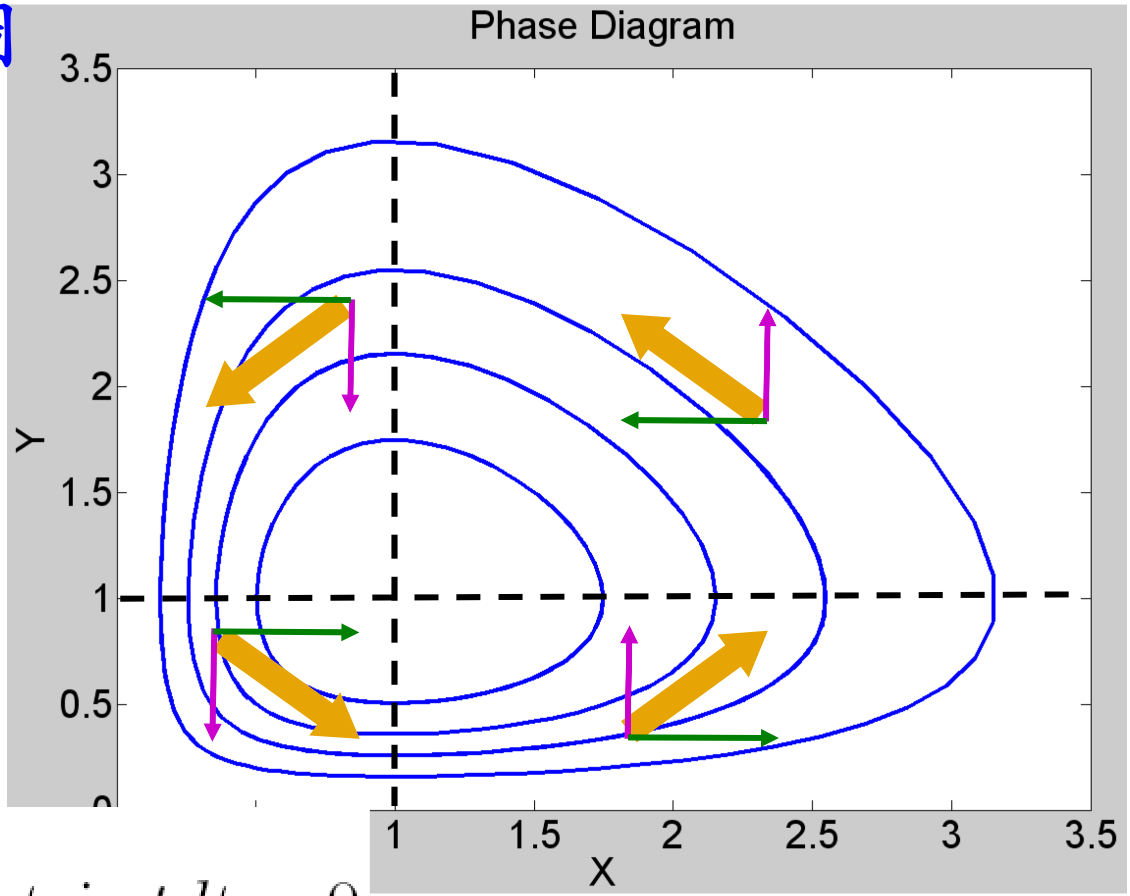
$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

相位圖

愛情動力系統

相位圖



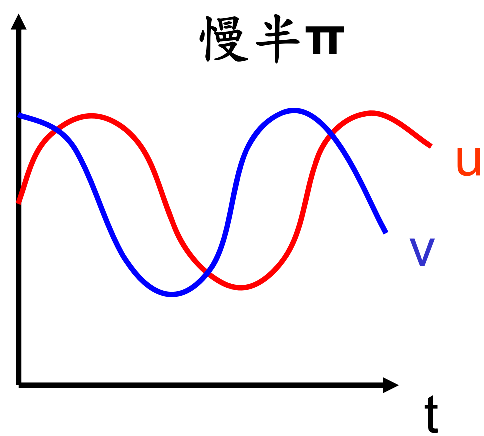
dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

慢半π



$$\int_0^{2\pi} \underset{u}{\cos t} \underset{v}{\sin t} dt = 0$$

Cos 和 **Sin** 零相關、不來電！

動力系統

鴛鴦蝴蝶夢，一段愛情故事；

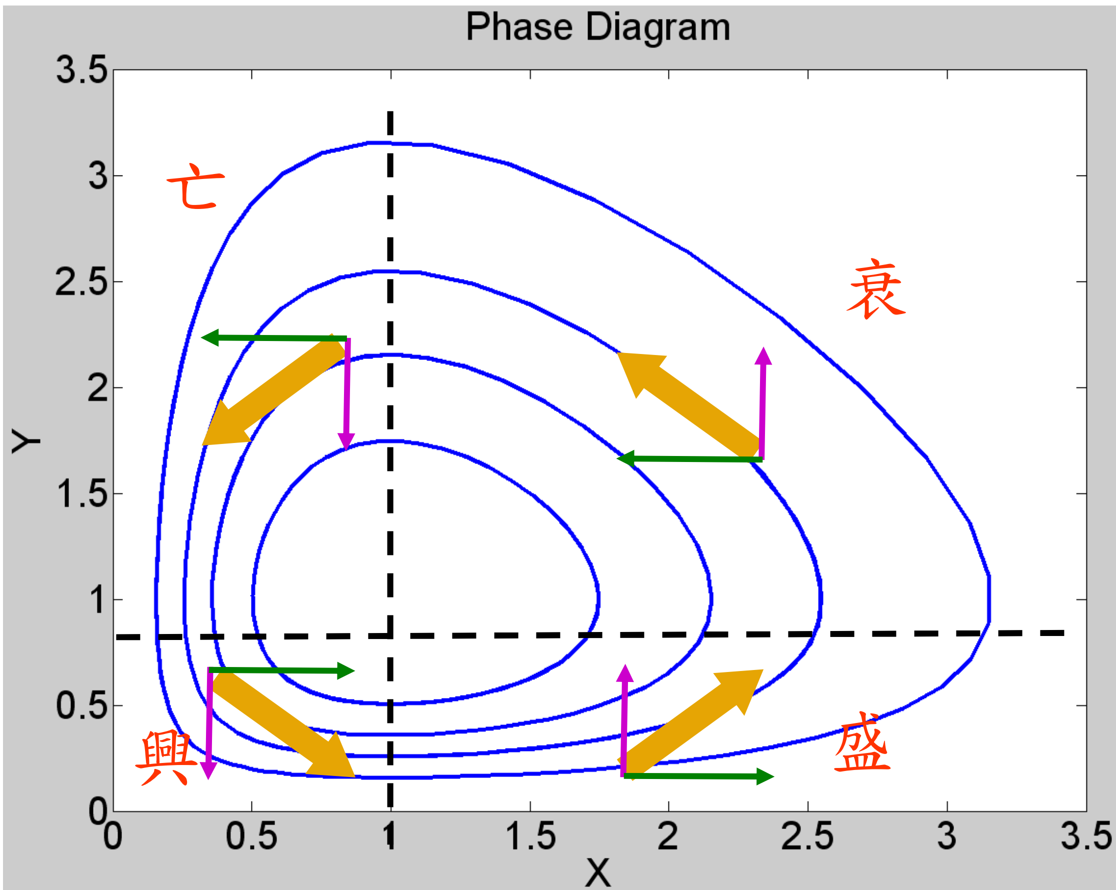
千古興盛衰亡。

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

掠食者

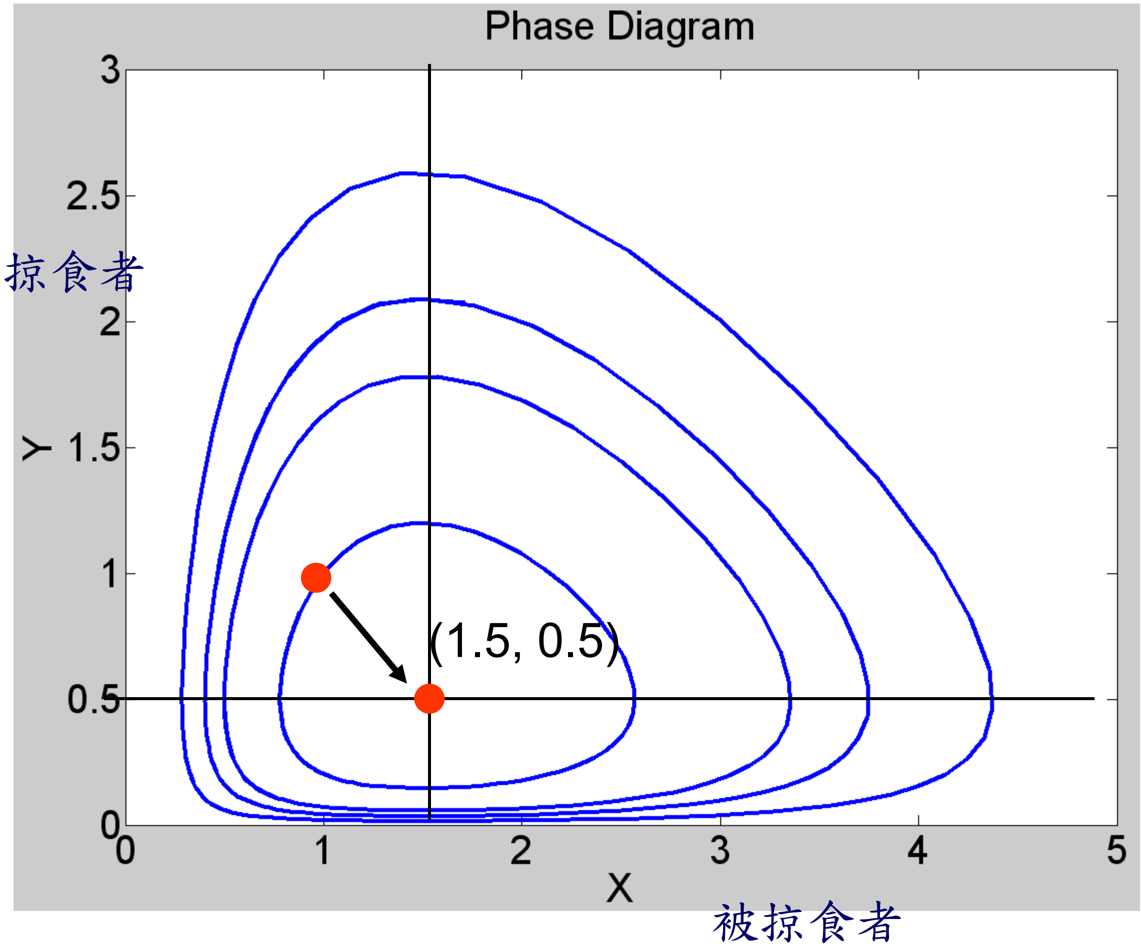
（官僚）



被掠食者（善良人民）

$$\begin{cases} \dfrac{dx}{dt} = x - xy - hx \\ \dfrac{dy}{dt} = -y + xy - hy \end{cases}$$

h=0.5



Harvest effect
對掠食者與被掠食者同時進行獵取殺害，可以使生態往對被掠食者有利方向移動。

1960 DDT 事件

無間道警察
水滸傳哲學

$$
\begin{cases}
\dfrac{dp}{dt} = K_1 p(1 - \dfrac{p}{c_1}) - \alpha_1 pq \\
\dfrac{dq}{dt} = K_2 q(1 - \dfrac{q}{c_2}) - \alpha_2 pq
\end{cases}
$$

原方程式變數：

K_1	c_1	α_1
K_2	c_2	α_2

令

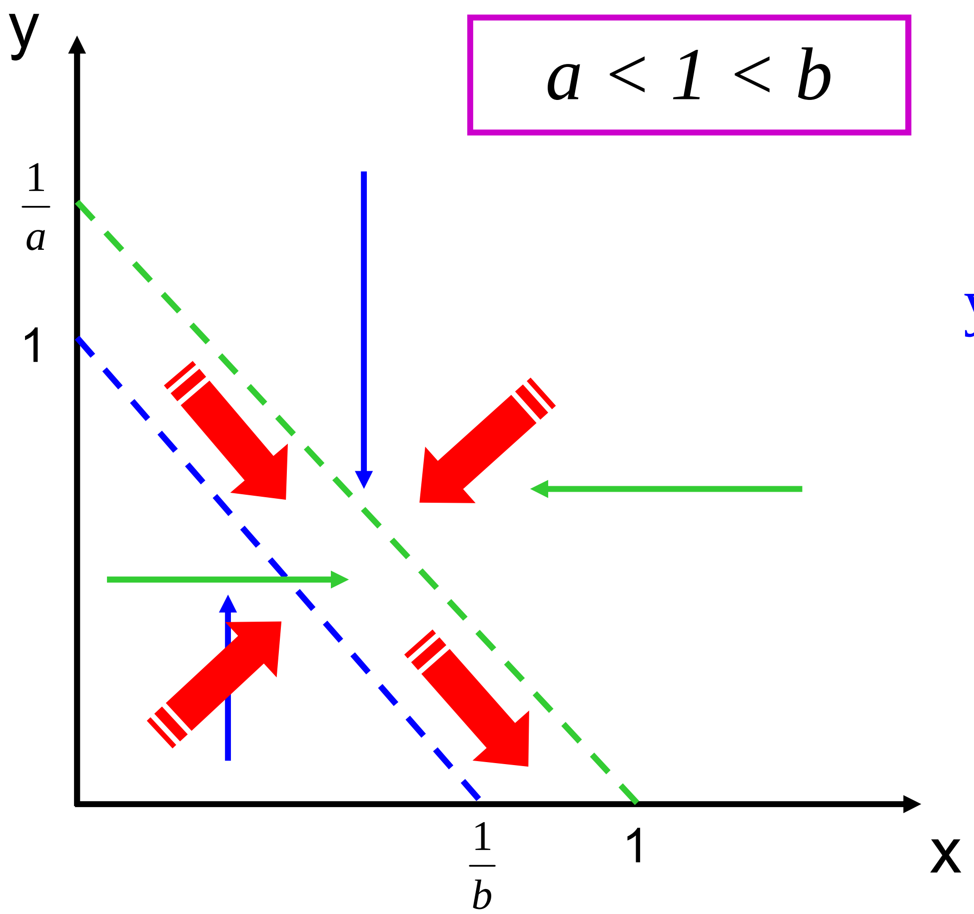
$x = \dfrac{p}{c_1}$　　$\dfrac{\alpha_1 c_2}{K_1} = a$

$y = \dfrac{q}{c_2}$　　$\dfrac{\alpha_2 c_1}{K_2} = b$

$$
\begin{cases}
\dfrac{dx}{dt} = K_1 x(1 - x - ay) \\
\dfrac{dy}{dt} = K_2 y(1 - y - bx)
\end{cases}
$$

強弱參數＝攻擊力/修護力， 攻擊力＝戰鬥力＊族群數目

$$\begin{cases} \dfrac{dx}{dt} = K_1 x(1 - x - ay) \\ \dfrac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$

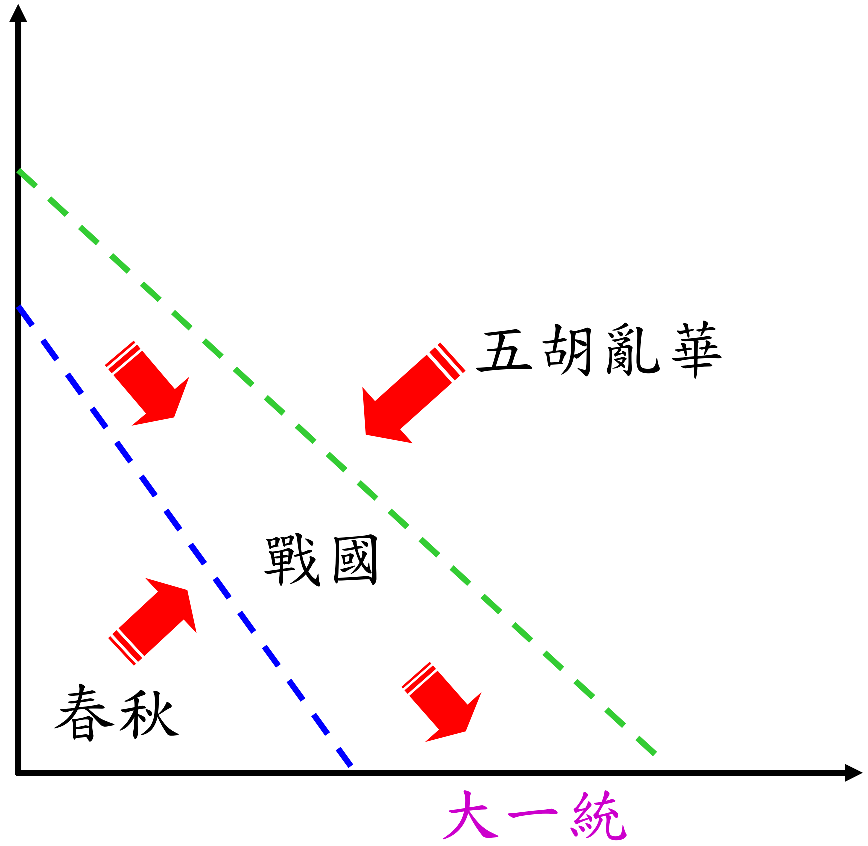


$a < 1 < b$

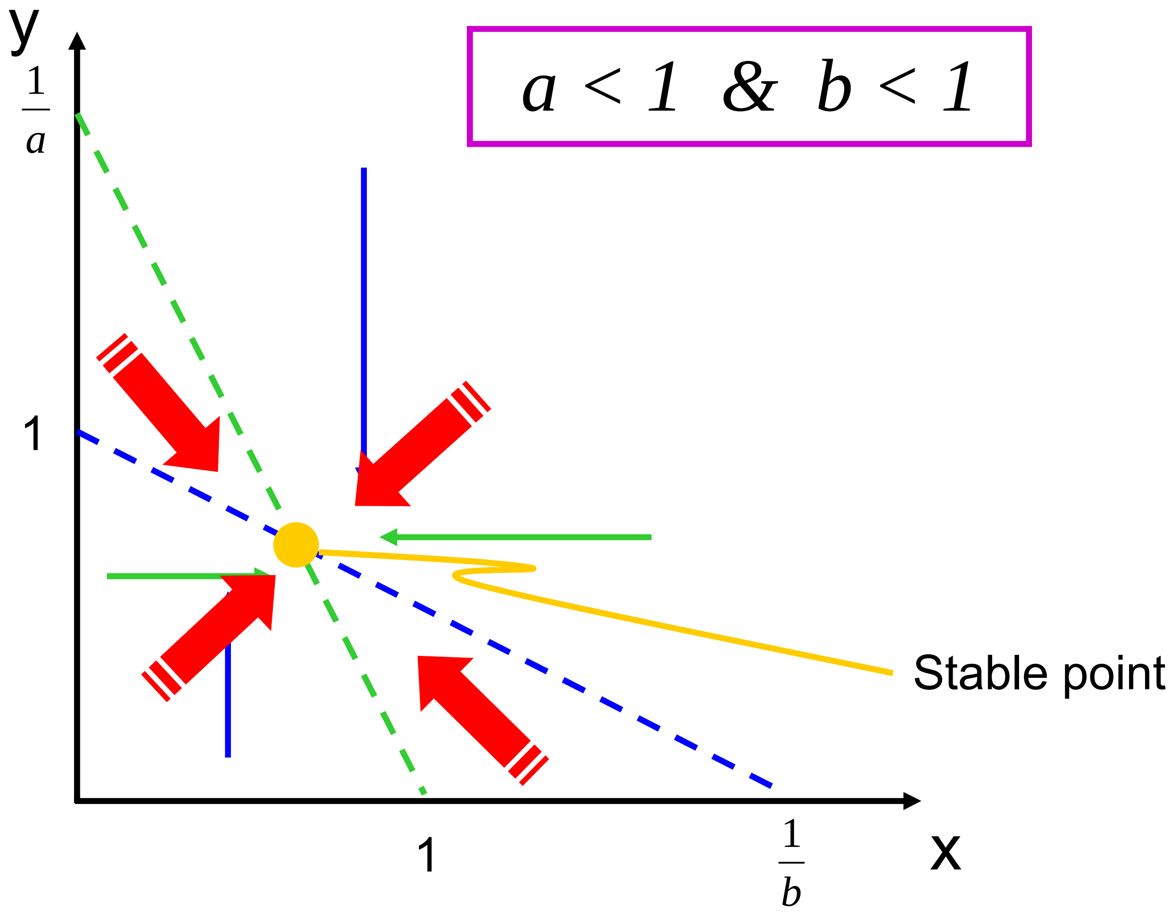
X 比 Y強

y 滅絕

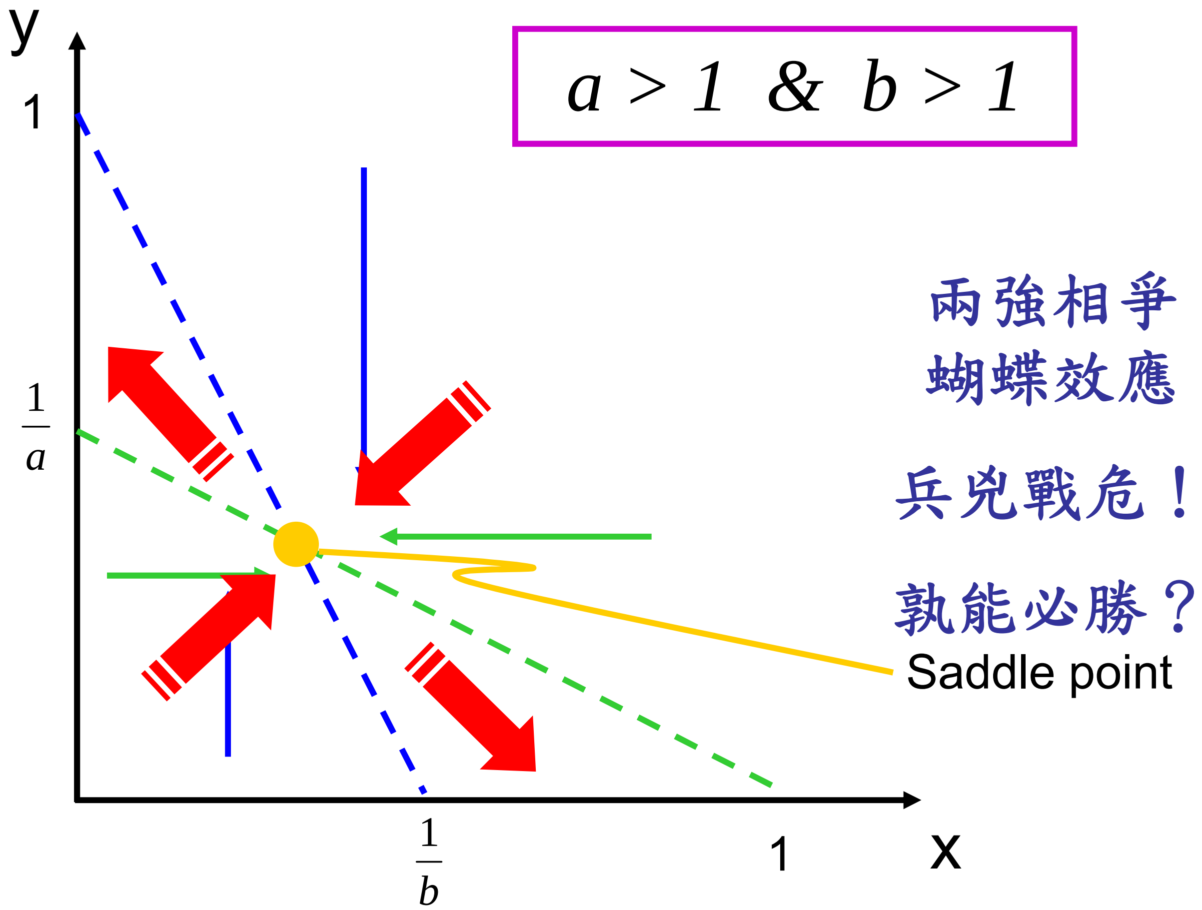
強弱之爭



$$\begin{cases} \dfrac{dx}{dt} = K_1 x(1 - x - ay) \\ \dfrac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$



$$\begin{cases} \dfrac{dx}{dt} = K_1 x(1 - x - ay) \\ \dfrac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$



君子務本 本立道生

關鍵基礎能力 語文能力 能專精方能跨領域

誠不以富 亦祇以異

不同立場有不同地位 特色 專業水準 眼光

做自己有興趣且有長處的事業

It is easy to say!! 需要許多過程，自我追尋、自我瞭解、自我訓練，才能找到自己的路。

好的工作習慣

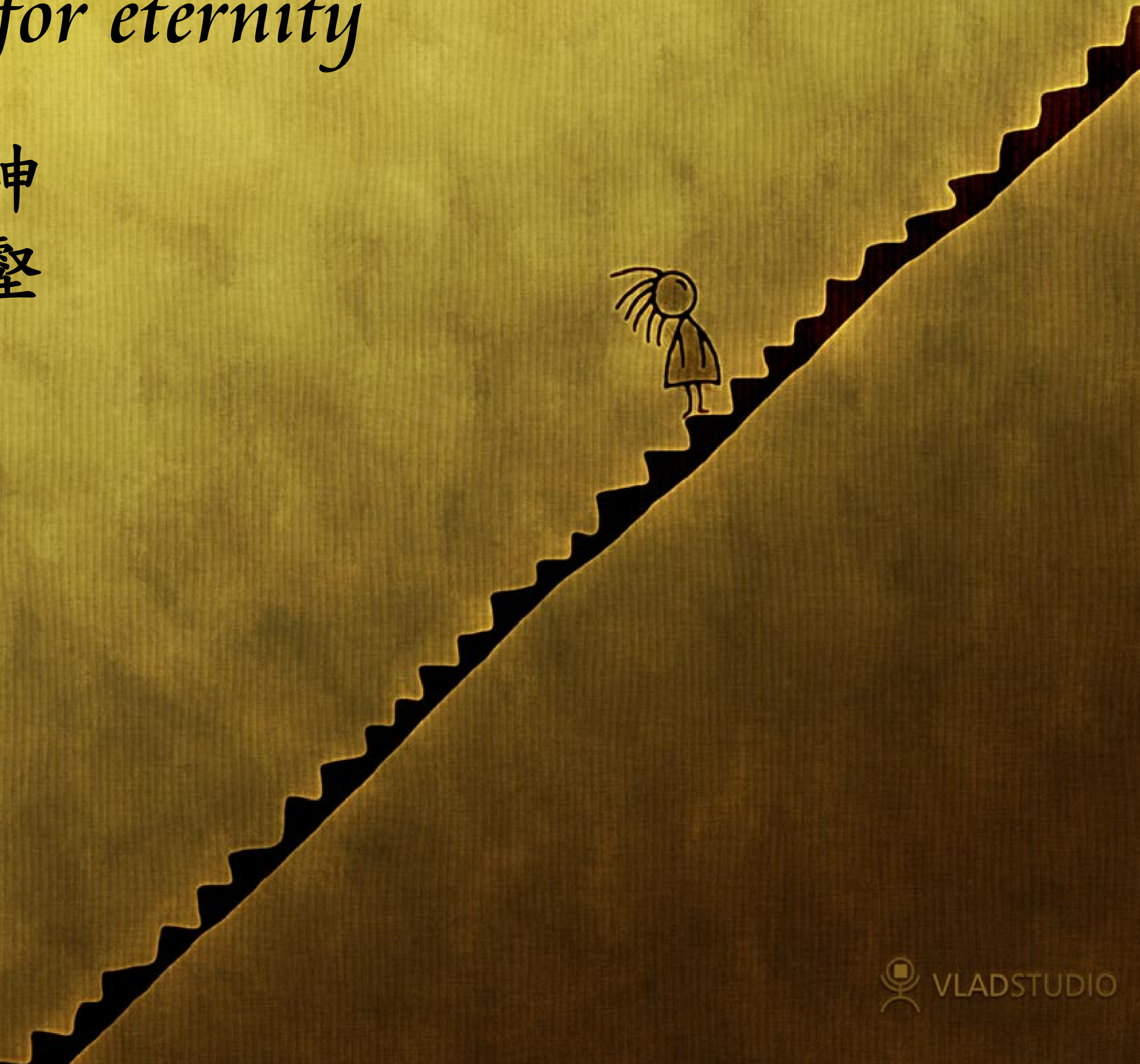
好的基礎功夫-------讀、算、寫

眼界、Vision、選重要的問題

2008和林長壽老師的對談

Politics are for the moment
An equation is for eternity

但覺高歌有鬼神
不知餓死填溝壑



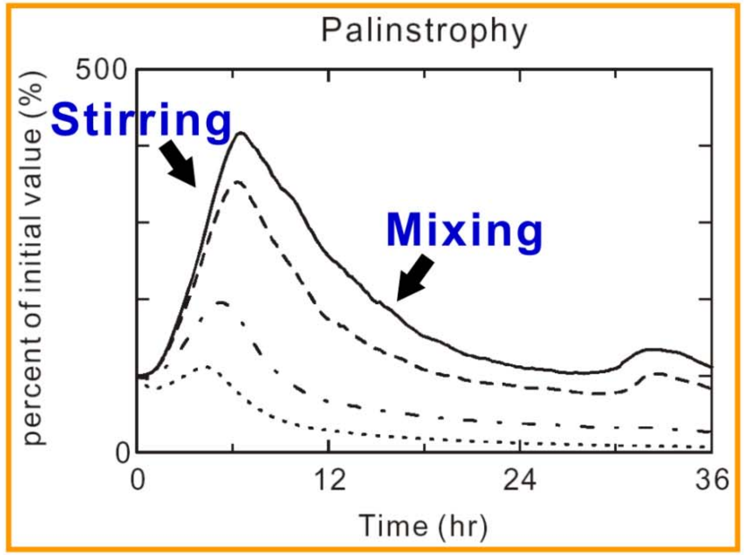
A coffee lover's dream:
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

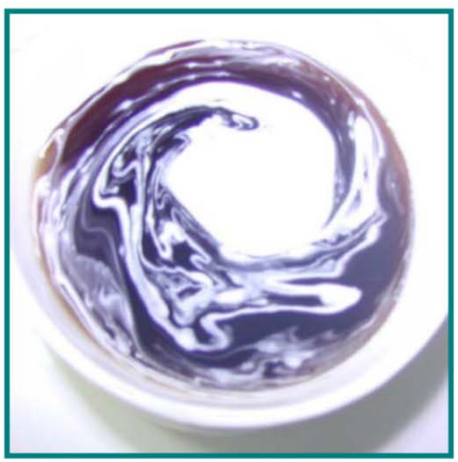
$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta \; dV$$

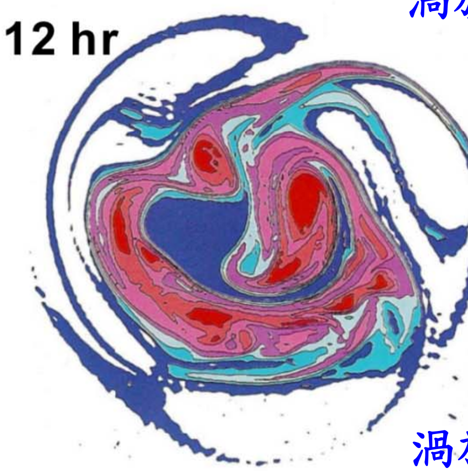
$$\frac{dC}{dt} = \int (\vec{V}\cdot\nabla\theta)\nabla^2\theta \; dV - \nu\int (\nabla^2\theta)\, dV$$

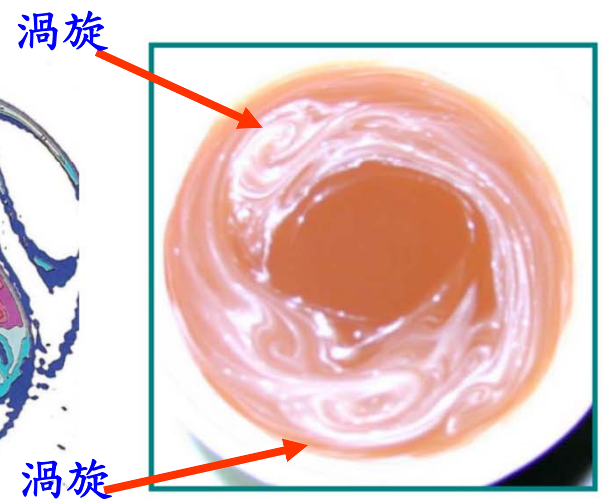
Stirring **Mixing**



Coffee with white







$$\frac{d\,x}{d\,t} = x(1-x) - h$$

h: constant rate of population harvested.

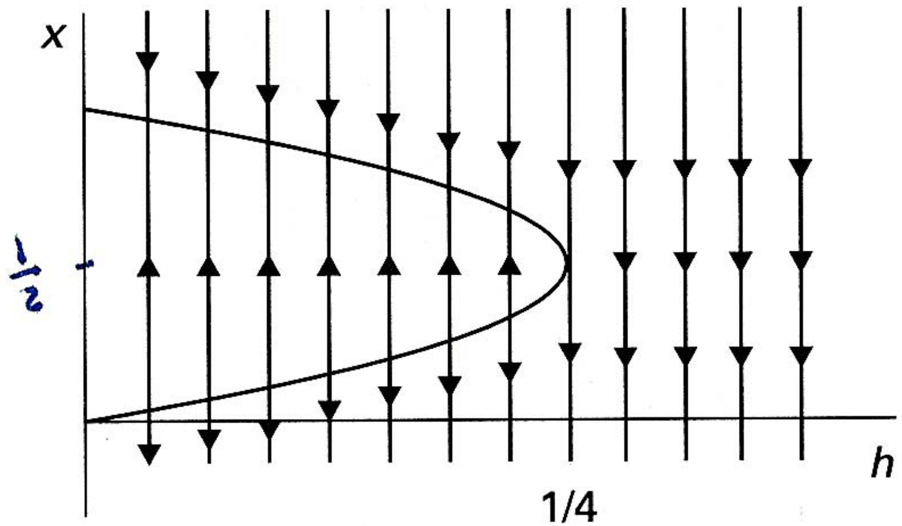
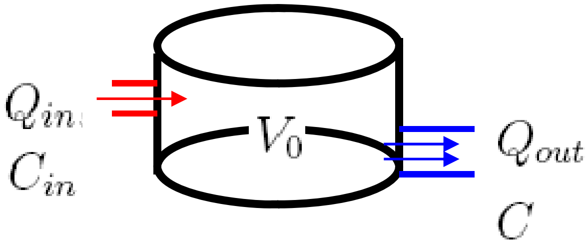


Figure 1.7 The bifurcation diagram for $f_h(x) = x(1-x) - h$.

Small changes in harvesting rate can lead to disastrous changes in population has been observed many times in real situations on earth.



$$\frac{dV}{dt} = Q_{in} - Q_{out} = -\triangle Q,$$

$$\frac{dVC}{dt} = Q_{in}C_{in} - Q_{out}C,$$

$$C(0) = 0, \quad V(0) = V_0.$$

$$V(t) = V_0 - \triangle Q t.$$

$$V\frac{dC}{dt} = Q_{in}(C_{in} - C),$$

$$\frac{dC}{dt} = \frac{Q_{in}(C_{in} - C)}{V_0 - \triangle Q t}.$$

$$C = C_{in}\left(1 - \left(1 - \frac{\triangle Q}{V_0}t\right)^{Q_{in}/\triangle Q}\right)$$

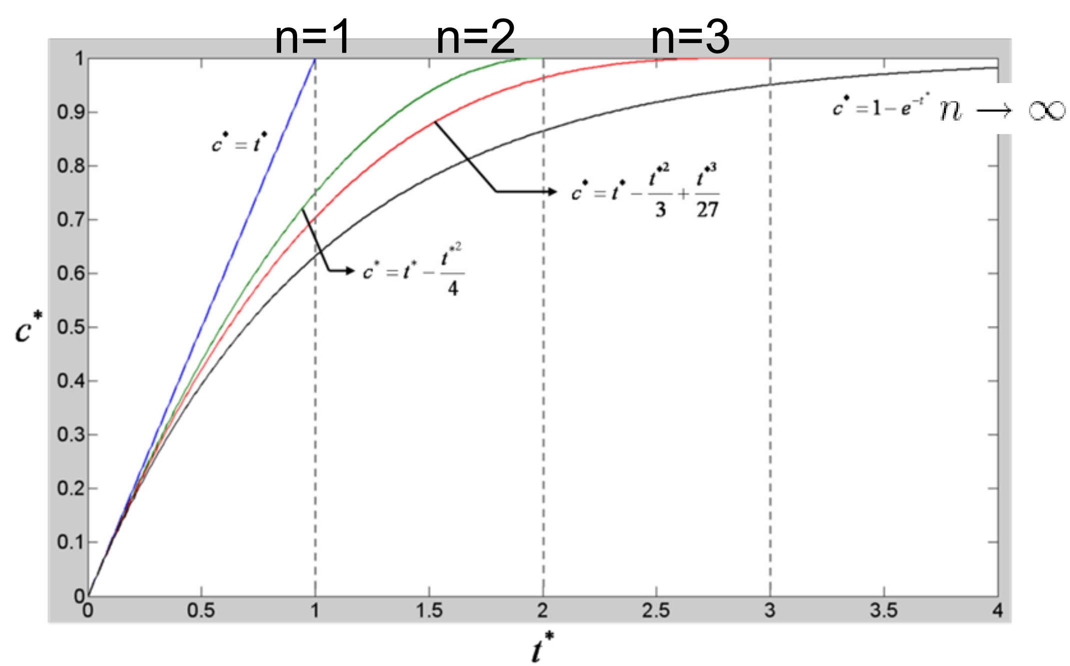
$$t = \frac{V_0}{Q_{in}}t^*, \quad C = C_{in}C^* \quad n = \frac{Q_{in}}{\triangle Q}$$

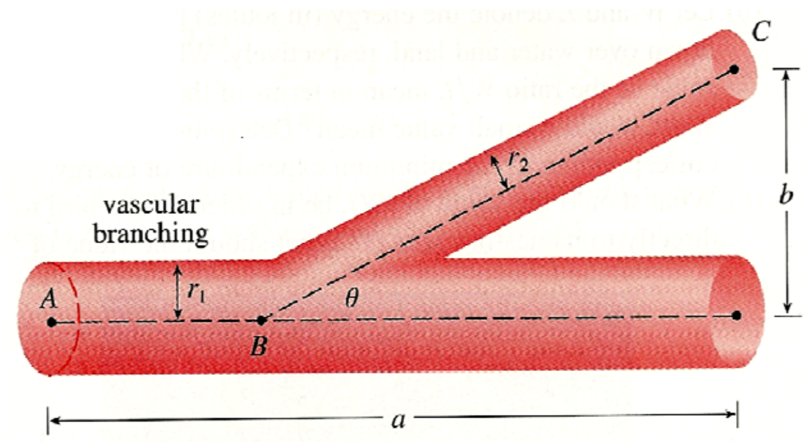
$$\frac{dC^*}{dt^*} = \frac{1 - C^*}{1 - t^*/n}$$

$$\triangle Q = 0,\ n \to \infty$$

$$C^* = 1 - \left(1 - \frac{t^*}{n}\right)^n.$$

$$C^* = 1 - \exp(-t^*)$$





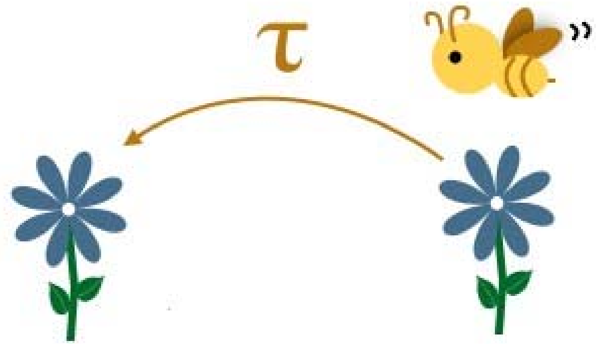
Resistance: $$R = C\frac{L}{r^4}$$

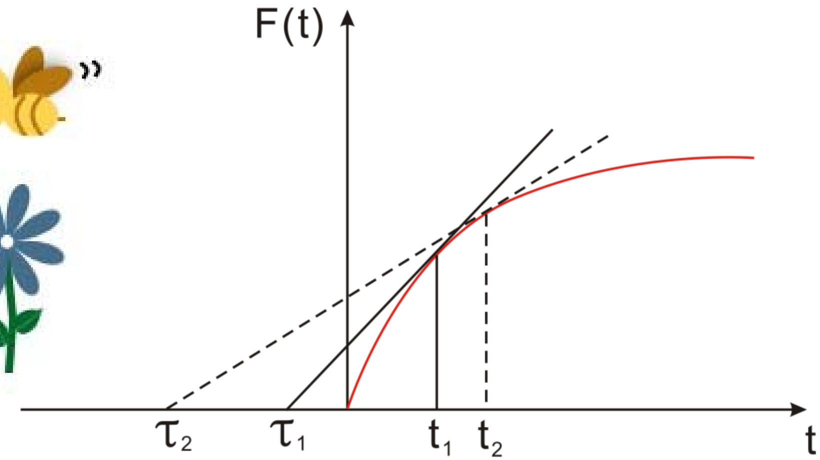
Total Resistance: $$R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

The Resistance is Minimized when:

$$\cos\theta = \frac{r_2^4}{r_1^4}$$

術　方法





一朵花停留的時間：t

蜜峰從第一朵花到下一朵花的飛行時間：τ

$$g(t)=\frac{f(t)}{t+\tau} \qquad \text{nectar collection rate} = \frac{\text{food per visit}}{\text{time}}$$

$$g'(t)=0$$ 有最大採集效率

$$\Rightarrow \frac{f'(t)}{t+\tau}-\frac{f(t)}{(t+\tau)^2}=0 \quad \Rightarrow f'(t)=\frac{f}{t+\tau}$$

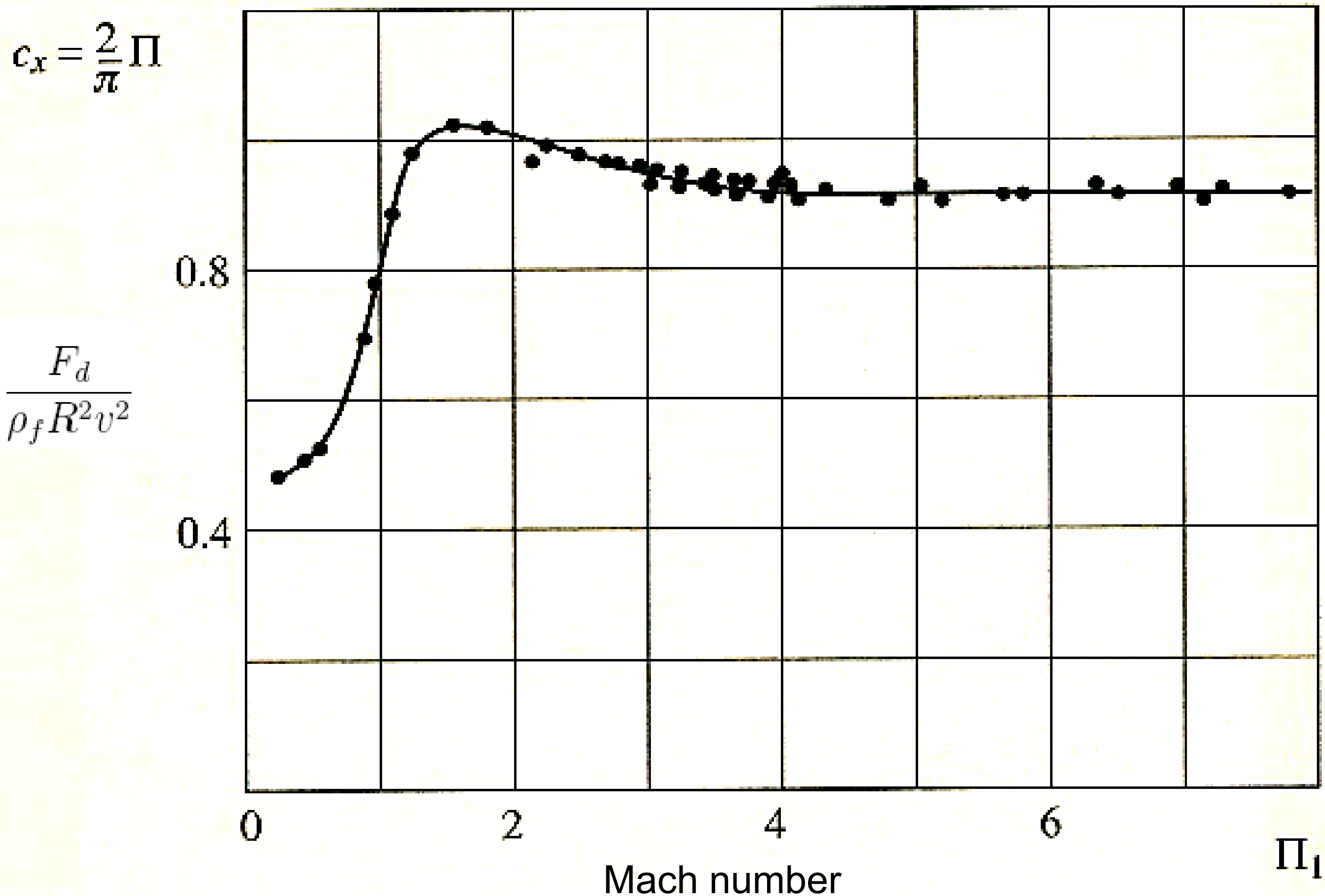
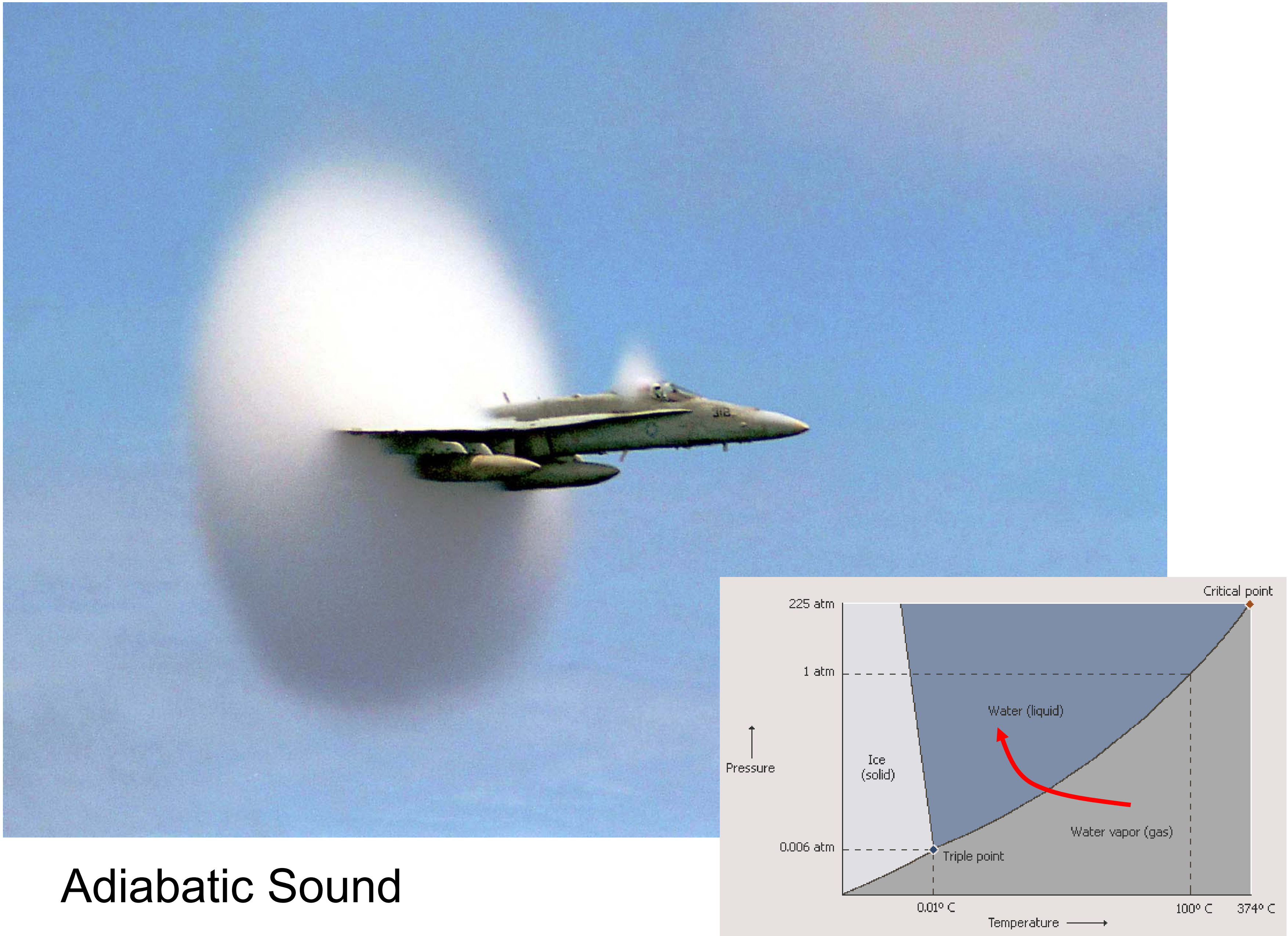
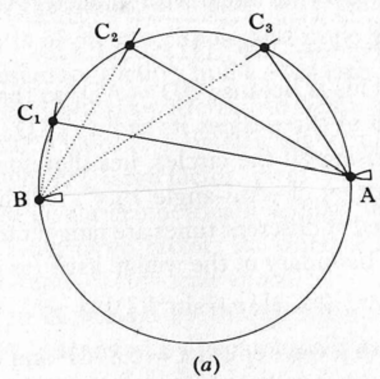


Figure 1.11. The dimensionless drag on a sphere, Π (times $2/\pi$), as a function of the dimensionless governing parameter $\Pi_1 = U/c$, the Mach number (Chernyi 1961). The quantity Π approaches a constant for large values of Π_1.

Sonic Boom



Adiabatic Sound



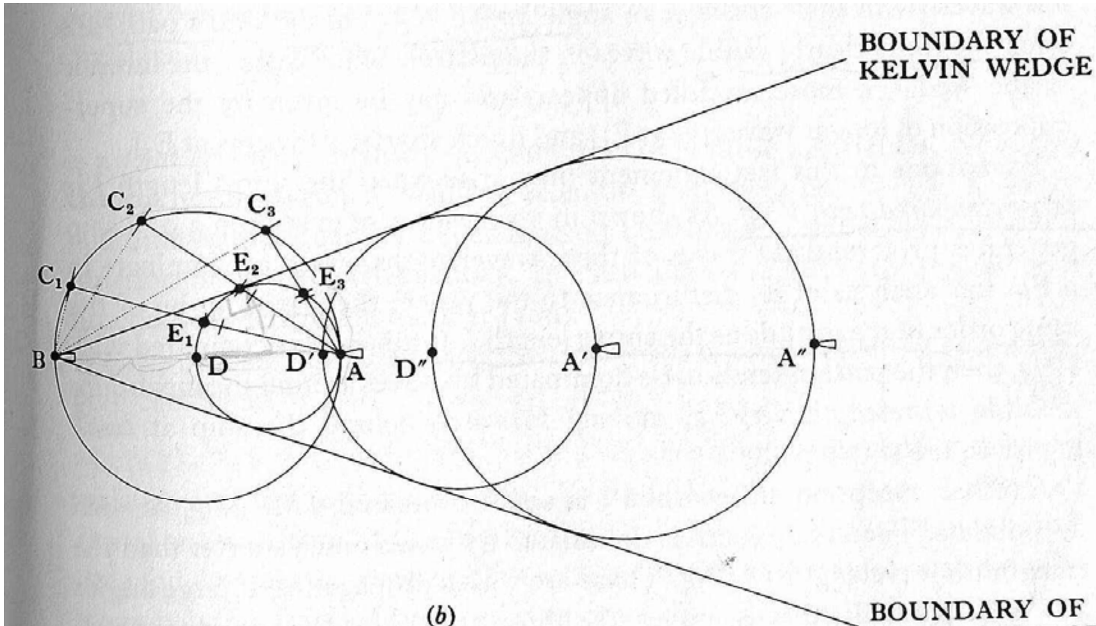


Figure 68. Waves generated in deep water by a ship B. Case (*a*): positions C_1, C_2, C_3 of any waves generated t_g seconds ago (when the ship was at A) if their energy had travelled a distance ct_g. Case (*b*): the real positions E_1, E_2, E_3 of the same waves, taking into account that their energy has only travelled a distance $\frac{1}{2}ct_g$. At each, the dependence of wavelength on direction of emission, as inferred from equation (183), is shown. The circle with diameter AD is the locus of all such waves. Other such circles, with diameters A′D′ and A″D″, are where waves generated when the ship was at A′ and A″ are now to be found. All such circles lie within the Kelvin wedge of semi-angle (184).

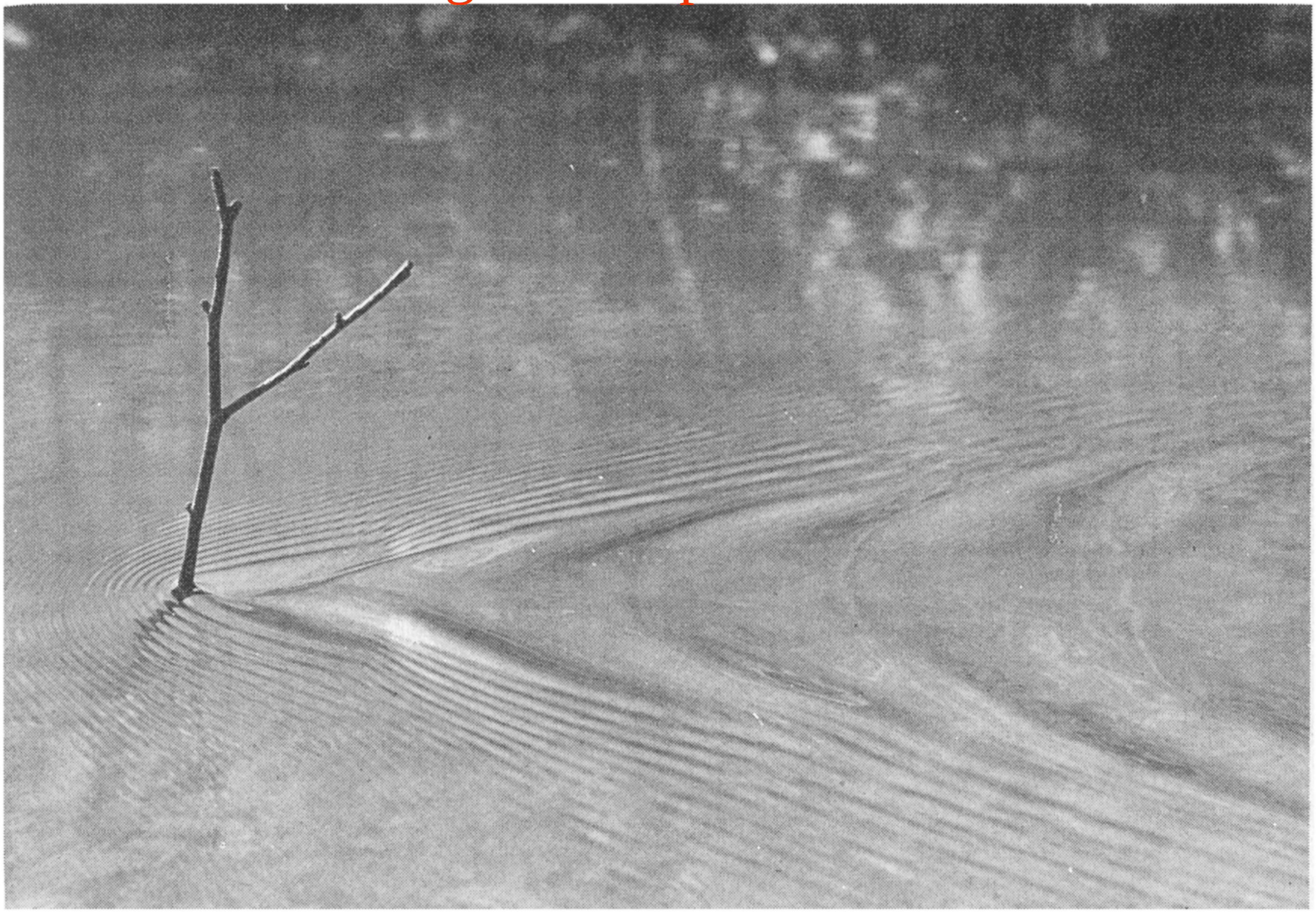
Kelvin edge   deep water waves

< Ship-wave pattern >

$$2 * \sin^{-1}\left(\frac{1}{3}\right) = 2 * 19.5^{o}$$

[*Courtesy of Aerofilms Ltd.*]

Kelvin wedge deep water waves



[*From V. A. Tucker, “Waves and Water Beetles,”*
Physics Teacher **9**, 10-14 19 (1971), Fig.3
(Copyright 1971 by American Assoc. Phys. Teachers)]

Dead-water near the coast

Wave drag problem

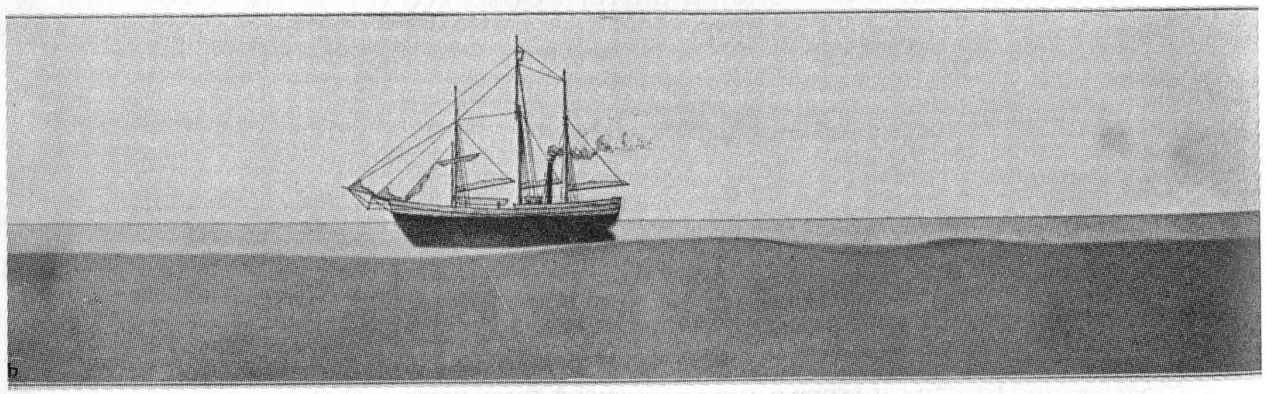
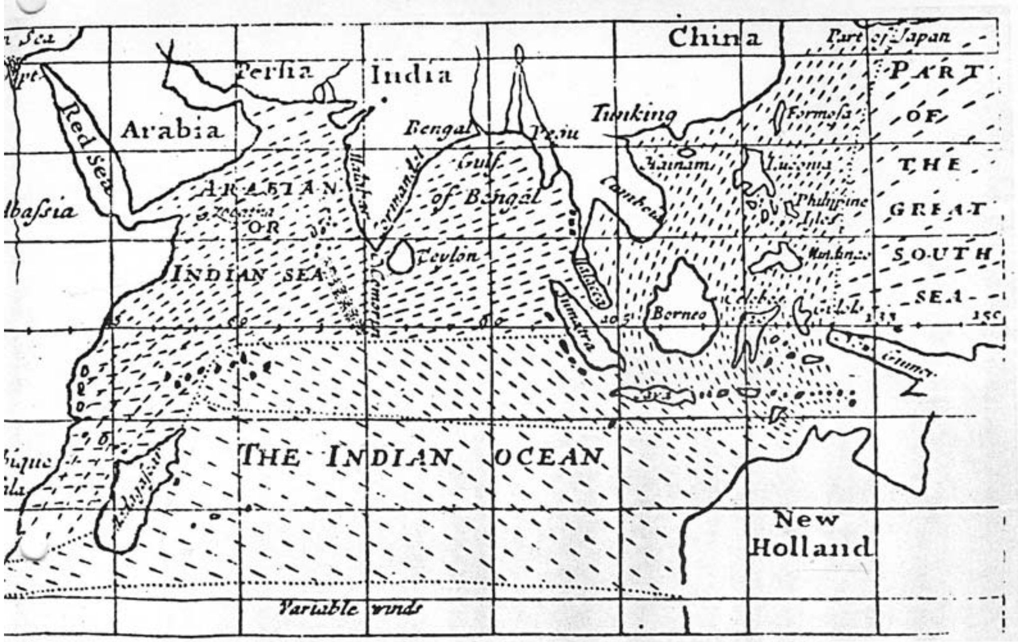


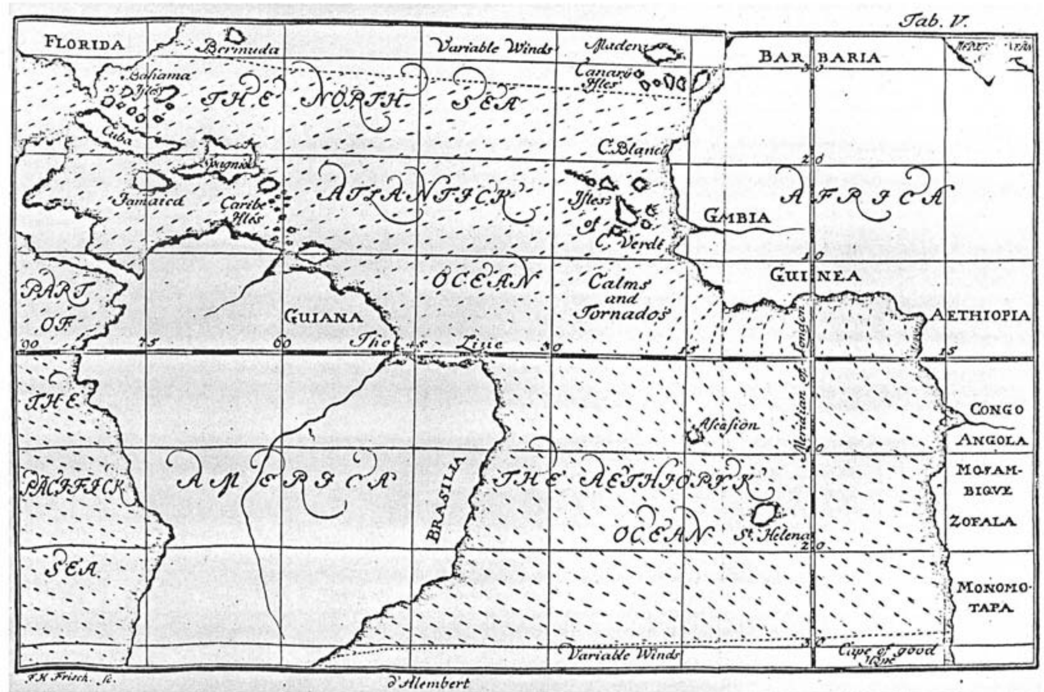
Fig. 6.2. (a) Surface "slicks" showing the presence of internal waves in the wake of a ship in Bute Inlet, British Columbia. The vessel was traveling at 0.5 m s^{-1} in a surface layer of almost fresh water only slightly deeper than its 3.4 m draft. The internal waves caused horizontal motion at the surface that affects the ripple pattern and so renders the internal wave pattern visible at the surface during calm conditions. [Photo courtesy of Defence Research Establishment Pacific, Victoria, British Columbia.] (b) A laboratory experiment [from Ekman (1904)], showing internal waves being generated by a model ship. The tank is filled with two fluids of different density, the heavier one being dyed to make the interface clearly visible. The model ship (the superstructure of the "Fram" has been drawn in subsequently) is towed from right to left, causing a wake of waves on the interface.

Halley 1686



Halley's World Map of Wind Circulations

Fig. 8.1

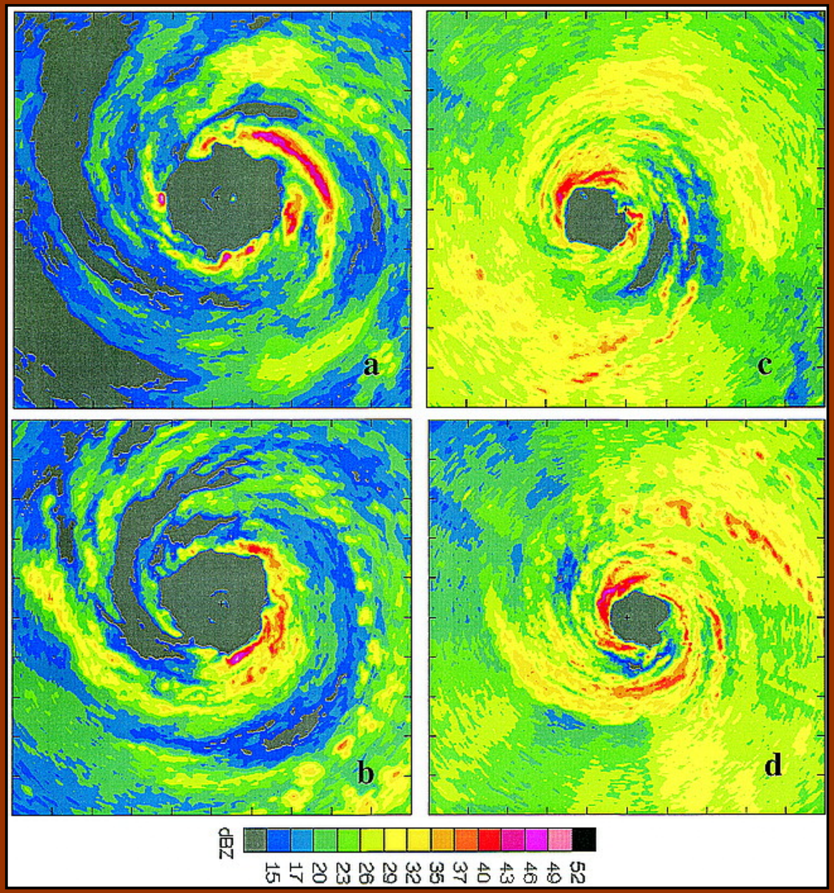


D'Alembert 1746

D'Alembert's Map of the Winds in the Lower Latitudes

Fig. 8.2

Spiral Band in Hurricane and Galaxy



Airborne-radar reflectivity in Hurricanes
Guillermo (1997) (left panels) and Bret (1999) (right panels).

Kossin and Schubert 2001

Bowmen and Mangus (1993)

大氣化學和大氣動力學的跨領域合作

臭氧洞衛星觀測

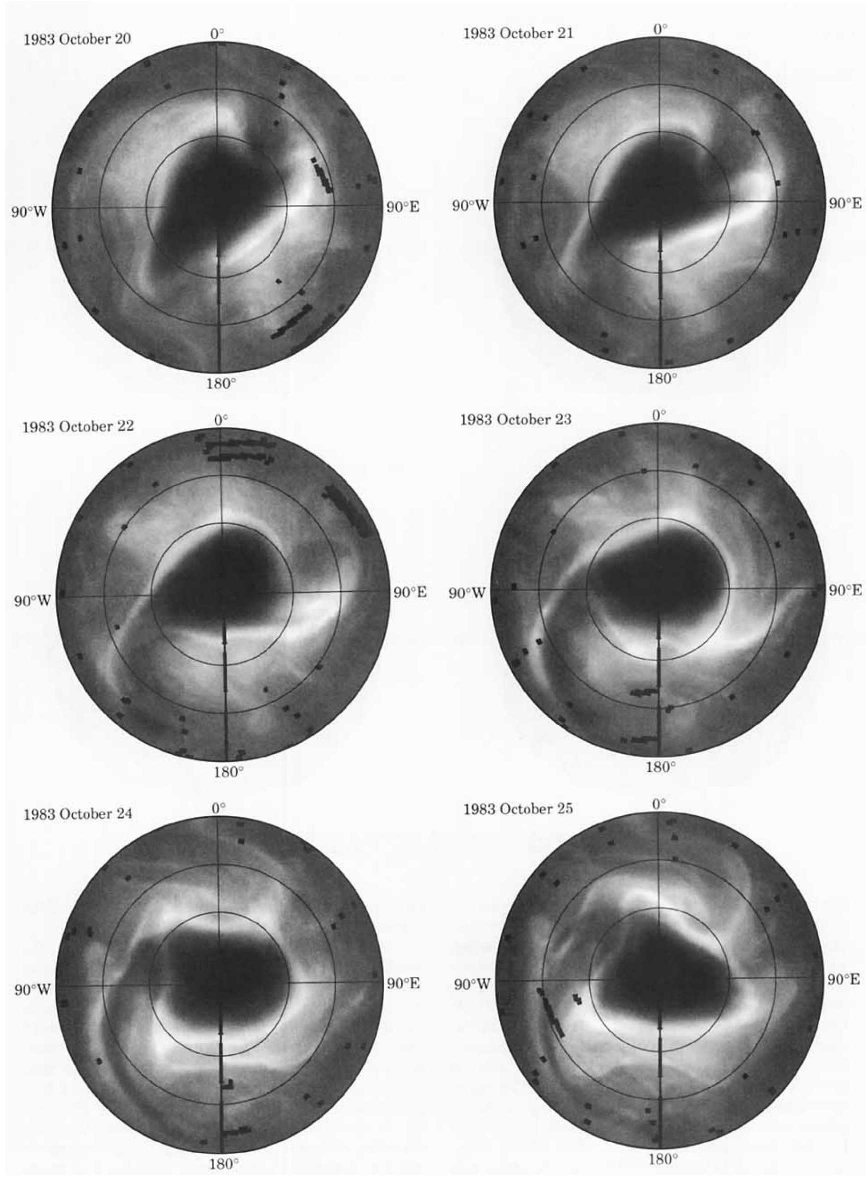
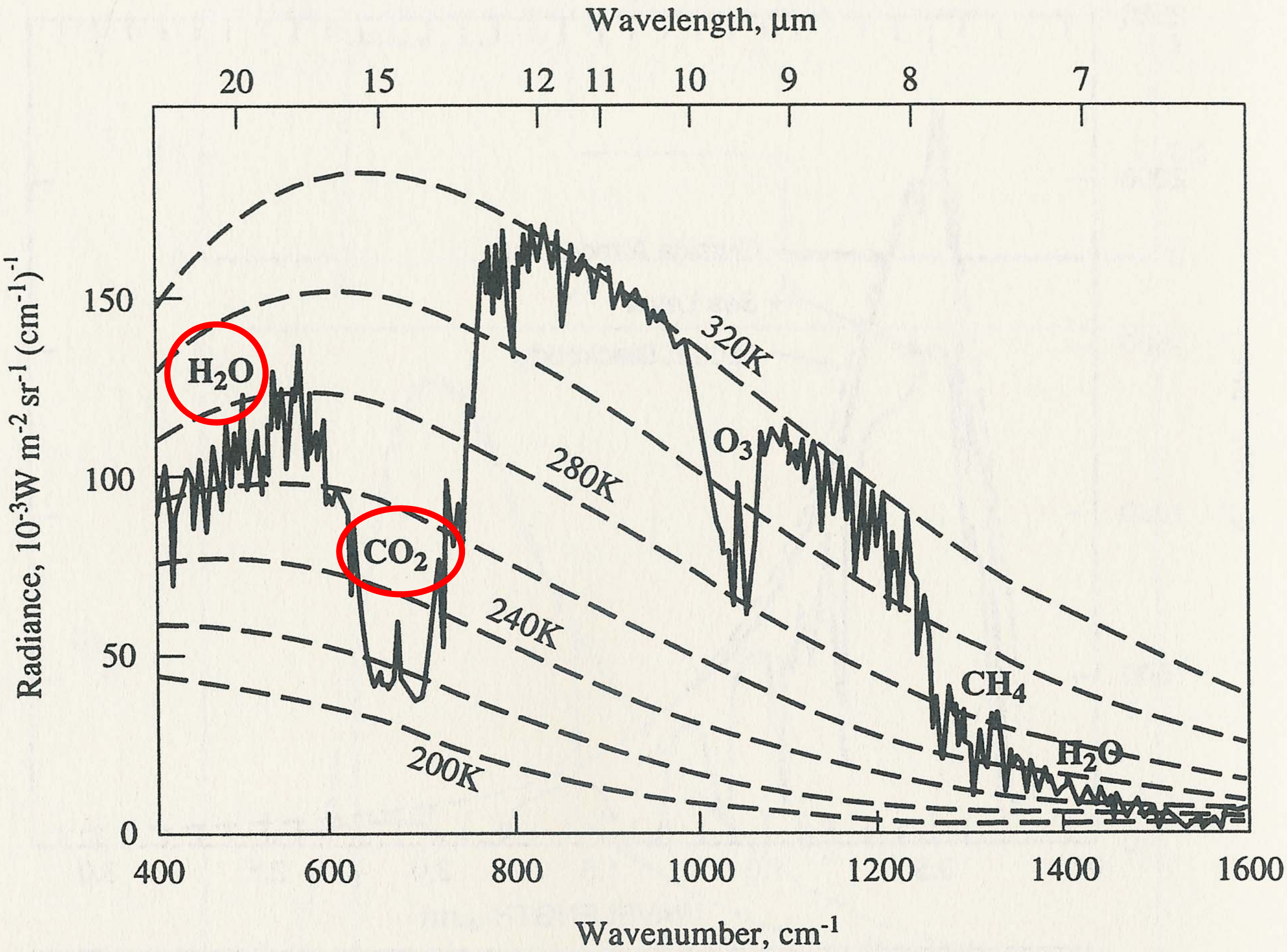
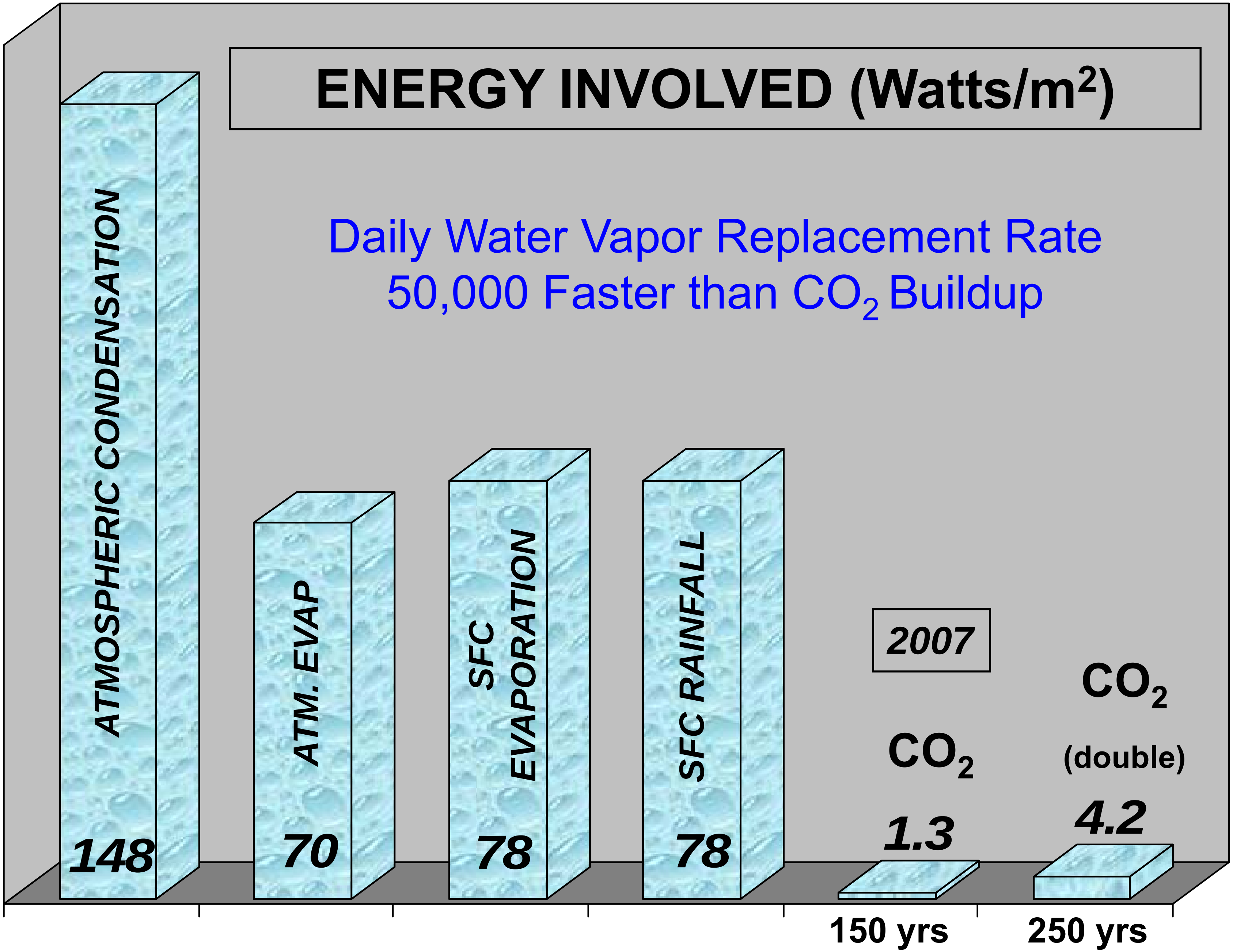


Fig.1: Daily TOMS images of total ozone in the Southern Hemisphere for six consecutive days in October 1983. Latitude circles are drawn at 40°,60°, and 80 °S. The outermost latitude is 20 °S.



Wavelength, μm
20
15
12
11
10
9
8
7
150
100
50
0
Radiance, 10^{-3}W m^{-2} sr^{-1} $(cm^{-1})^{-1}$
H_2O
CO_2
O_3
CH_4
H_2O
320K
280K
240K
200K
400
600
800
1000
1200
1400
1600
Wavenumber, cm^{-1}



ENERGY INVOLVED (Watts/m²)
Daily Water Vapor Replacement Rate
50,000 Faster than CO2 Buildup
ATMOSPHERIC CONDENSATION
148
ATM. EVAP
70
SFC EVAPORATION
78
SFC RAINFALL
78
2007
CO2
1.3
150 yrs
CO2
(double)
4.2
250 yrs

Henri Poincaré

(1854～1912)

French
Mathematician & Physicist

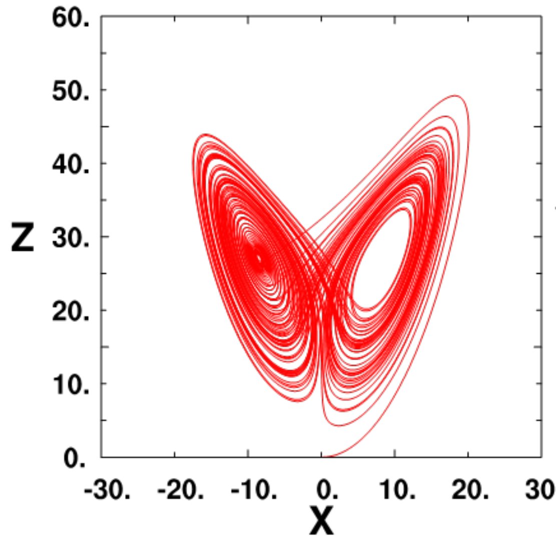
Chaotic deterministic system

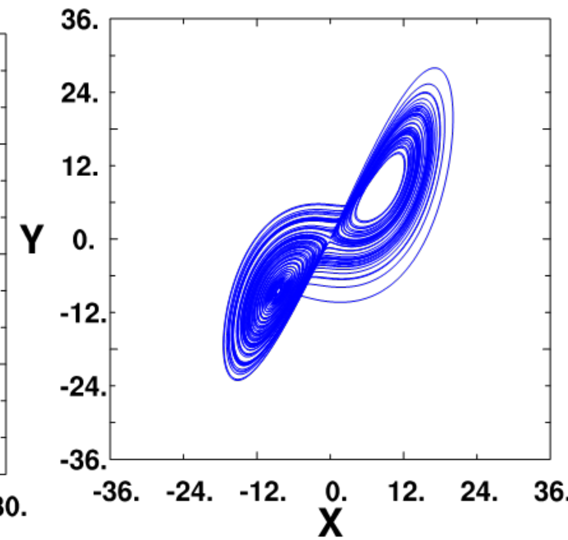
最早探索非線性混沌動力系統，以相位圖展示動力系統。

Science and Hypothesis 書

Edward Norton Lorenz (1917～2008)

American Mathematician & Meterologist

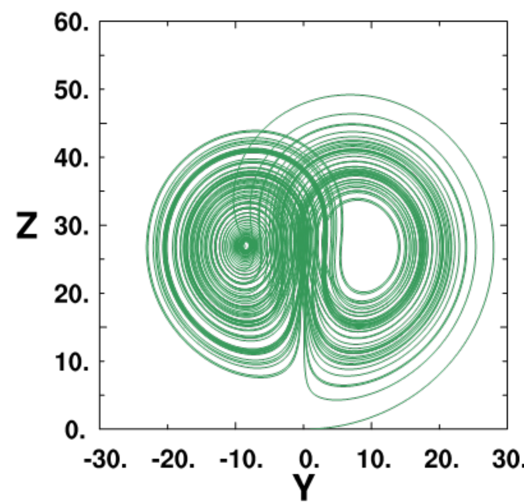




$$\frac{dX}{dt} = -\sigma X + \sigma Y$$

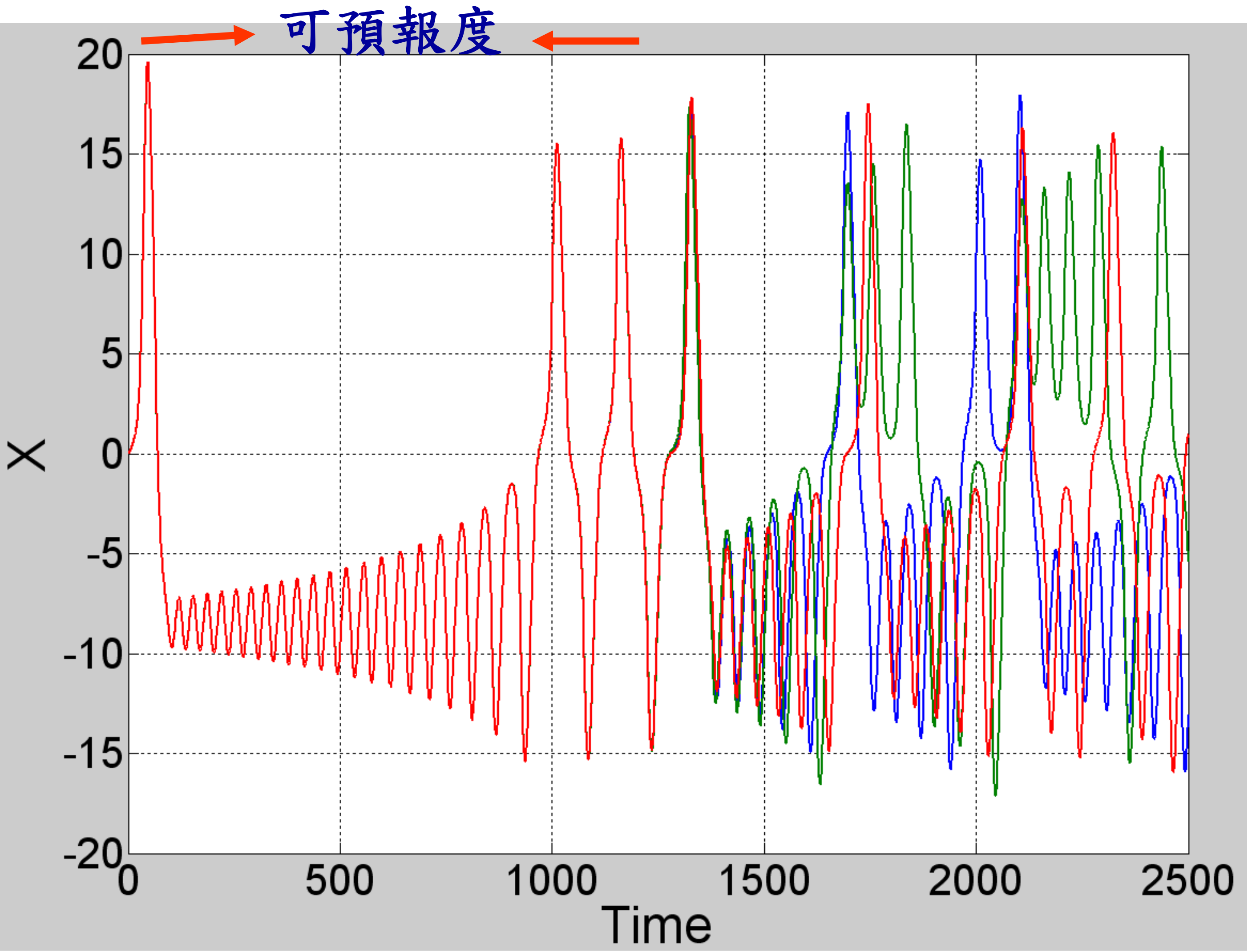
$$\frac{dY}{dt} = -XZ + rX - Y$$

$$\frac{dZ}{dt} = XY - bZ$$



$$\sigma = 10,\ r = 28,\ b = \frac{8}{3}$$

Time Series – Initial Condition Slightly Difference



(x,y,z)=(0,1,1)

(x,y,z)=(0,1.001,1)

(x,y,z)=(0,0.999,1)

蝴蝶效應 Butterfly Effect Chaos 混沌

混沌 非線性 $y = x^{30}$ 精確度有限

非線性

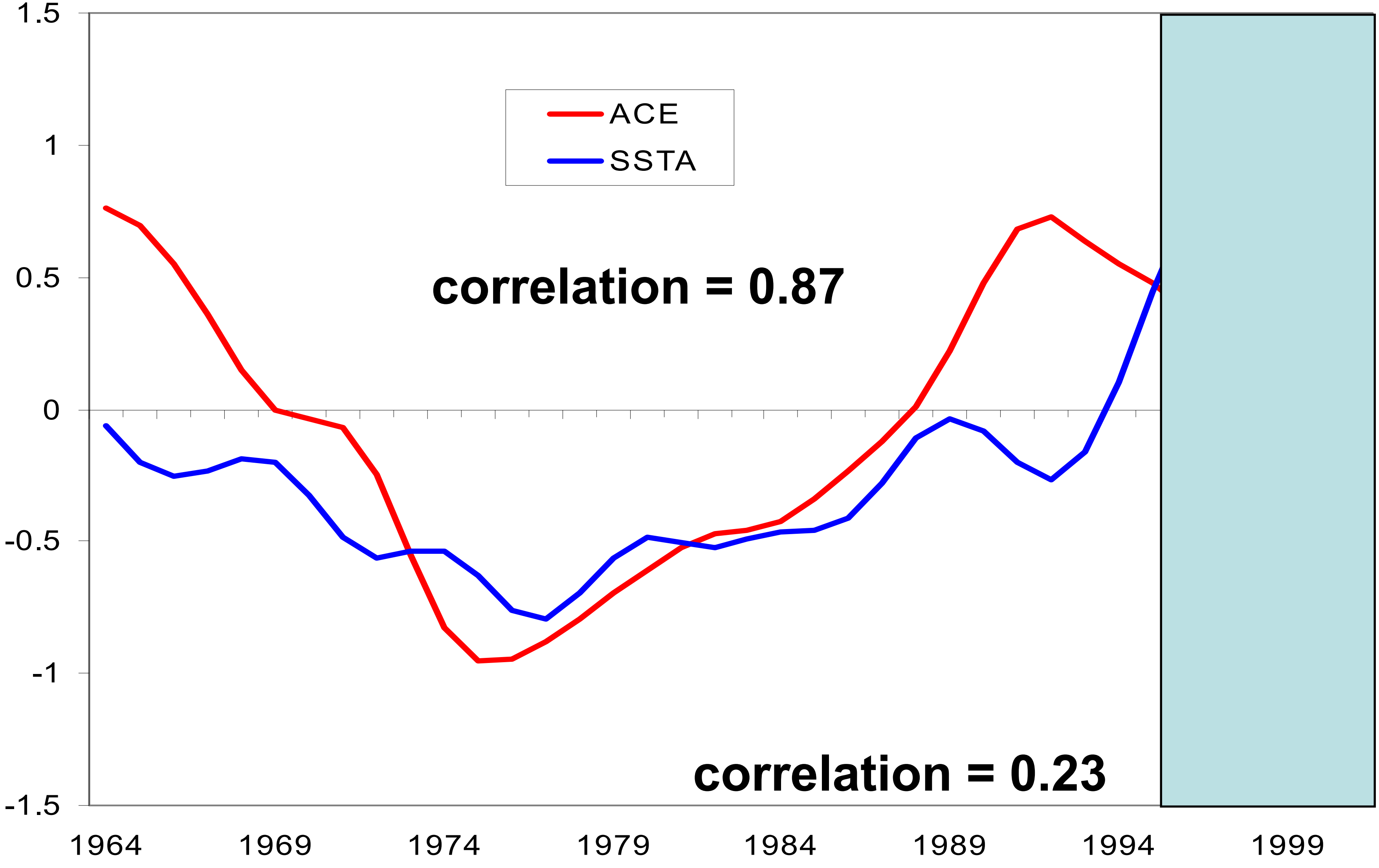
$$0.02 \begin{cases} x = 0.99 \\ x = 1.01 \end{cases} \qquad 0.61 \begin{cases} y \approx 0.74 \\ y \approx 1.35 \end{cases}$$

預報能力的喪失！！

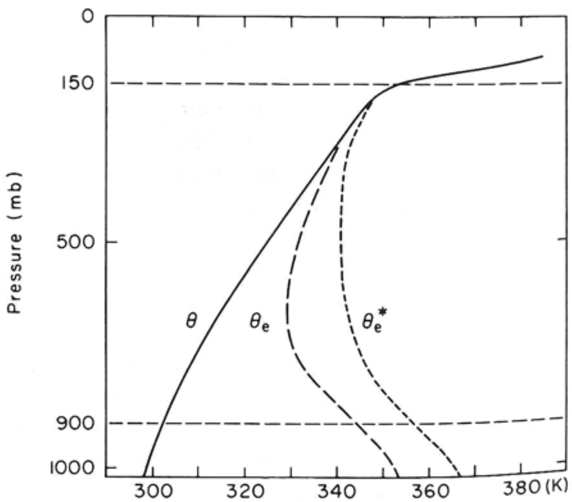
"Sensitivity dependence on initial condition."

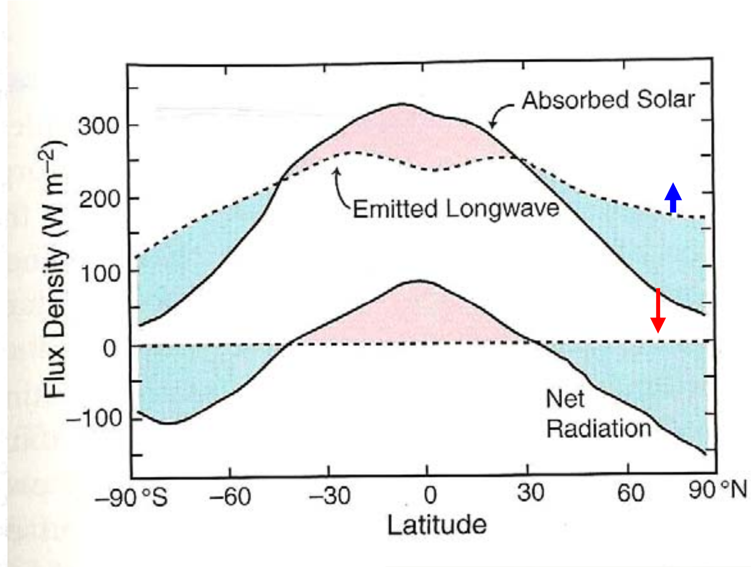
H Poincare

ACE vs May-Nov SSTA (5-30°N, 120-180°E)
[10-year Gaussian-filtered; standardized]



Conditional unstable atm.

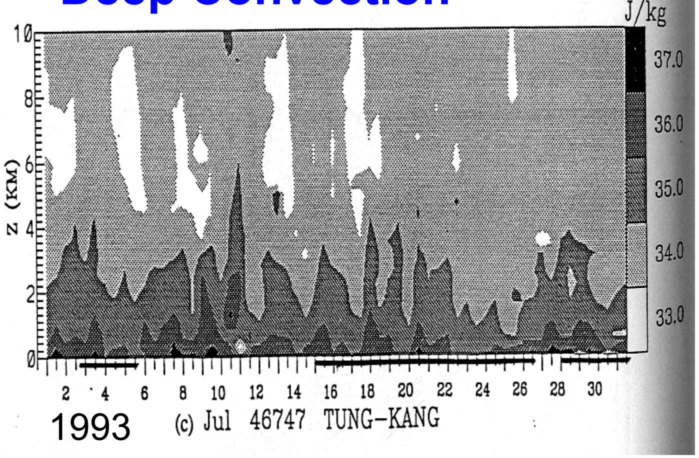




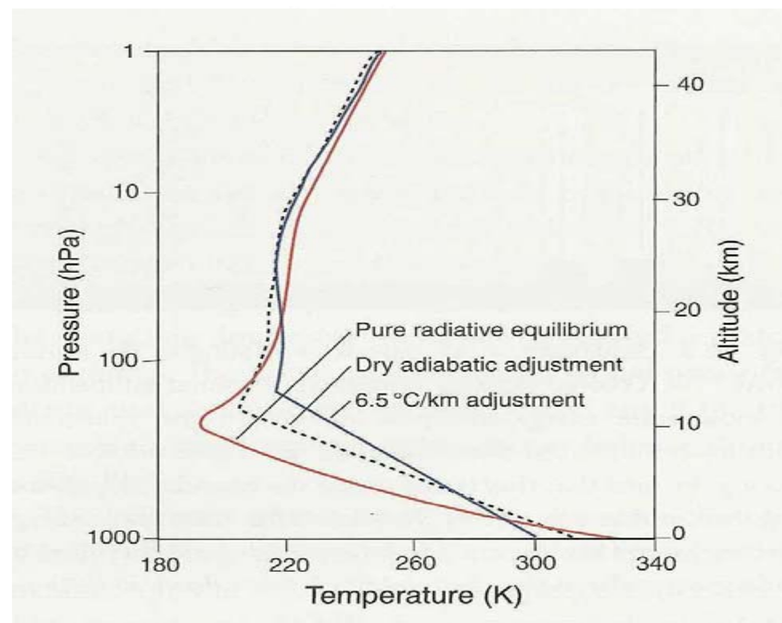
Solar + IR
(Green house gas)

Finite amplitude forcing
Rooted in BL
High h* for instability

Deep Convection



東港 7月



Radiative Convective Adjustment
水文循環

1K/day cooling
1m/year precipitation