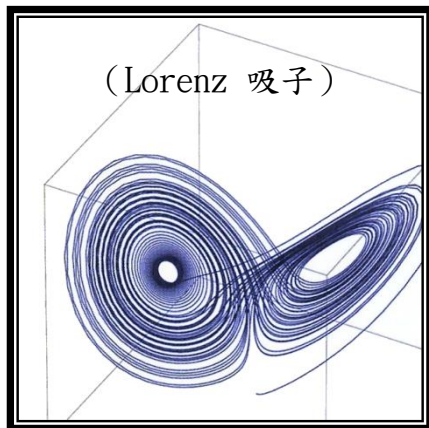
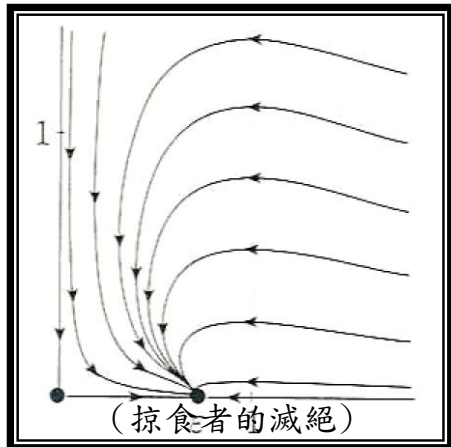


數學可以大不同

輕鬆看數學建模

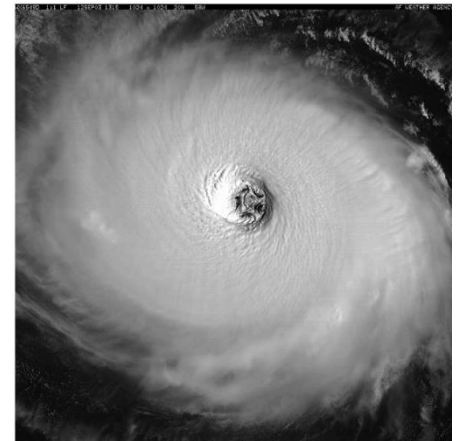
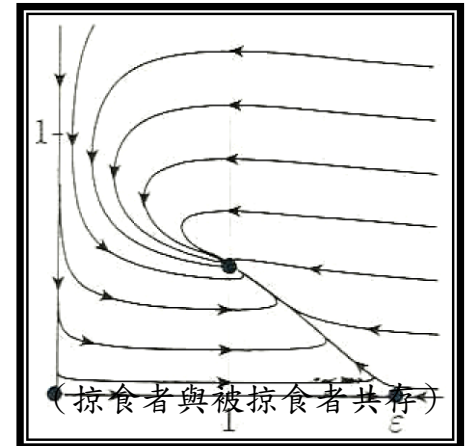


郭鴻基 教授

03/17/2019

南部高中數學科學
人才培育計畫

第11屆教育部國家講座教授
臺灣大學講座教授



科學理性與方法 ○ ○ 3/17上的課程題目

數學思維 ○

遲滯與同步化 ○

跨尺度相似

千變萬化-變動不居 多尺度變化

指數與週期函數

無限大與零共舞 ○

面對時代挑戰-複雜系統

認識時代：

數位資訊科技

Information tech + digital

全球化與文化衝突

Globalization and Clash of Civilization

亞洲崛起

Asia Emerging

環境變遷

Climate Change

Scientific Rationalism

"regards reason as the chief source and test of knowledge" or "any view appealing to reason as a source of knowledge or justification".

More formally, rationalism is defined as a **methodology** or a **theory** "in which the criterion of the truth is not sensory but intellectual and deductive"

Artificial Intelligence

AlphaGo AI 打破人類圍棋 3000 年來的盲點

AlphaGo 被 100:0 完敗

AlphaGo team 解散

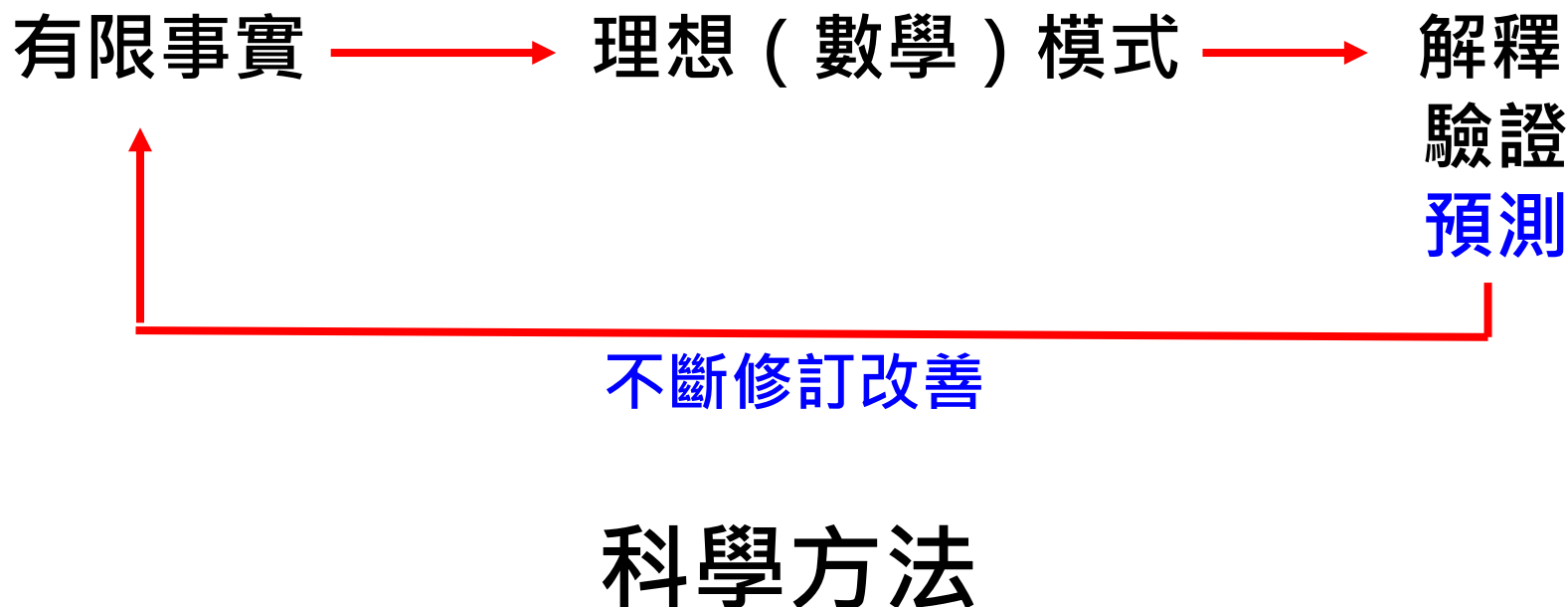


AI 顯學(硬體、數據、演算、domain knowledge)

Not all intelligence is artificial!

以特殊事實為憑藉，逐漸推廣引伸，成立概念式定律的系統，以便籠罩更複雜更廣泛的對象，科學家依據事實為前提來證明普遍的結論。

方東美



False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness. Darwin, The Origin of Man, chapter 6

觀點或可時時更新

資料嚴謹驗證不容妥協

- Ability to identify assumptions 找出假說
- Test Hypothesis 檢驗假說
- Draw relationship between variables 變數關係

• Test Hypothesis

實驗檢驗假說

- (1) 解決**問題**的實驗設計，
- (2) 評估實驗的**數據結果**，
- (3) **解釋**實驗的結果。

Formulation 方程式 模式

Solution Methods 解 計算

Interpretation 詮釋

五個步驟的方法論：

(1) 定義問題

(2) 設計實驗

(3) 執行實驗

(4) 檢驗與解釋實驗結果

(5) 完整立論與外部連結



Sir Isaac Newton
(1642-1727)

Isaac Newton

Principia 1687

**Nature and nature's law
lay hid in night,
God said,
Let Newton be,
and all was light.**

A. Pope

數學思維

Math Thinking

Computational Thinking

Darwin 生在維多利亞時代，
當時人們相信小孩特性完全由父母而來，
(恐怕現在還有許多人也是如是相信。)
一個簡單數學式子 **小屁孩 = (父+母)/2**。
Darwin 不是數學家，據說他數學不好，
但他對於維多利亞的遺傳觀念，
特別憂心和他的物種演化理論衝突，
因物種演化需要大量複雜物種的數目。
終其一生，他無法得到滿意的答案。

這問題直到孟德爾的豌豆實驗才得到解決。

**“Six monkeys, set to strum
unintelligently on typewriters
for millions of years,
Would be bound in time
to write all the books
in the British Museum.”**

Huxley 1860

On June 30, 1860, at the Oxford Union in England, Anglican Archbishop of Oxford University, Samuel Wilberforce, and evolutionist and agnostic Thomas Huxley were engaged in the “Great Debate.” Huxley declared that given enough time all the possible combinations of matter, including those necessary to produce a man, will eventually occur by chance molecular movement.

“Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum.” Huxley 1860

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。

通典

共**49**個字，假設中文常用字為**1000**字，共有 **10^{147}** 個選擇

地球歷史 **10^{18}** sec

10^{10} 一百億隻猴子在打字，假設每秒鐘打一萬字 **10^4** ，

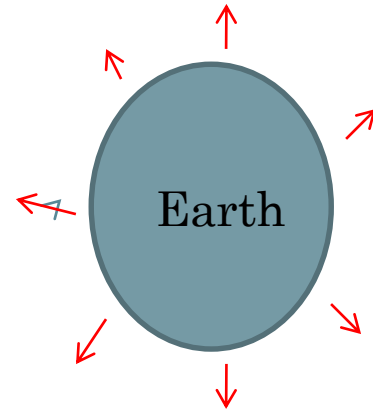
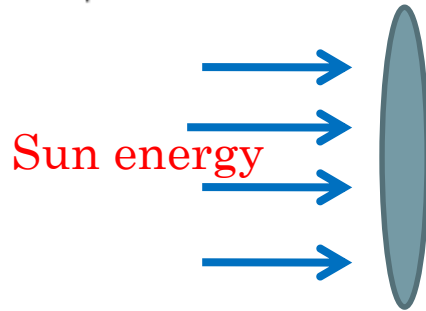
$$10^{10} * 10^{18} * 10^4 = 10^{32}$$

$10^{32} / 10^{147} = 10^{(-115)} \sim 0$ 機率為零，不可能的巧合！

進 出

$$C \frac{dT}{dt} = S \downarrow - IR \uparrow$$

比熱*質量



$$S \downarrow = \pi a^2 s(1 - \alpha)$$

$$IR \uparrow = 4\pi a^2 \epsilon \sigma T^4$$

反照率 albedo

比熱 * 質量

海水 深層海水

反照率

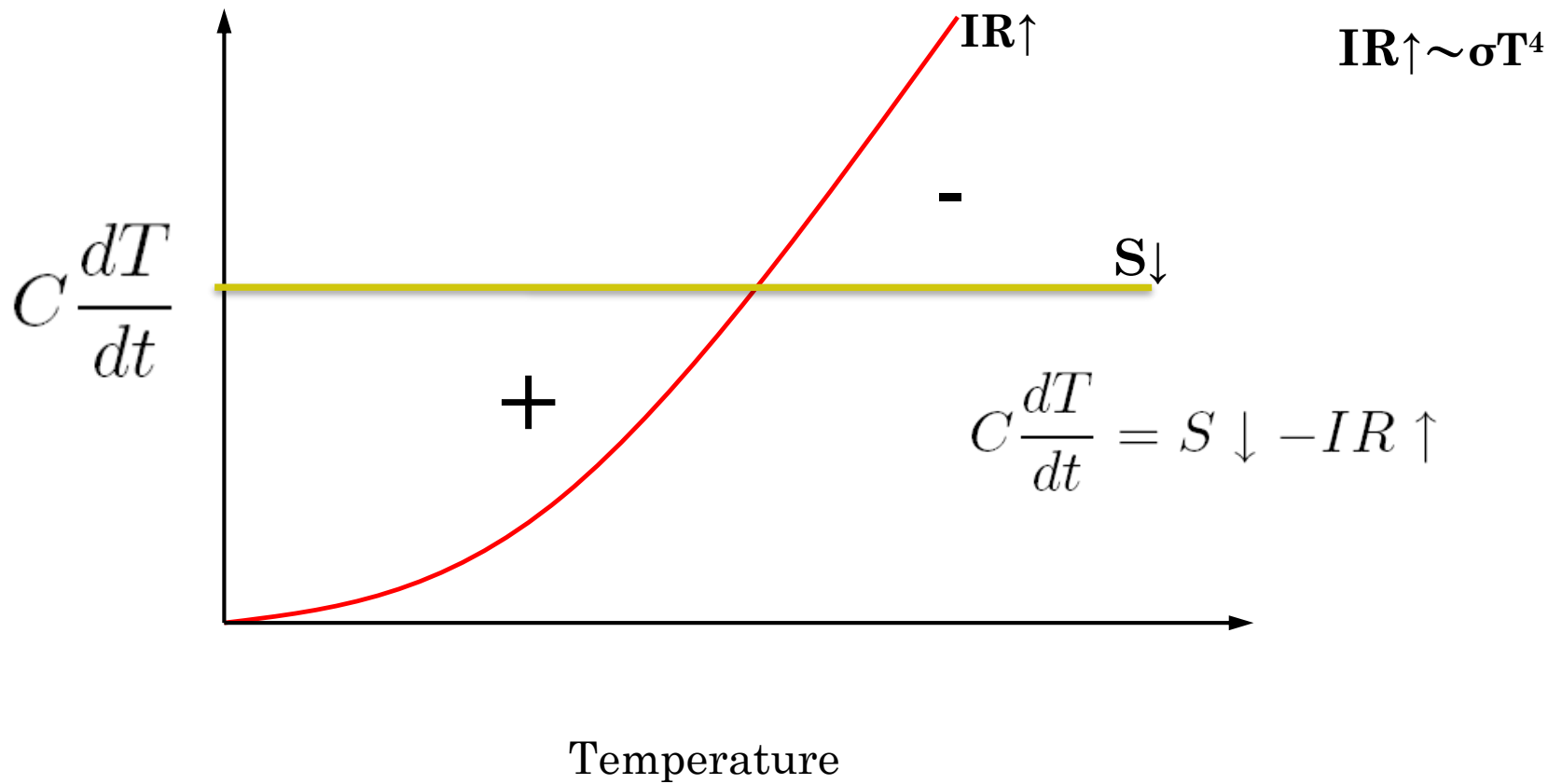
冰雪

雲 (IPCC沒討論的因素，氣象最大的挑戰)

太陽常數

天文因素

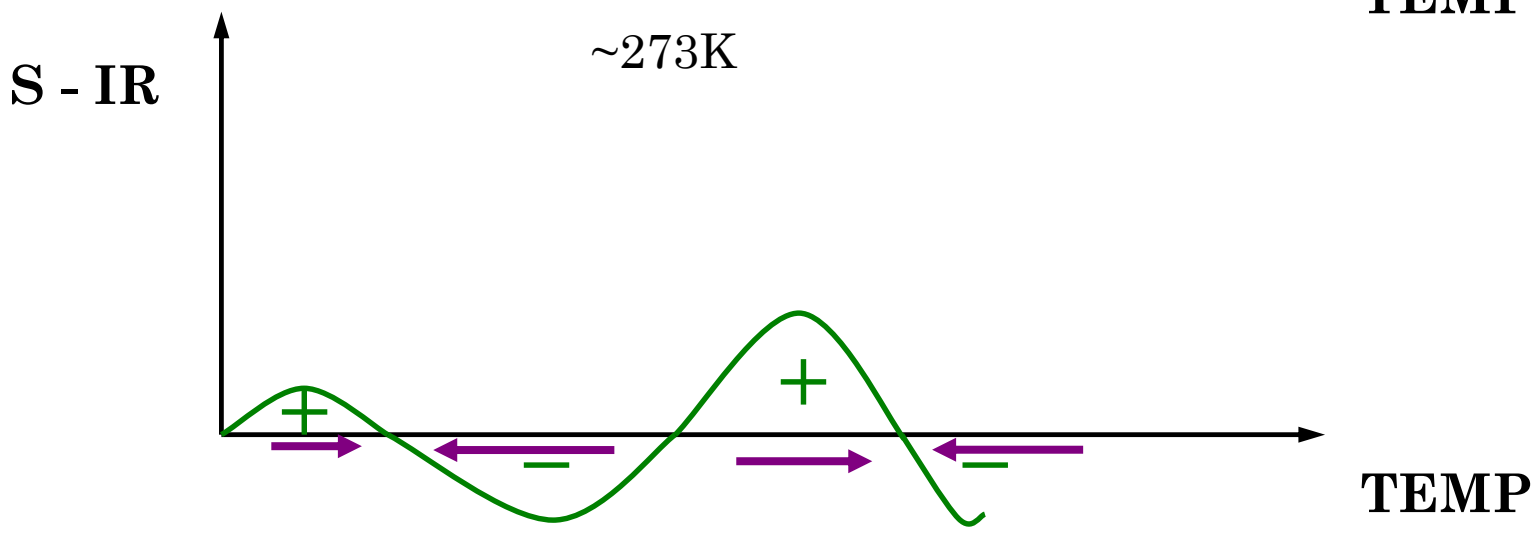
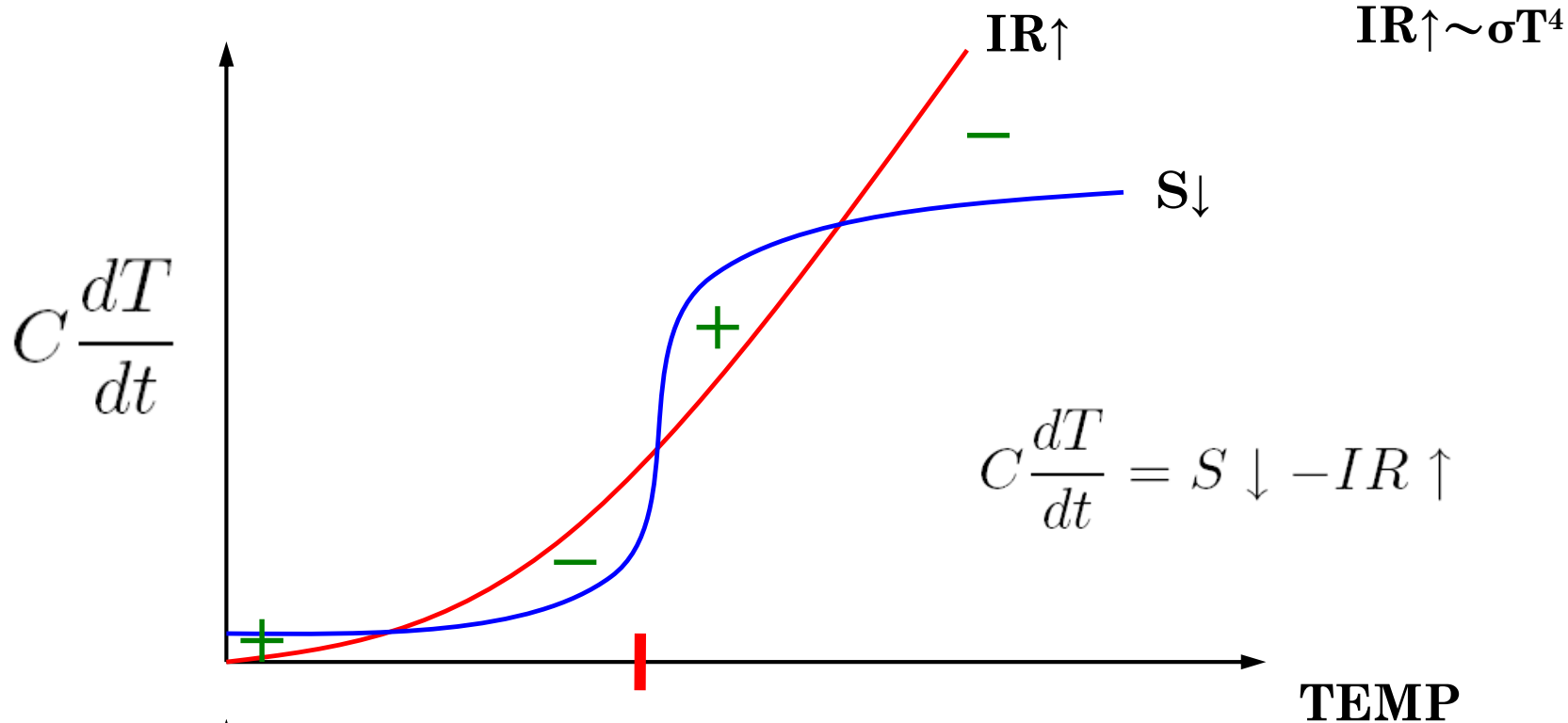
太陽物理



Don't Worry! Be Happy!

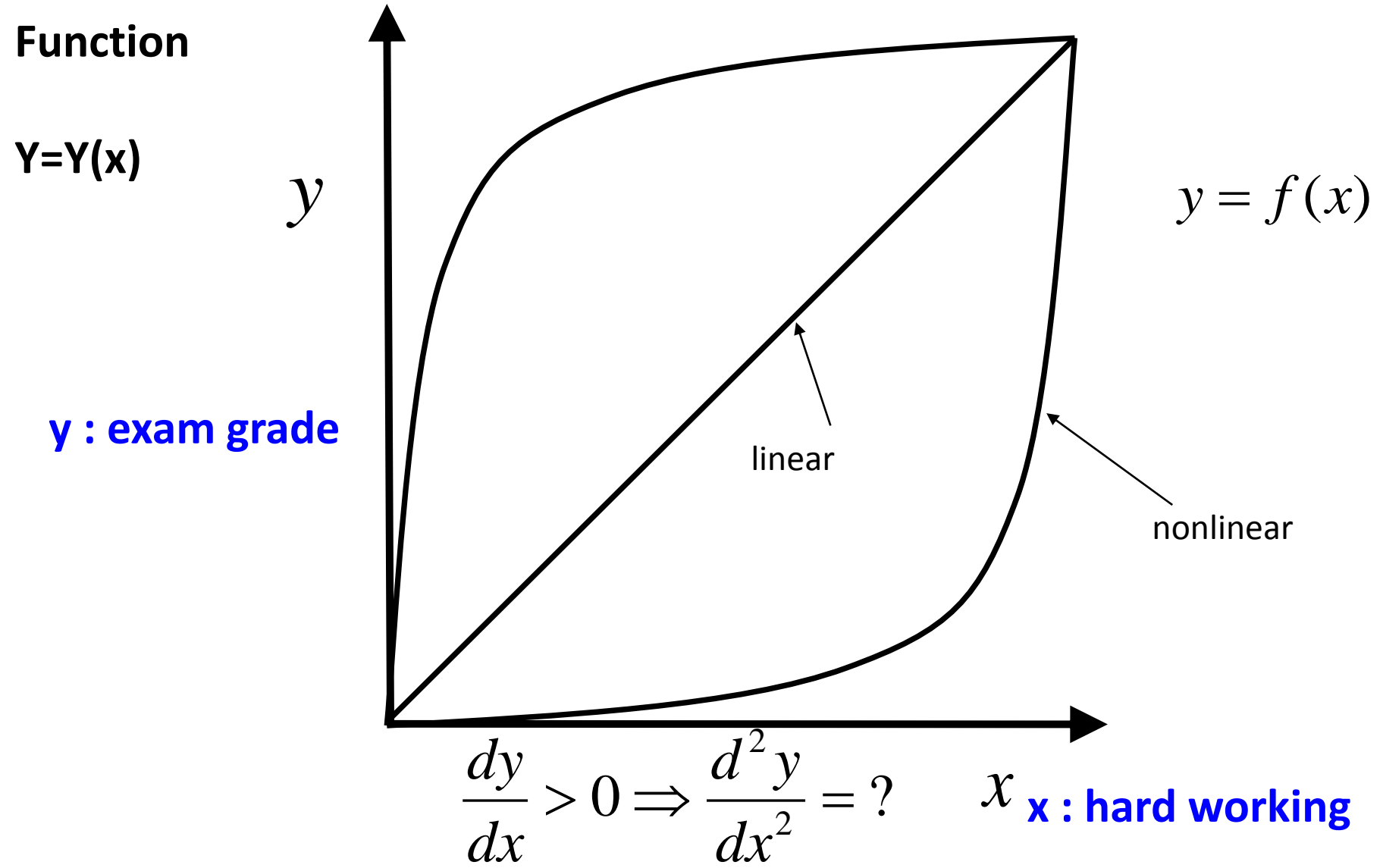
天行有常 不為堯存 不為桀亡
荀子

接收之太陽能量只和太陽有關
和地球溫度無關



2 stable multiple equilibria

“whatsoever a man soweth,
that he shall also reap”



世紀之謎？1715-2010年

Bode's Law of Astronomy 1778

0 3 6 12 24 48 96 192 384

4 7 10 16 28 52 100 196 388

0.4 0.7 1.0 1.6 2.8 5.2 10 19.6 38.8

Mercury	0.4	(0.39)	Venus	0.7	(0.72)	Earth	1.0	(1.0)
Mars	1.6	(1.52)	Asteroids	2.8	(2.77)	Jupiter	5.2	(5.2)
Saturn	10	(9.54)	Uranus	19.6	(19.19)	Neptune	38.8	(30.07)
Pluto	fails	(39.60)						

“Plutoed” To demote or devalue someone or something.

American Dialect Society, word of the year 2007



PLANET PLUTO

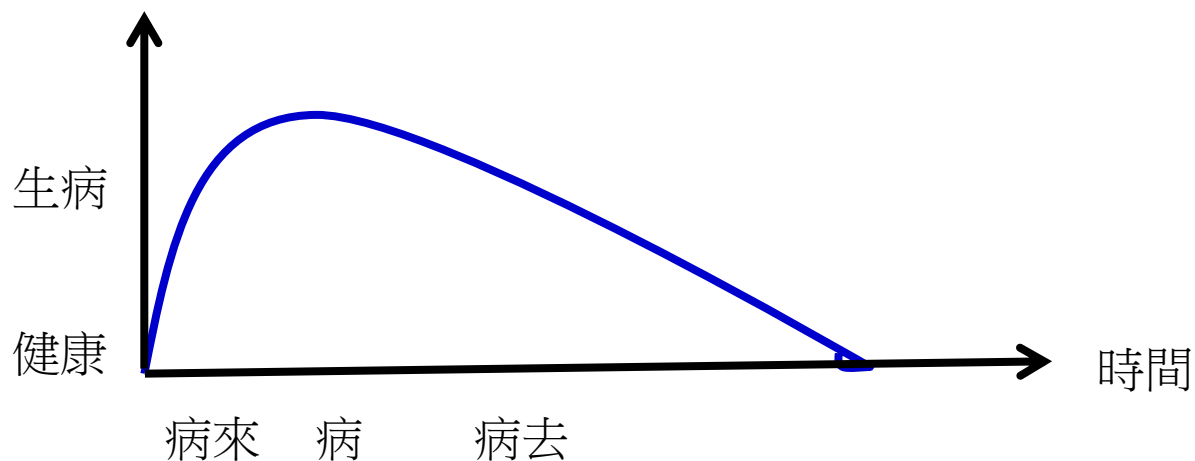
FEB 1930 - AUG 2006

Hysteresis

遲滯反應

同步化

病來如山倒 病去如抽絲

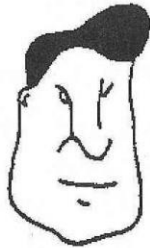


來去時間之前後，病是時間參考點

1



2



3



4



5



6



7



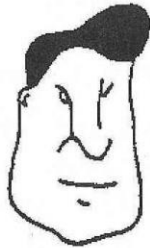
8



1



2



3



4



5



6



7



8



Number of Students Perceiving

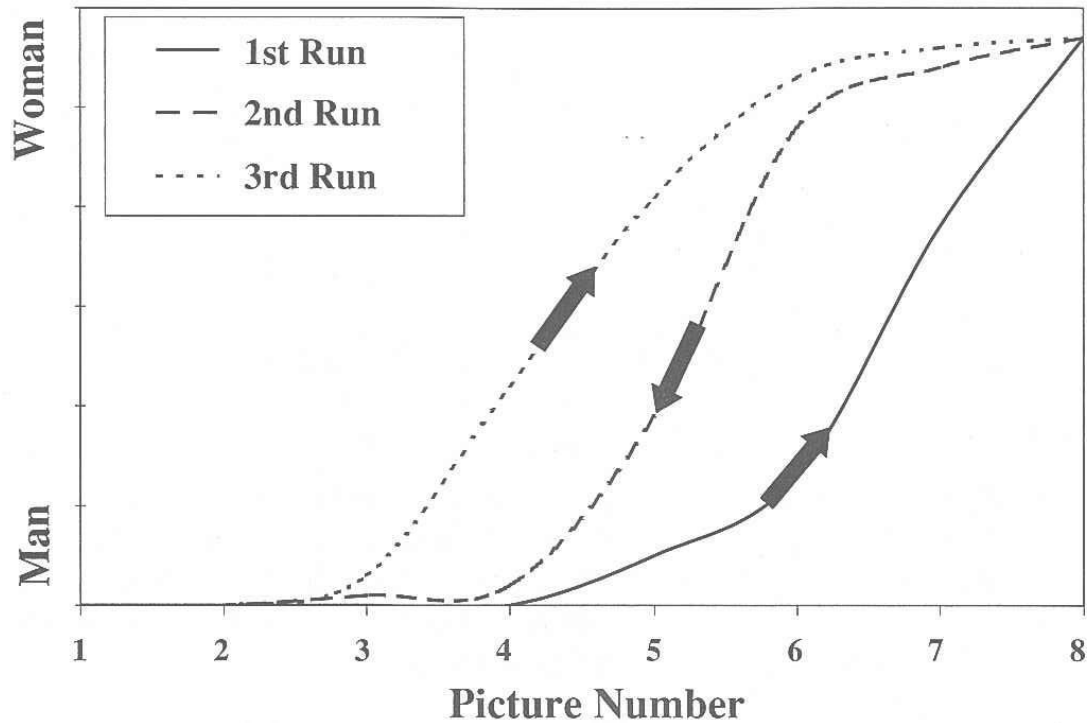
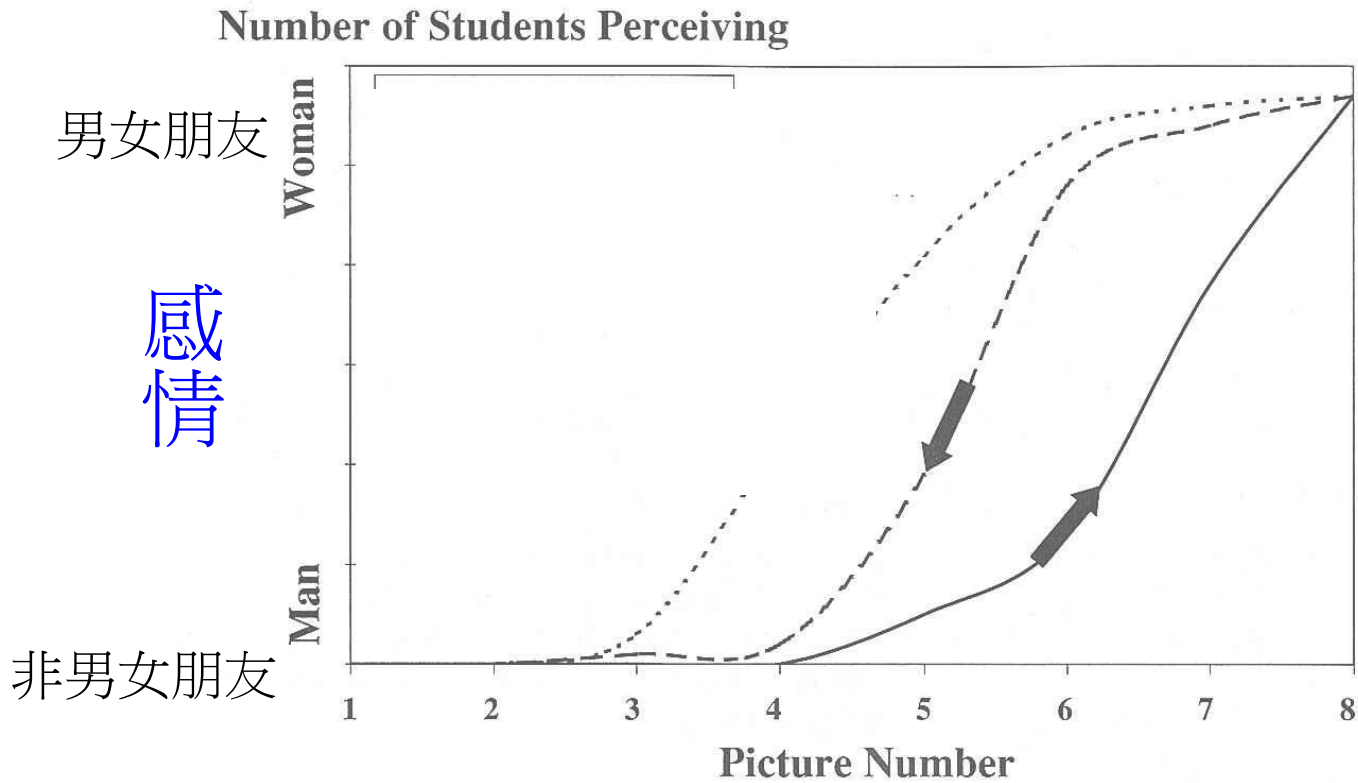


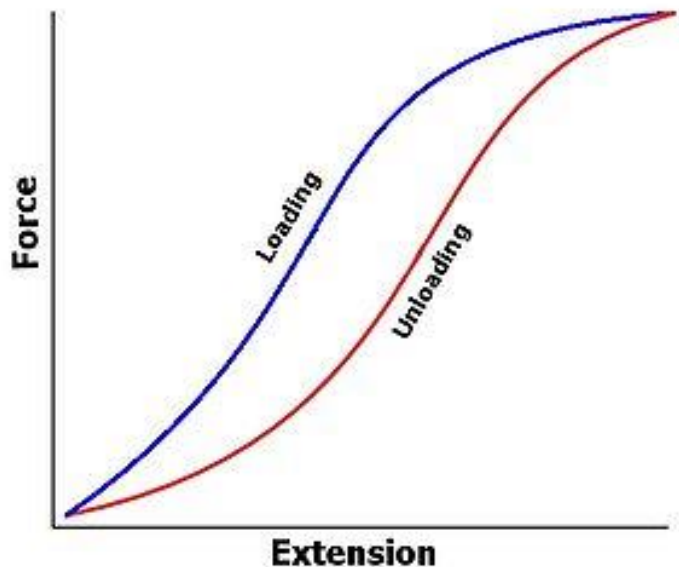
Figure 6.8: Schematic representation of the visual catastrophe based on the data in table 6.1 on three runs (1234567876543212345678) through the series of pictures in figure 6.7. On the vertical axis is plotted the number of students perceiving a change in their perception. At the level “man,” all of the students perceive a man, and at the level “woman,” all of the students perceive a woman. On the first run (solid line), all of the students see a man’s face until the fifth picture, where a few students suddenly see a woman. The biggest jump occurs at the seventh picture. By the eighth picture, all of the students are seeing a seated woman. On the second run, most of the students continue to see a woman until the fifth and fourth pictures are presented, after which point most of the class is seeing a man’s face. A similar pattern appears in the line for the third run, but the shifts in perception do not occur in the same place as they did for the second run. Thus these results are a good illustration of one-time hysteresis.

Hysteresis 遲滯



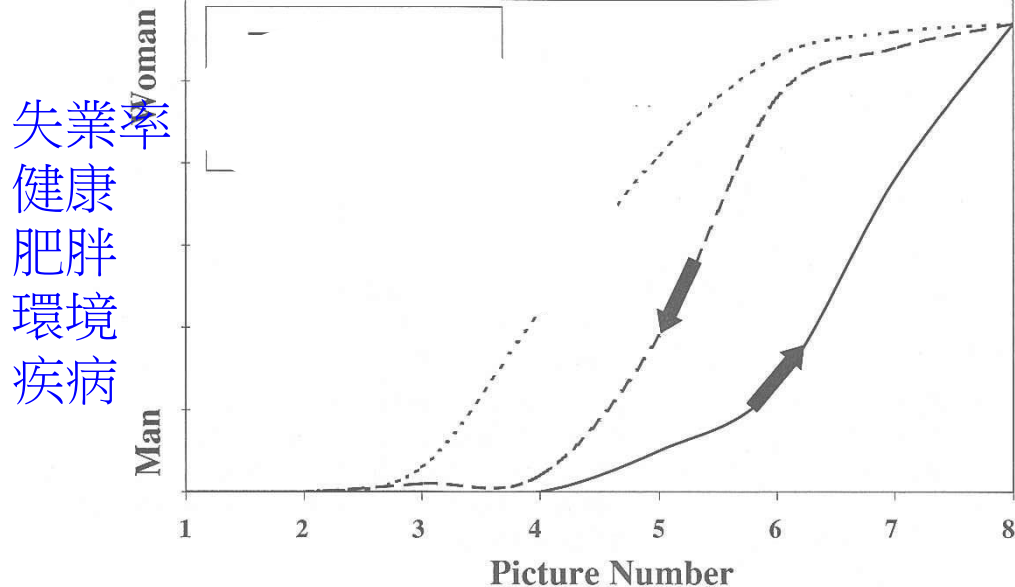
約會頻率

負載與減載的形變不同 防震阻尼



In one sense the rubber band was harder to stretch when it was being loaded than when it was being unloaded.

Number of Students Perceiving



失業率
健康
肥胖
環境
疾病

經濟條件 (通貨膨脹)
身體條件
環境條件
公衛條件

時來天地皆同力
運去英雄不自由

同步化

Pendulum Synchronized 單擺同步擺動

<https://www.youtube.com/watch?v=W1TMZA-SCR-I>

倫敦千禧橋開幕時的擺動

https://www.youtube.com/watch?v=eAXVa__XWZ8

<https://www.youtube.com/watch?v=gQK21572oSU>

London Millennium Bridge



London's Millennium Bridge is the first **pedestrian** river crossing over the Thames in central London for more than a century.

It is a **325m** steel bridge linking the City of London at St. Paul's Cathedral with the Tate Modern Gallery at Bankside.

“Nice” lateral vibrations (**20 cm S shape wobble, 1 cycle per second**) like on Tacoma Bridge developed on the day (June 12, 2000) of the opening.....

無限大與零
的共舞

微積分數學

the Mathematics of Change 變化率

$$u = u(x, y)$$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt}$$

偏微分

$$\frac{\partial x^2 y}{\partial y} = x^2$$

只對y變數微分，不改變x變數

莊子：一尺之錘、日取其半、萬世不絕。

逼近於零不等於零

$$\Delta x \rightarrow 0$$

Rate of Change

$$\lim_{\Delta \rightarrow 0} \frac{f(\Delta)}{g(\Delta)} = \frac{df}{dg}$$

$$\lim_{\epsilon \rightarrow 0} \frac{\sin \epsilon}{\epsilon} = ?$$

一尺之錘、日取其半、
萬世不絕。

莊子

0

$\Delta x \rightarrow 0$ 逼近於零不等於零

$$\lim_{\Delta \rightarrow 0} \frac{f(\Delta)}{g(\Delta)} = \frac{df}{dg}$$

$$\lim_{\epsilon \rightarrow 0} \frac{\sin \epsilon}{\epsilon} = ?$$

面積固定值，
但是總邊長是無限
體積固定值，
但是總表面積無限

有可能嗎？

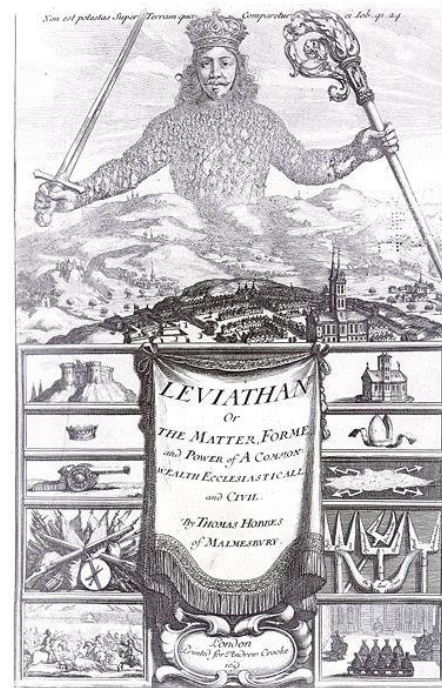
Evangelista Torricelli (1608-1647)

無限面積但有限體積

— $xy = 1$ 對 x 軸旋轉; 類似聖經故事的Gabriel's horn

“**Torricelli's paradox funnel**”

1672, 英國哲學家Thomas Hobbes (霍布斯) 宣稱只有神經病才會相信 Torricelli的無限面積.



霍布斯的政治原則是「不要傷害」，他的道德黃金律是「己所不欲，勿施於人」

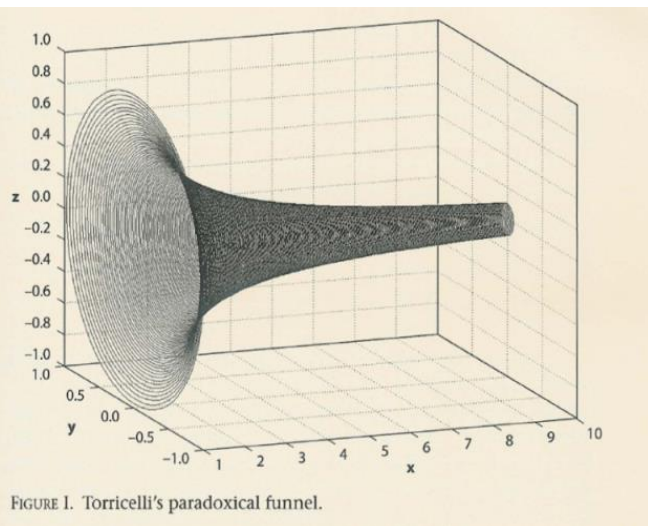


FIGURE I. Torricelli's paradoxical funnel.

$$\Delta V \approx \pi y^2 \Delta x$$

$$V = \int dV = \pi \int_a^\infty y^2 dx$$

$$V = \pi \int_a^\infty \frac{dx}{x^2} = \frac{\pi}{a}$$

$$A = \int_a^\infty y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$A = \int_a^\infty \frac{1}{x} \sqrt{1 + \frac{1}{x^4}} dx = \int_a^\infty \frac{\sqrt{x^4 + 1}}{x^3} dx > \int_a^\infty \frac{\sqrt{x^4}}{x^3} dx = \int_a^\infty \frac{1}{x} dx \sim \infty$$

Richardson is not a great lecturer, never won big award, but he is truly beyond his time!



Richardson's Game Theory 賽局理論

Could an arms-race end without fighting?

“Yes, without a shot being fired,”

Arm Races

if one side outspent the other on armaments and the weaker nation bankrupted itself.”

$$\frac{dx}{dt} = -a(x - x_0) + by$$

Relaxation to basics

$$\frac{dy}{dt} = -a(y - y_0) + bx$$

Positive feedback in arms race

x, y war potential

$1/a < 1/b$ Truce

(Peace?)

短文「軍備競賽可否不產生戰爭？」
此文在Richardson逝世後，
1953年受到重視。

Richardson認為讓雙方加強軍備競賽，
但因雙方都無必勝把握暫時不開戰，
最後沒效率的國家因軍備競賽而經濟破產。

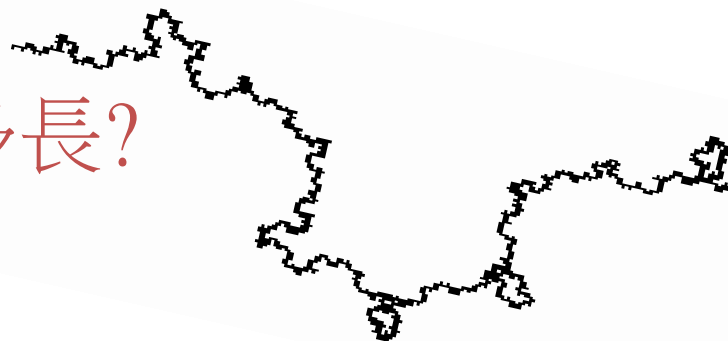
A relation between the probability of two countries going to war and the length of their common border.
戰爭可能性和共同邊界長短成比例。



相同邊界 不同長度

海岸線有多長?

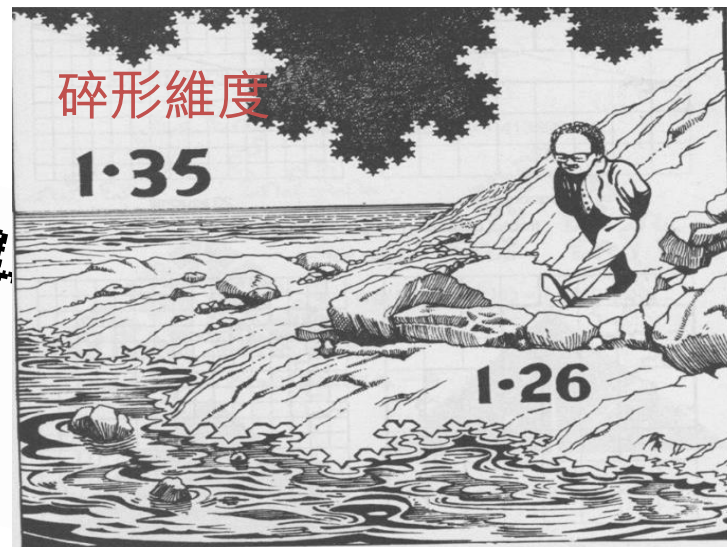
碎形源起



碎形維度

1.35

1.26



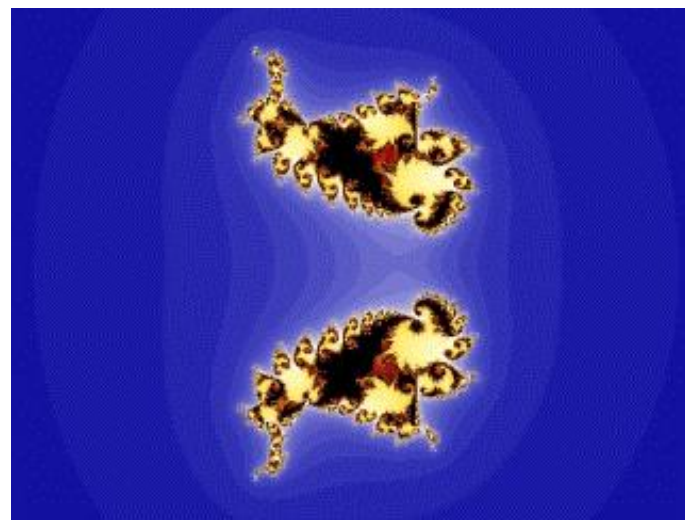
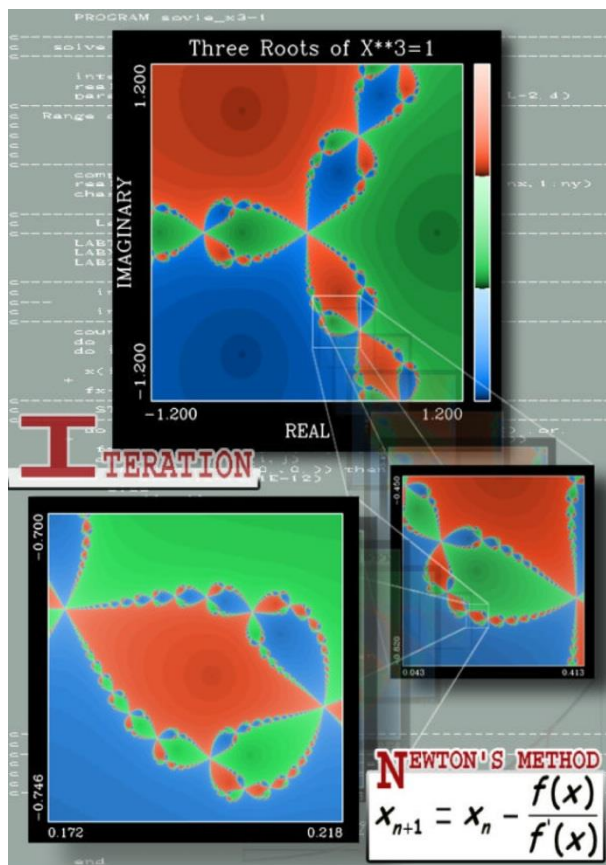
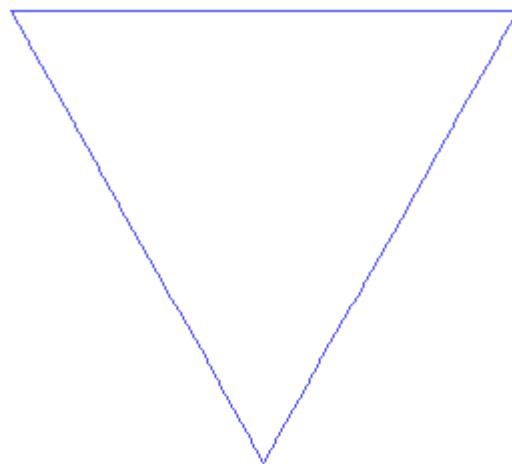
自然的形狀近似於碎形

Fractal 碎形

自我相似性

奇異吸子

碎形維度



The best part of waking up, is to see the vortex in your cup!

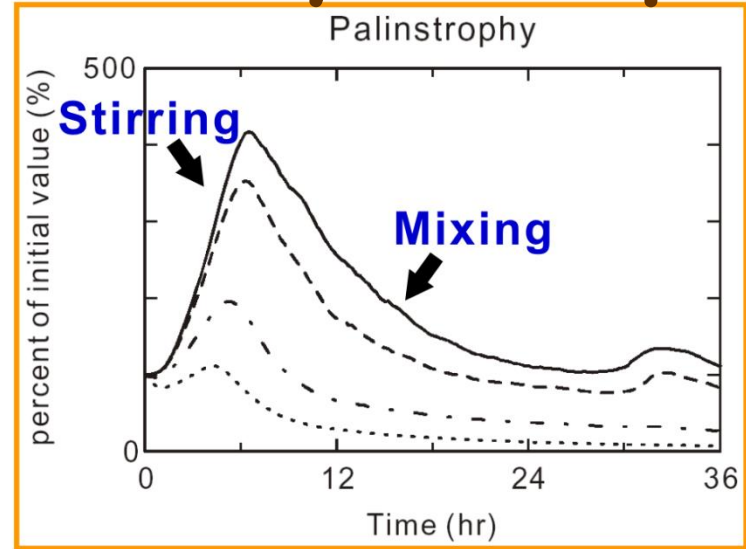
$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V} \cdot \nabla\theta = v\nabla^2\theta$$

$$C = \frac{1}{2} \int \nabla\theta \cdot \nabla\theta \, dV$$

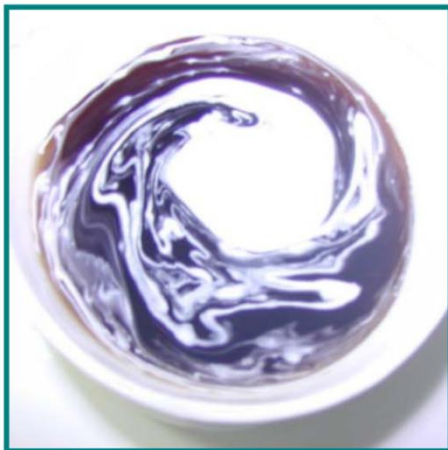
$$\frac{dC}{dt} = \int (\vec{V} \cdot \nabla\theta) \nabla^2\theta \, dV - v \int (\nabla^2\theta) \, dV$$

Stirring

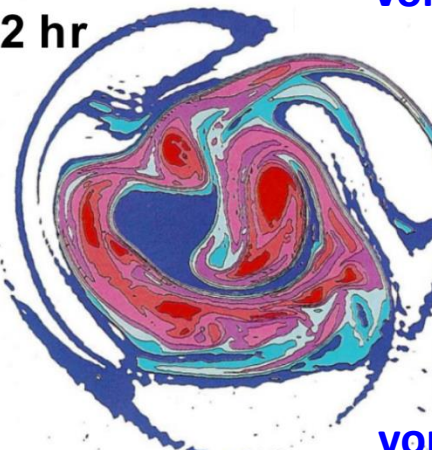
Mixing



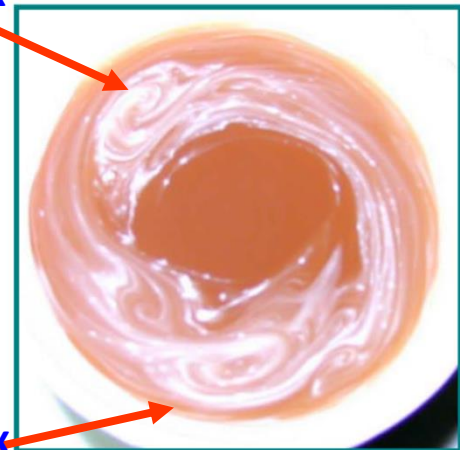
Coffee with white



12 hr



vortex



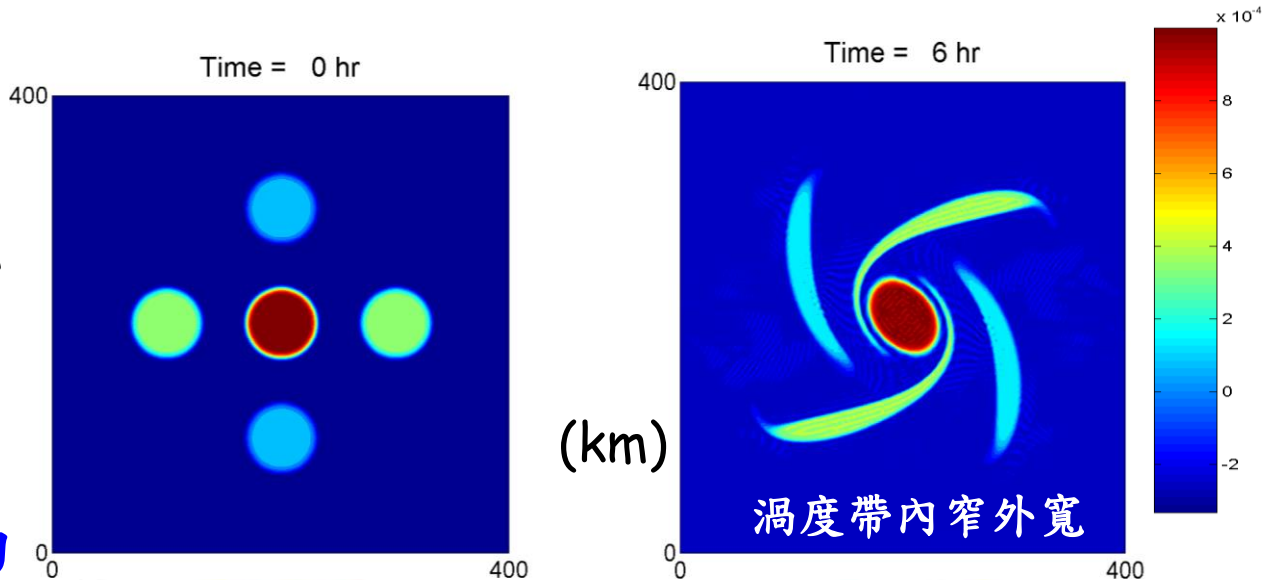
vortex



2012 Alan Berman Research Publication Award Best publication in the Naval Research Lab.

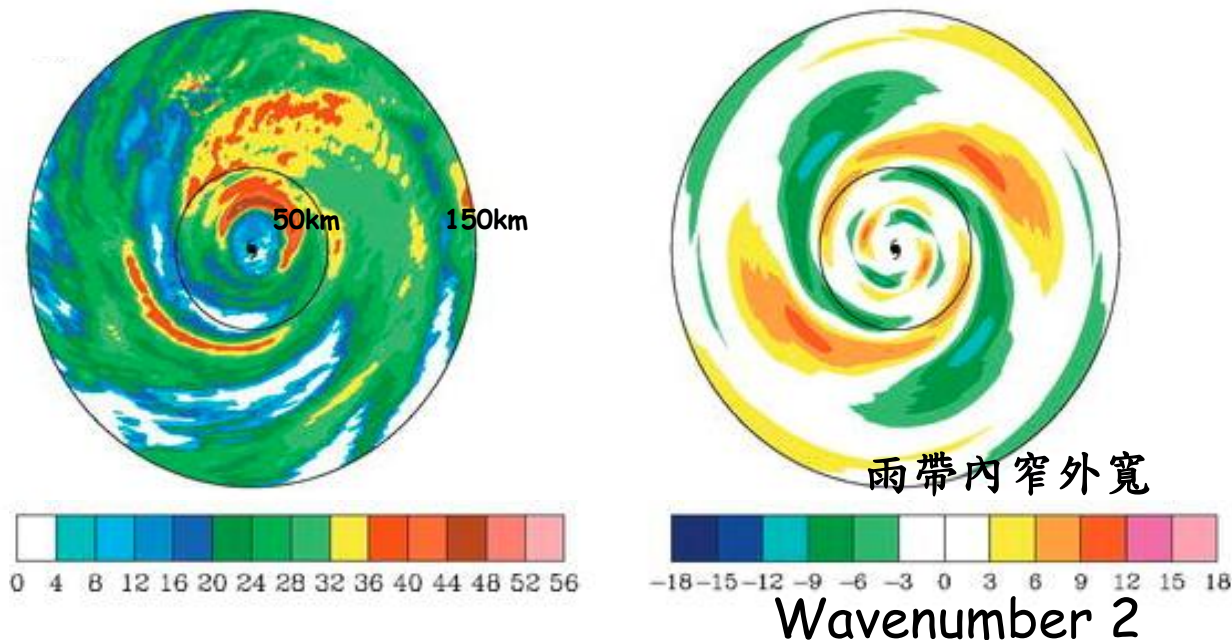
Vorticity Dynamics

颱風渦漩導致雨帶
內窄外寬形變。重要的
雨帶與颱風動力發現。



Radar Reflectivity

Hendricks et al. 2011
Corbosiero et al. 2006



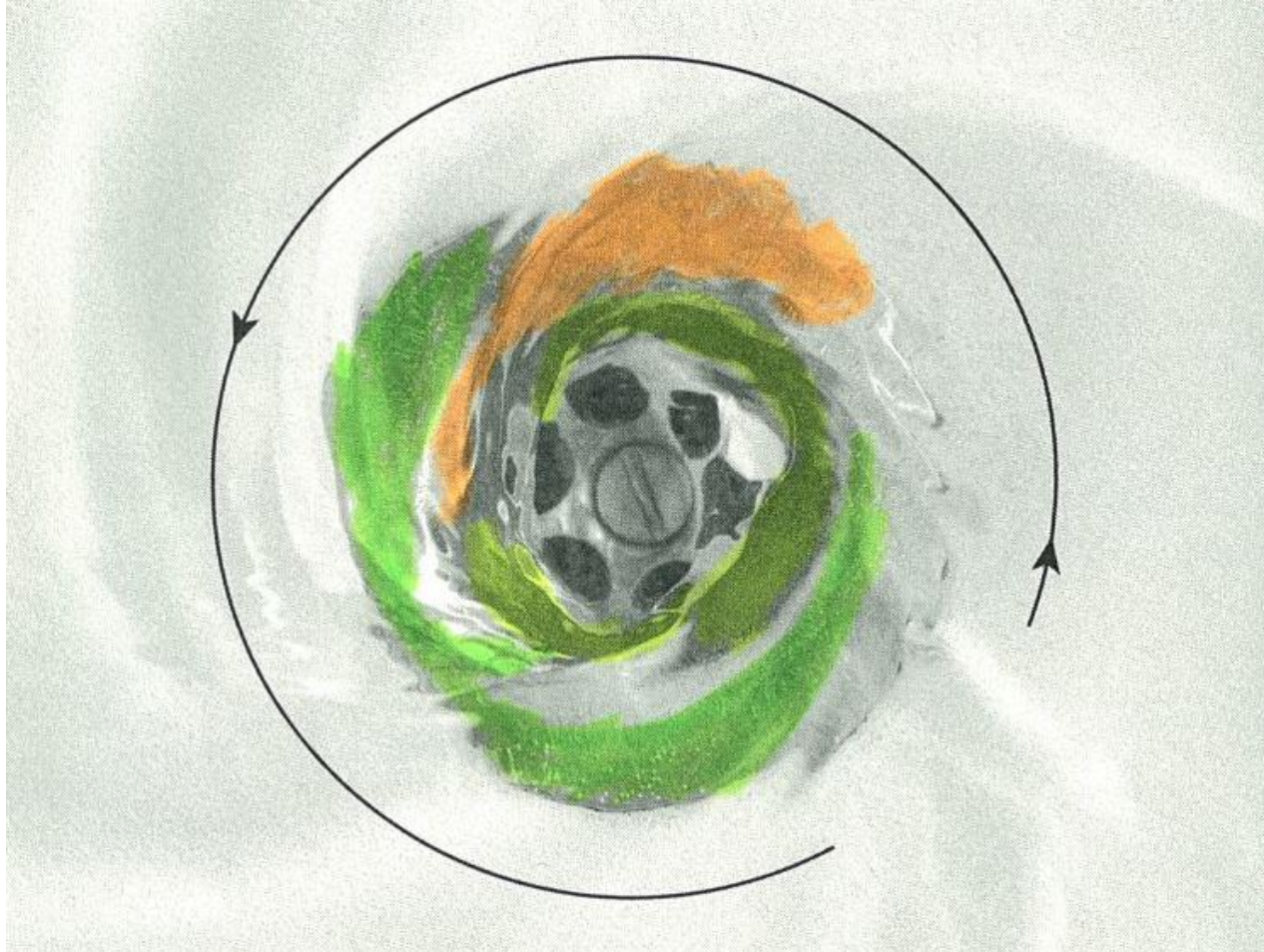


Figure 3.6. Swirling water flows out of a sink or bath by means of a central drain hole. As fluid parcels rotate and approach the drain hole, they speed up to conserve the circulation. © Pavel Losevsky / 123RF.COM.

逼近零 乘以 逼近無限大 = $\begin{cases} \text{零 或 無限大} \\ \text{有限值} \end{cases}$

真實世界中觀察的到的現象

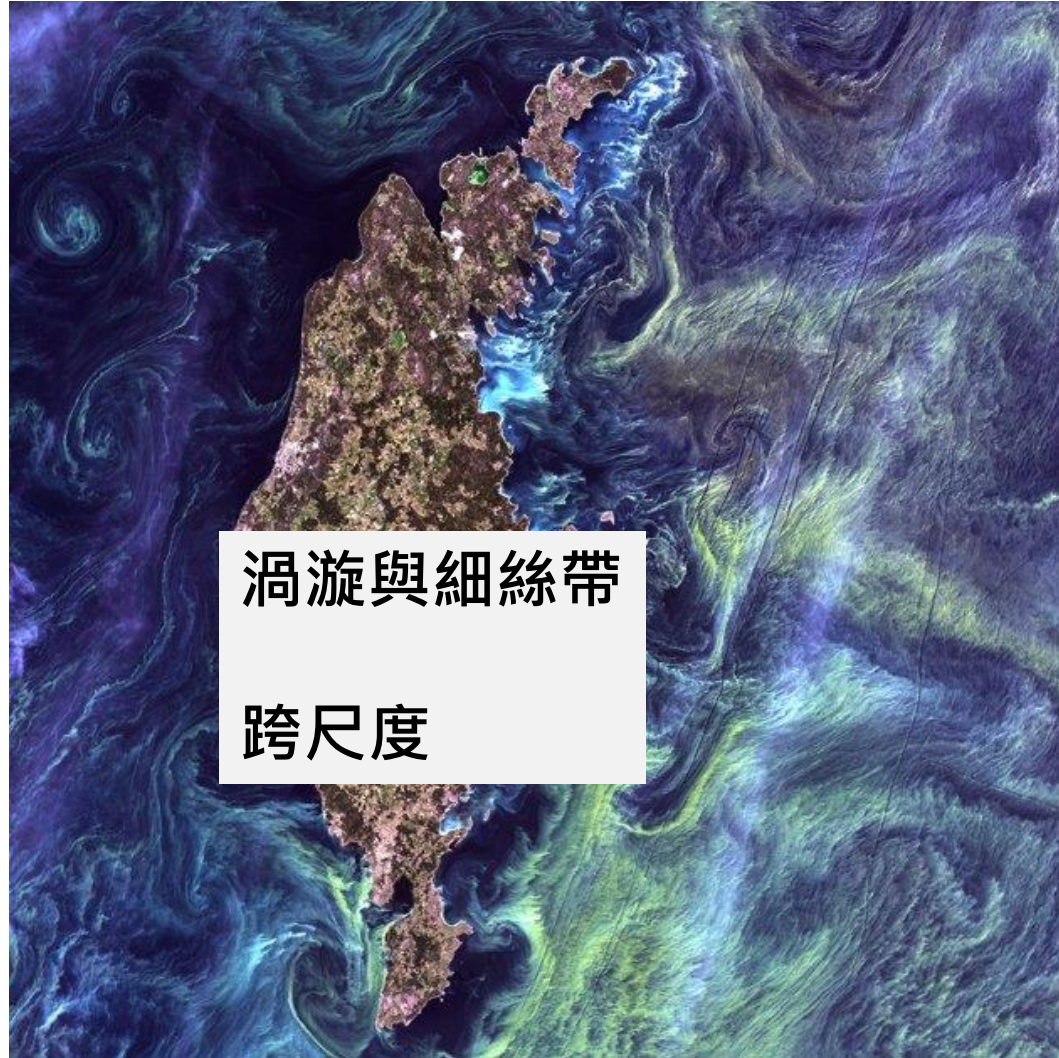
逼近零 乘以 逼近無限大 = 有限值

$0.0001 * 10000 = 1.0$

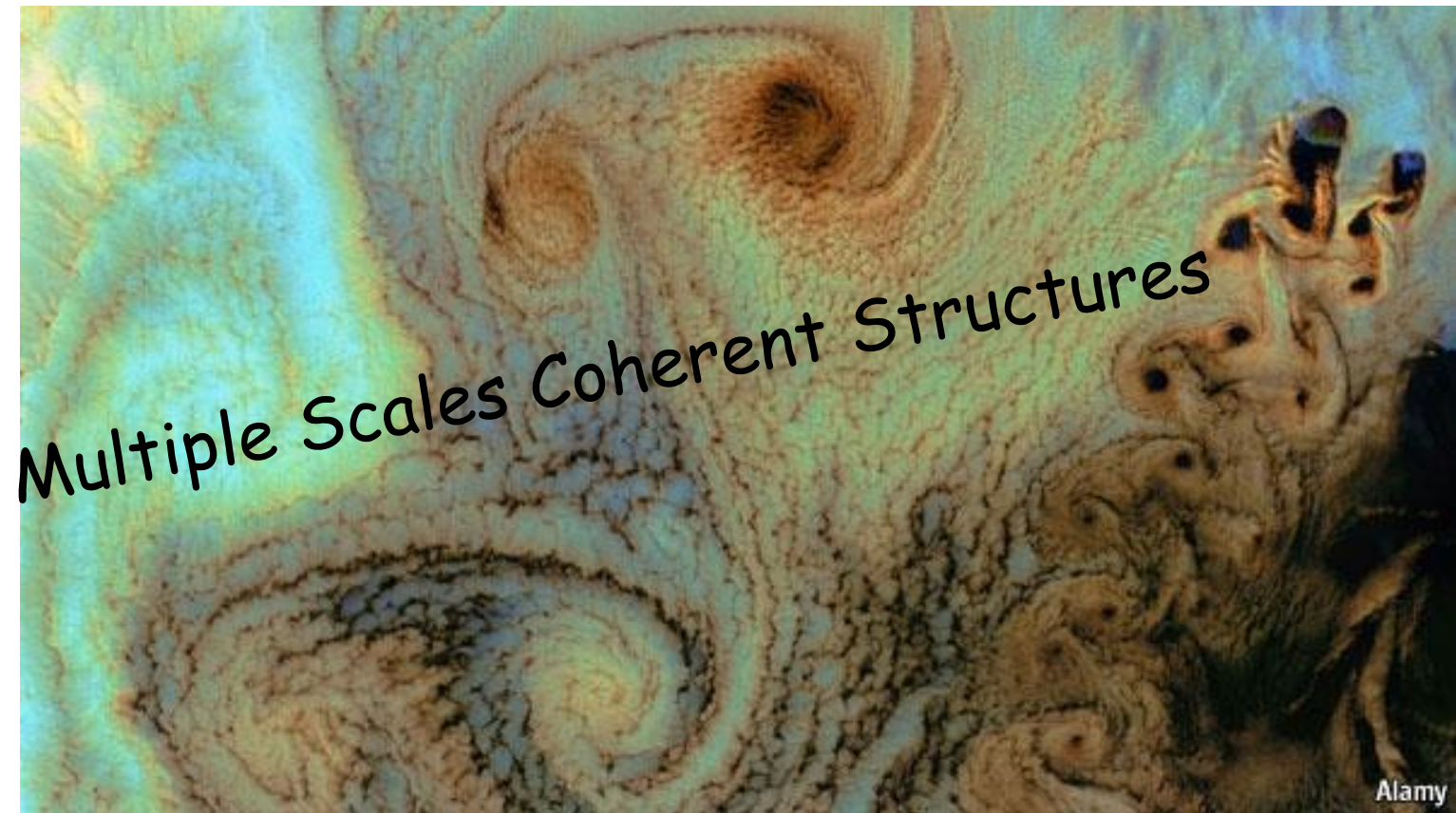
The profound study of nature is the most fertile source of mathematical discoveries. Fourier

Phytoplankton Swirls

Vortex & Filaments



In the style of Van Gogh's painting "Starry Night," massive congregations of greenish phytoplankton swirl in the dark water around Gotland, a Swedish island in the Baltic Sea. Population explosions, or blooms, of phytoplankton, like the one shown here, occur when deep currents bring nutrients up to sunlit surface waters, fueling the growth and reproduction of these tiny plants. (Credit: NASA Goddard Space Flight Center/USGS)



Theory points to three main mechanisms: mixing the ocean's surface layers (up to a few hundred metres) by wind; mixing of deeper layers by ocean currents; and eddies, swirls created when warm ocean currents meet cold ones, blending large swathes of the ocean 10-100km across.

Dr Salée and colleagues report in *Nature Geoscience*, eddies suck up as much carbon as the other two mechanisms do, something most current climate models fail to account for.

Economist, 12/08/2012

Statistical Dynamics 統計力學

Boltzmann mixing entropy

Constrained circulation and
angular momentum

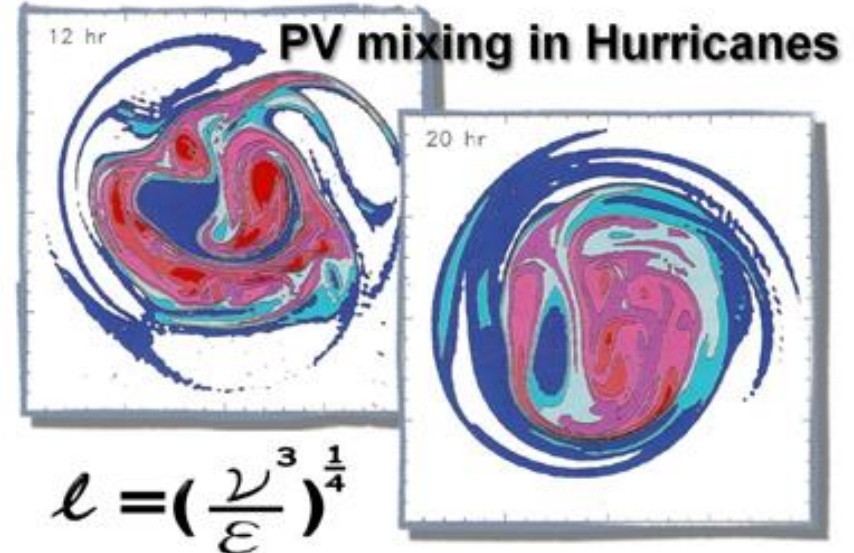
Max Entropy

End states prediction

Schubert et al. (1999)



Math Stirs !



Max Entropy Min Enstrophy

007⁵

Stirred but not shaken



007 likes multiscale in taste!



Thank you! Question?

