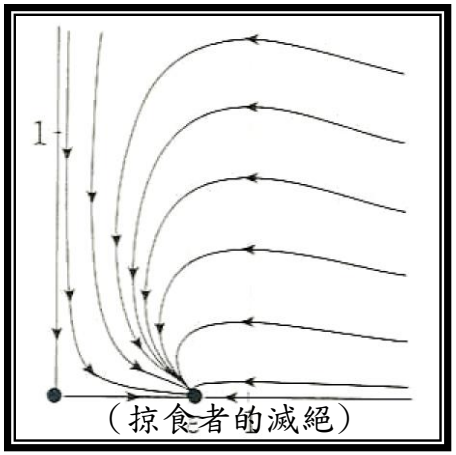
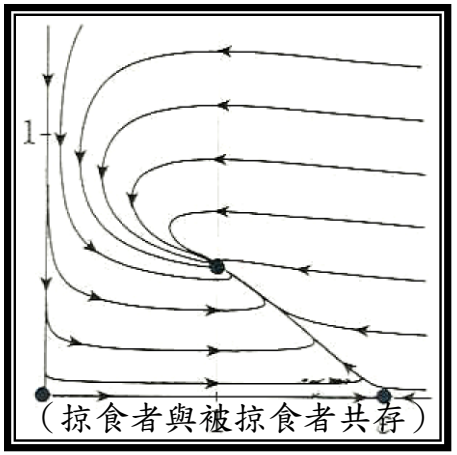
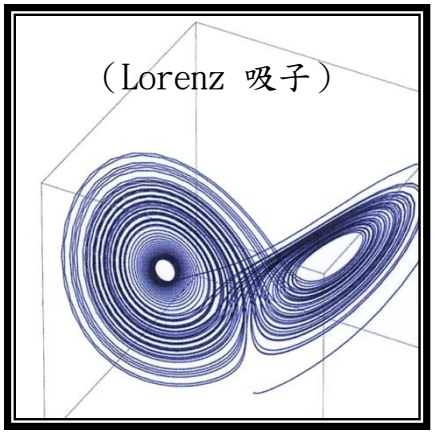
On the Mathematical Thinking
數學思維

郭鴻基 教授

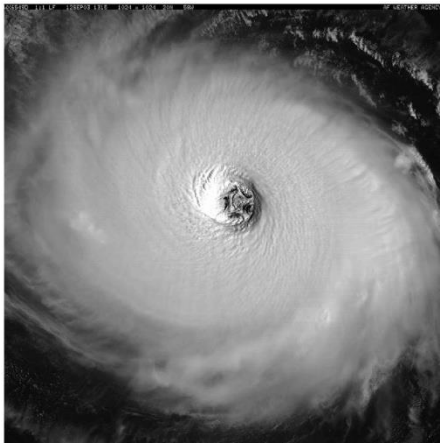
2018 0129 成大
數學建模







第11屆教育部國家講座教授
臺灣大學講座教授
臺大教務主任
臺大共同教育中心主任
2012 Alan Berman Publication Award



認識時代：

數位資訊科技

Information tech + digital

全球化與文化衝突

Globalization and Clash of Civilization

亞洲崛起

Asia Emerging

環境變遷

Climate Change

Artificial Intelligence
AlphaGo　AI 打破人類圍棋 3000 年來的盲點
AlphaGo　被 100:0 完敗
AlphaGo team 解散
AI 顯學(硬體、數據、演算、domain knowledge)

Not all intelligence is artificial!

數學思維

Math Thinking
Computational Thinking

Formulation 方程式 模式

Solution Methods解 計算

Interpretation 詮釋

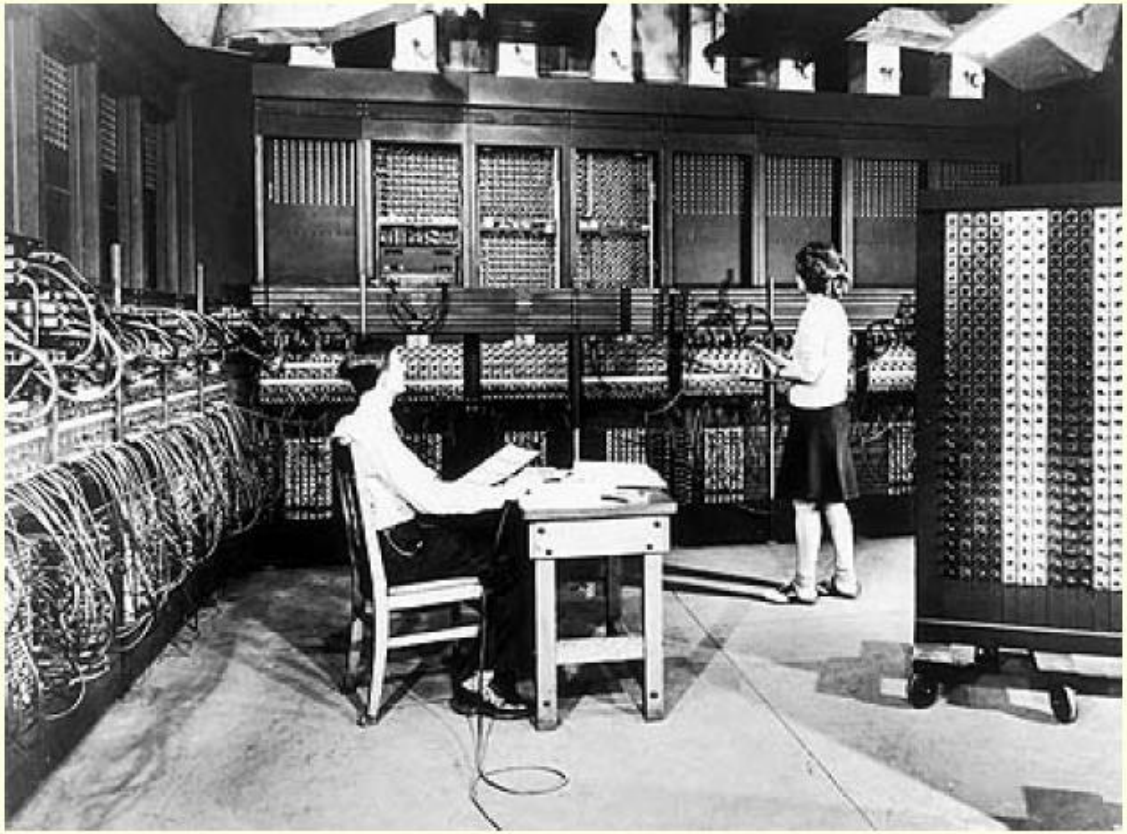
數據化與數學思維

Big Data and Math Thinking

(1)解決**問題**的實驗設計，
(2)評估實驗的**數據結果**，
(3)**解釋**實驗的結果。

The ENIAC

Electronic Numerical Integrator and Computer



18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

No high-level language
Assembly language

500 Flops
Function Table 0.001 s

3,700,000,000 times slower than current day large computer

第一部電腦　　氣象預報

一尺之錘、日取其半、
萬世不絕。 莊子

0

$\triangle x \to 0$ 逼近於零不等於零

$$\lim_{\triangle \to 0} \frac{f(\triangle)}{g(\triangle)} = \frac{df}{dg}$$

$$\lim_{\epsilon \to 0} \frac{\sin \epsilon}{\epsilon} = ?$$

面積固定值，
但是總邊長是無限
體積固定值，
但是總表面積無限

有可能嗎?

Evangelista Torricelli (1608-1647)

無限面積但有限體積

- xy =1 對 x 軸旋轉; 類似聖經故事的Gabriel's horn

"Torricelli's paradox funnel"

1672, 英國哲學家Thomas Hobbes (霍布斯）宣稱只有神經病才會相信 Torricelli的無限面積.

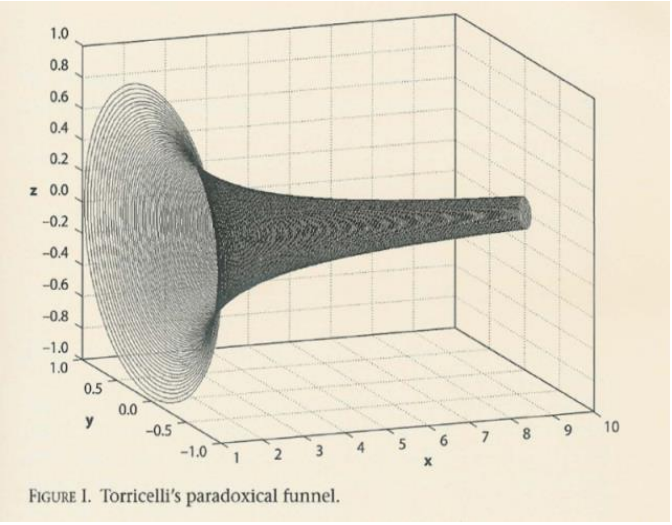


FIGURE 1. Torricelli's paradoxical funnel.

$$\triangle V \approx \pi y^2 \triangle x$$

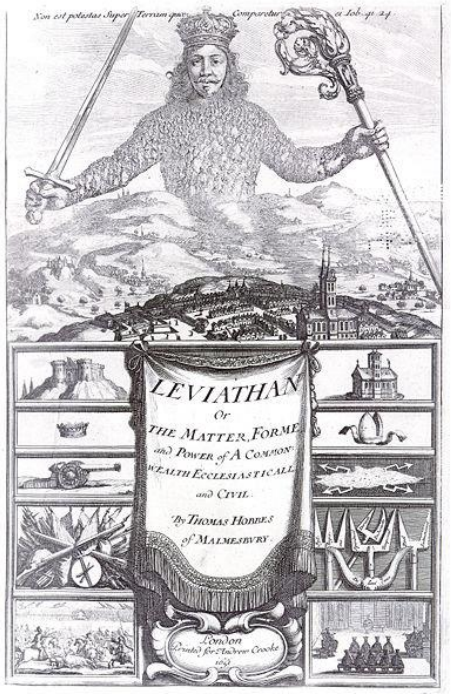
$$V = \int dV = \pi \int_a^\infty y^2 dx$$

$$V = \pi \int_a^\infty \frac{dx}{x^2} = \frac{\pi}{a}$$

$$A = \int_a^\infty y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$A = \int_a^\infty \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx = \int_a^\infty \frac{\sqrt{x^4+1}}{x^3} dx > \int_a^\infty \frac{\sqrt{x^4}}{x^3} dx = \int_a^\infty \frac{1}{x} dx \sim \infty$$



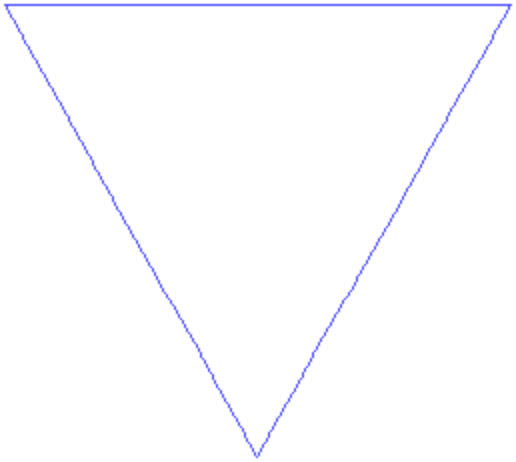
霍布斯的政治原則是「不要傷害」，他的道德黃金律是「己所不欲，勿施於人」

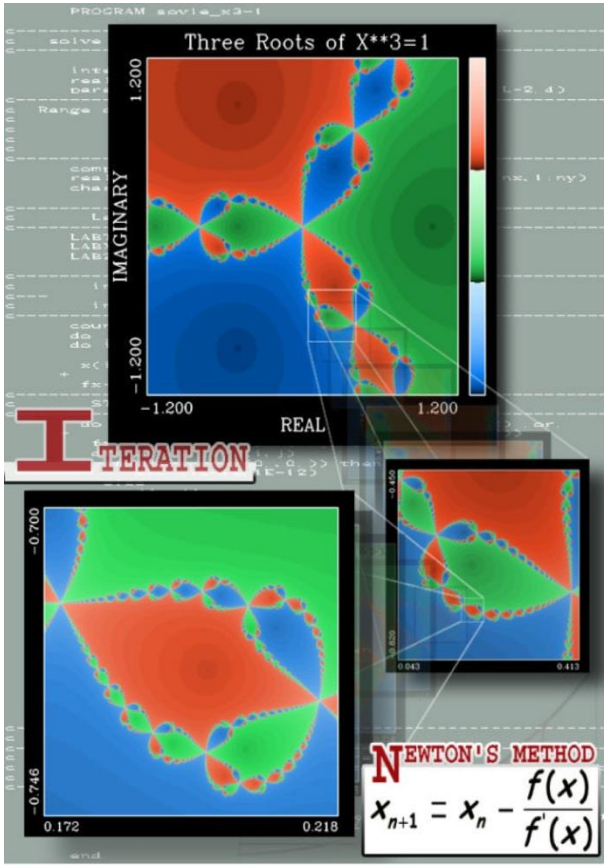
Fractal 碎形

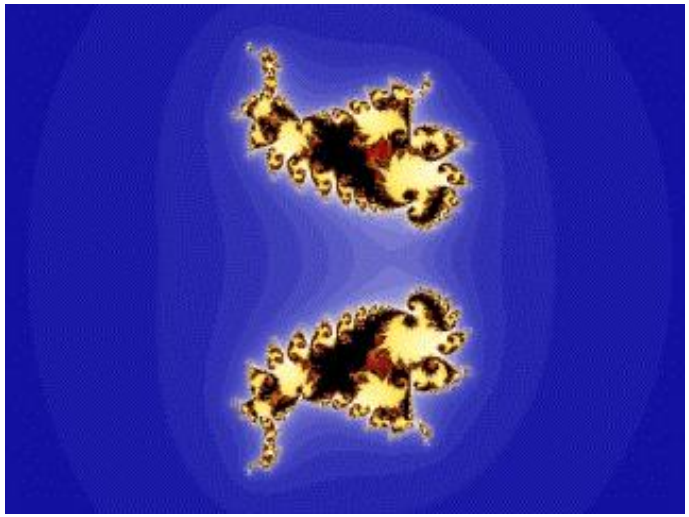
自我相似性

奇異吸子

碎形維度



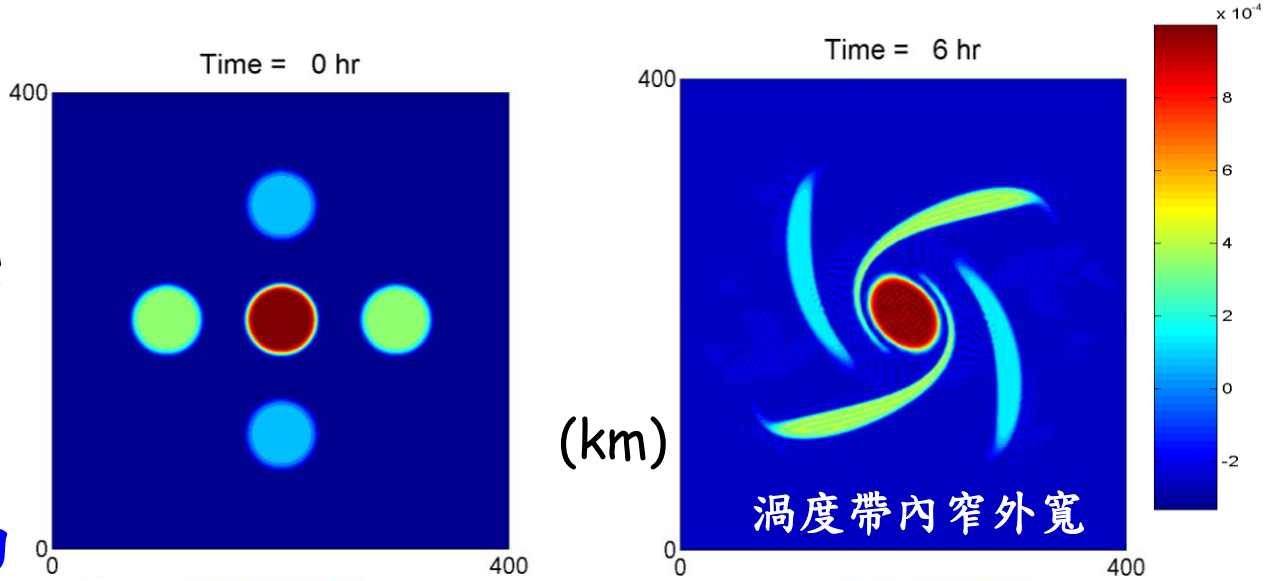




2012 Alan Berman Research Publication Award
Best publication in the Naval Research Lab.

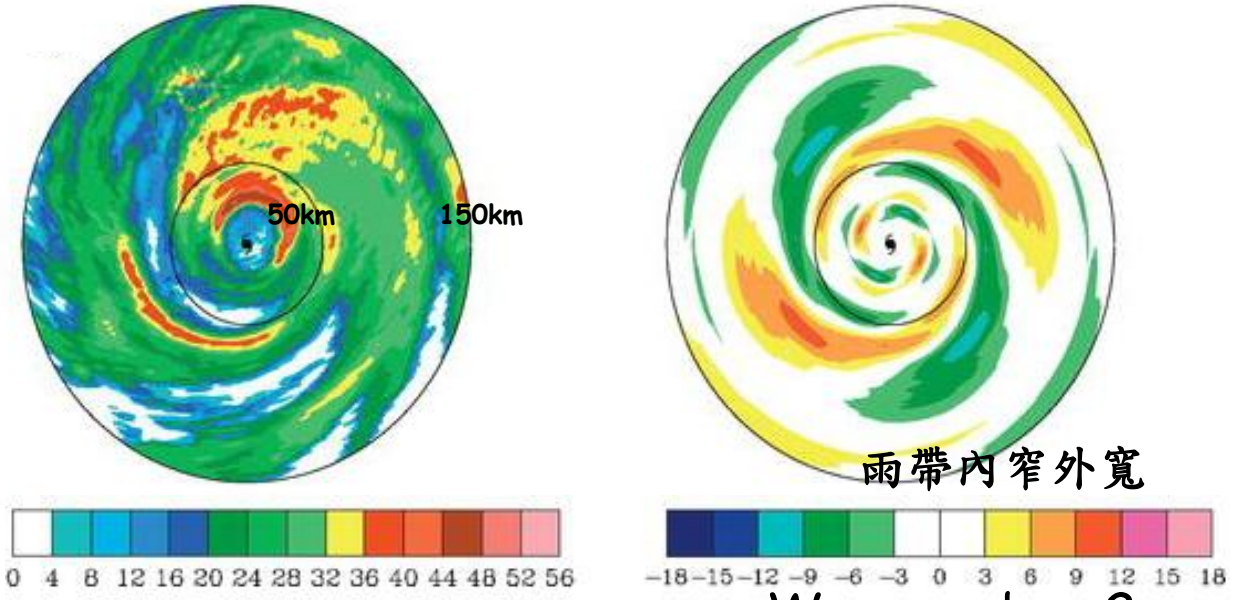
Vorticity Dynamics

颱風渦漩導致雨帶內窄外寬形變。重要的雨帶與颱風動力發現。



Radar Reflectivity

Hendricks et al. 2011
Corbosiero et al. 2006



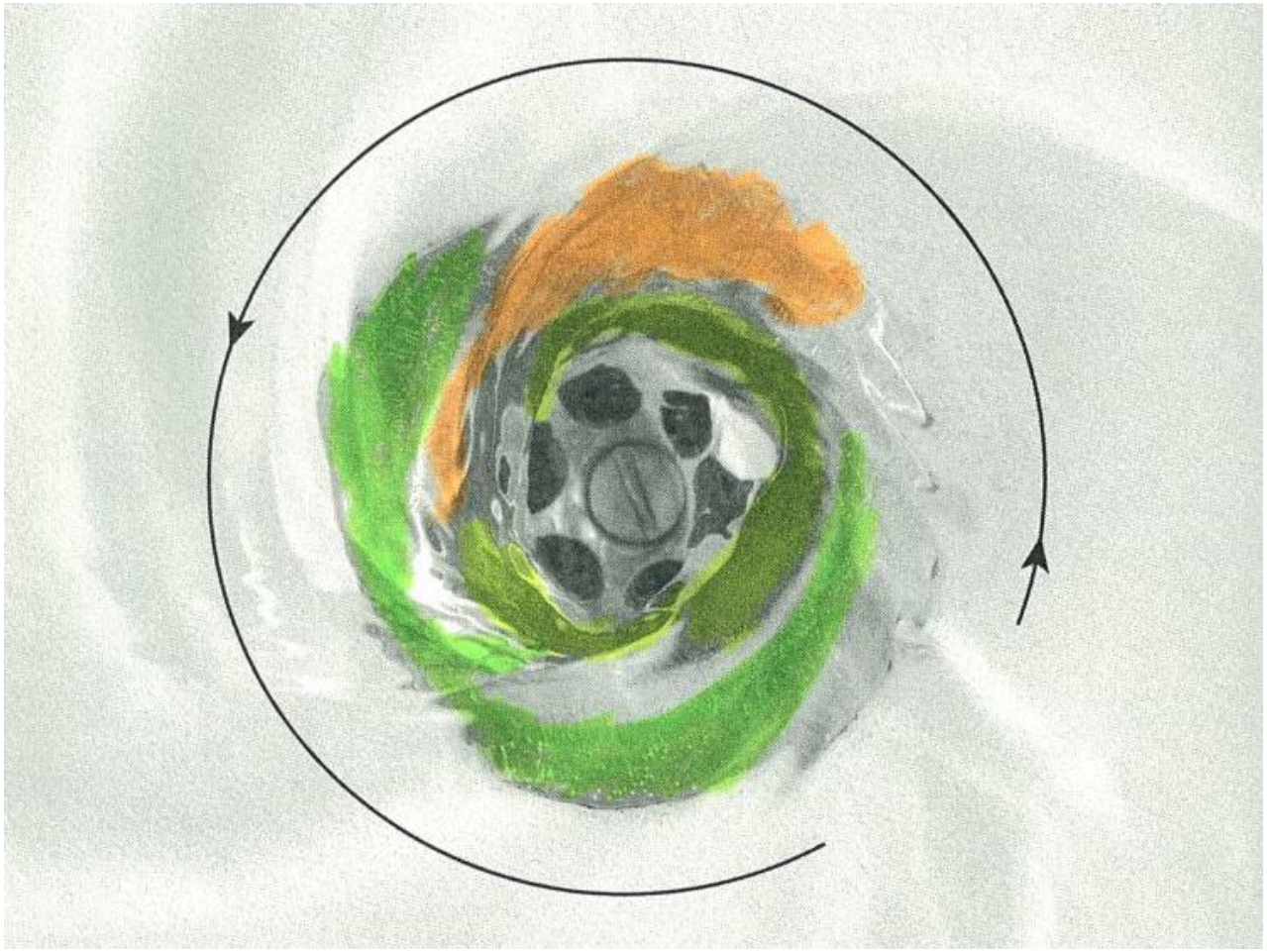


Figure 3.6. Swirling water flows out of a sink or bath by means of a central drain hole. As fluid parcels rotate and approach the drain hole, they speed up to conserve the circulation. © Pavel Losevsky / 123RF.COM.

逼近零 乘以 逼近無限大 = 零 或無限大

有限值

真實世界中觀察的到的現象

逼近零 乘以 逼近無限大 = 有限值

0.0001 * 10000 = 1.0

一杯咖啡，古今往事盡付笑談中。

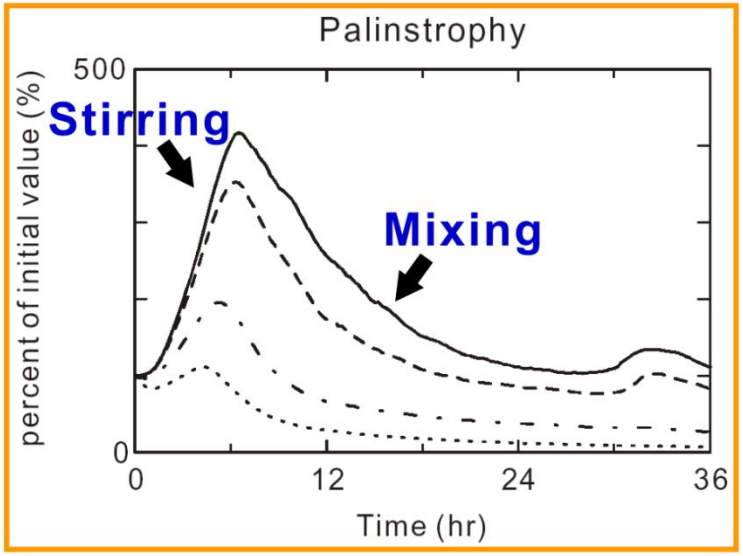
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

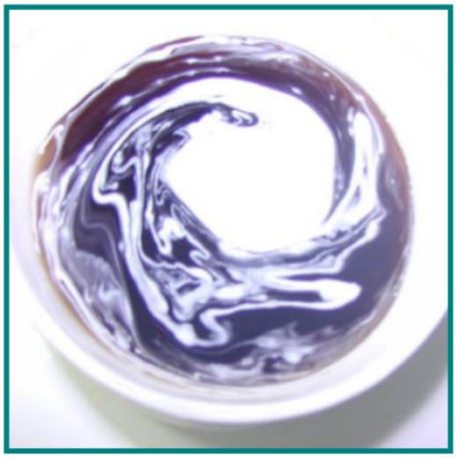
$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta \, dV$$

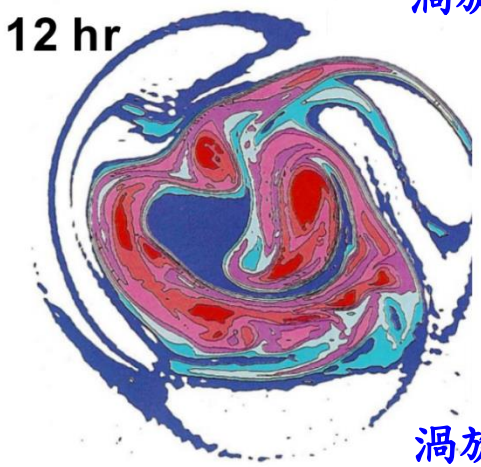
$$\frac{dC}{dt} = \int (\vec{V}\cdot\nabla\theta)\nabla^2\theta \, dV - \nu\int (\nabla^2\theta)\, dV$$

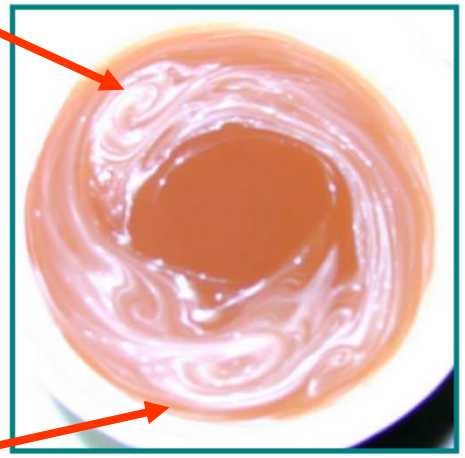
Stirring **Mixing**

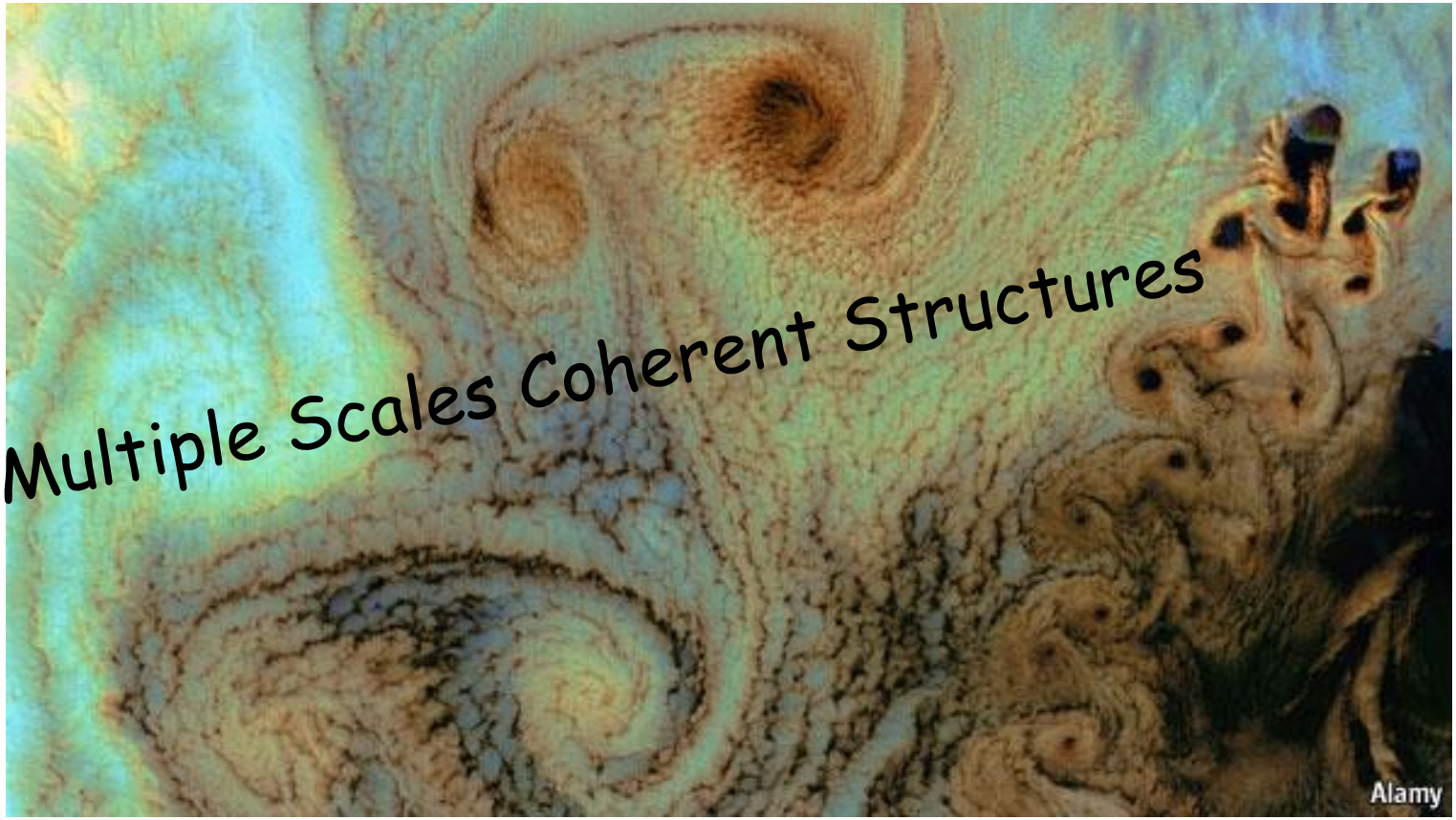


Coffee with white









Theory points to three main mechanisms: mixing the ocean's surface layers (up to a few hundred metres) by wind; mixing of deeper layers by ocean currents; and eddies, swirls created when warm ocean currents meet cold ones, blending large swathes of the ocean 10-100km across.

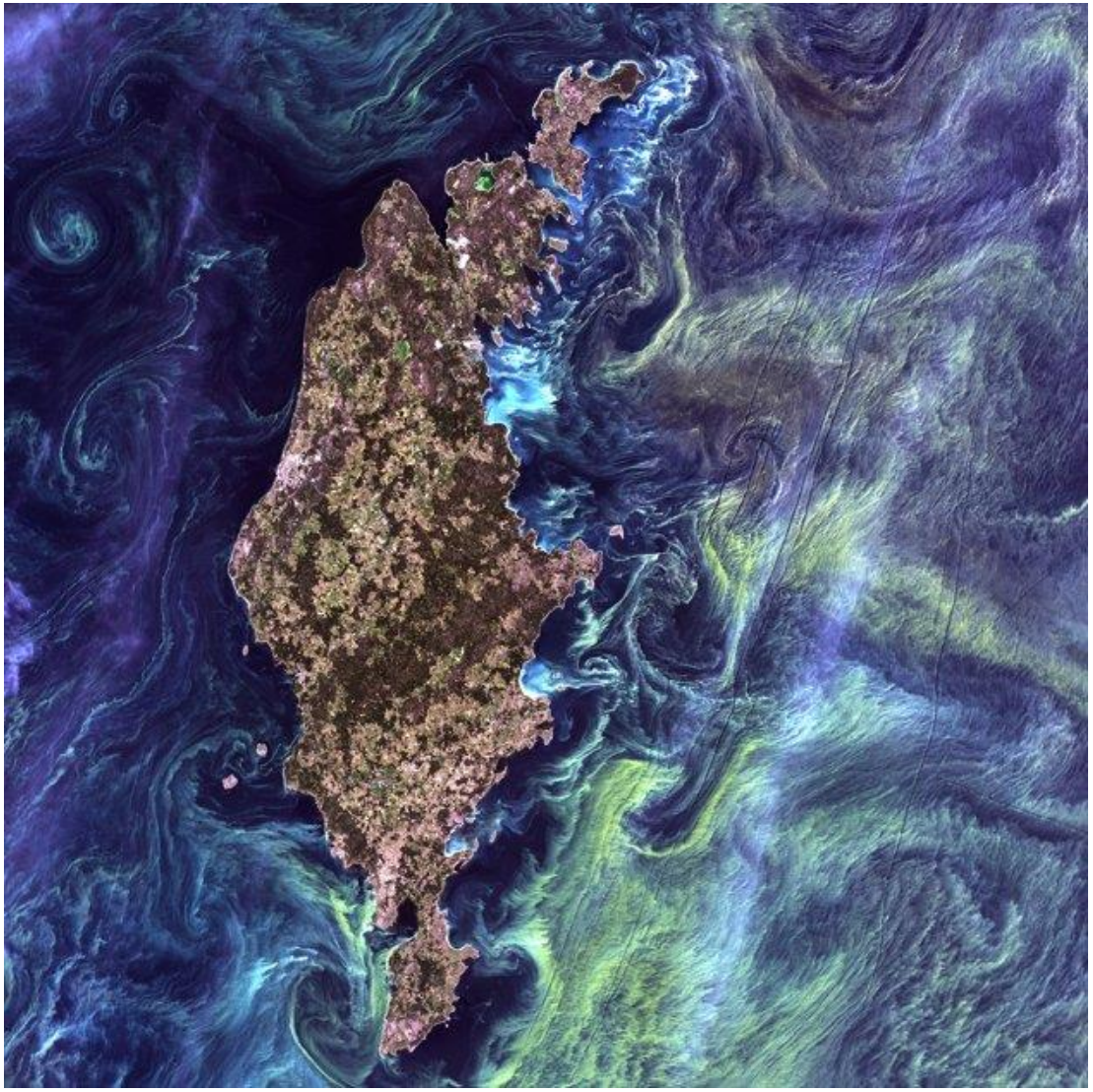
Dr Salée and colleagues report in in *Nature Geoscience,* eddies suck up as much carbon as the other two mechanisms do, something most current climate models fail to account for.

Economist, 12/08/2012

The profound study of nature is the most fertile source of mathematical discoveries. Fourier

Phytoplankton Swirls

Vortex & Filaments



In the style of Van Gogh's painting "Starry Night," massive congregations of greenish phytoplankton swirl in the dark water around Gotland, a Swedish island in the Baltic Sea. Population explosions, or blooms, of phytoplankton, like the one shown here, occur when deep currents bring nutrients up to sunlit surface waters, fueling the growth and reproduction of these tiny plants. (Credit: NASA Goddard Space Flight Center/USGS)

Statistical Dynamics
統計力學

Boltzmann mixing entropy

Constrained circulation and angular momentum

Max Entropy

End states prediction

Schubert et al. (1999)

Stirred but not shaken

He likes the multiple scales!

Darwin 生在維多利亞時代，
當時人們相信小孩特性完全由父母而來，
(恐怕現在還有許多人也是如是相信。)
一個簡單數學式子　**小屁孩 = (父+母)/2**。
Darwin 不是數學家，據說他數學不好，
但他對於維多利亞的遺傳觀念，
特別憂心和他的物種演化理論衝突，
因物種演化需要大量複雜物種的數目。
終其一生，他無法得到滿意的答案。

這問題直到孟德爾的豌豆實驗才得到解決。

“Six monkeys, set to strum unintelligently on typewriters for millions of years, Would be bound in time to write all the books in the British Museum.”

Huxley 1860

On June 30, 1860, at the Oxford Union in England, Anglican Archbishop of Oxford University, Samuel Wilberforce, and evolutionist and agnostic Thomas Huxley were engaged in the “Great Debate.” Huxley declared that given enough time all the possible combinations of matter, including those necessary to produce a man, will eventually occur by chance molecular movement.

“Six monkeys, set to strum unintelligently on typewriters for millions of years, wouldbe bound in time to write all the books in the British Museum.” Huxley 1860

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。 通典

共**49**個字，假設中文常用字為**1000**字，共有**10^147**個選擇

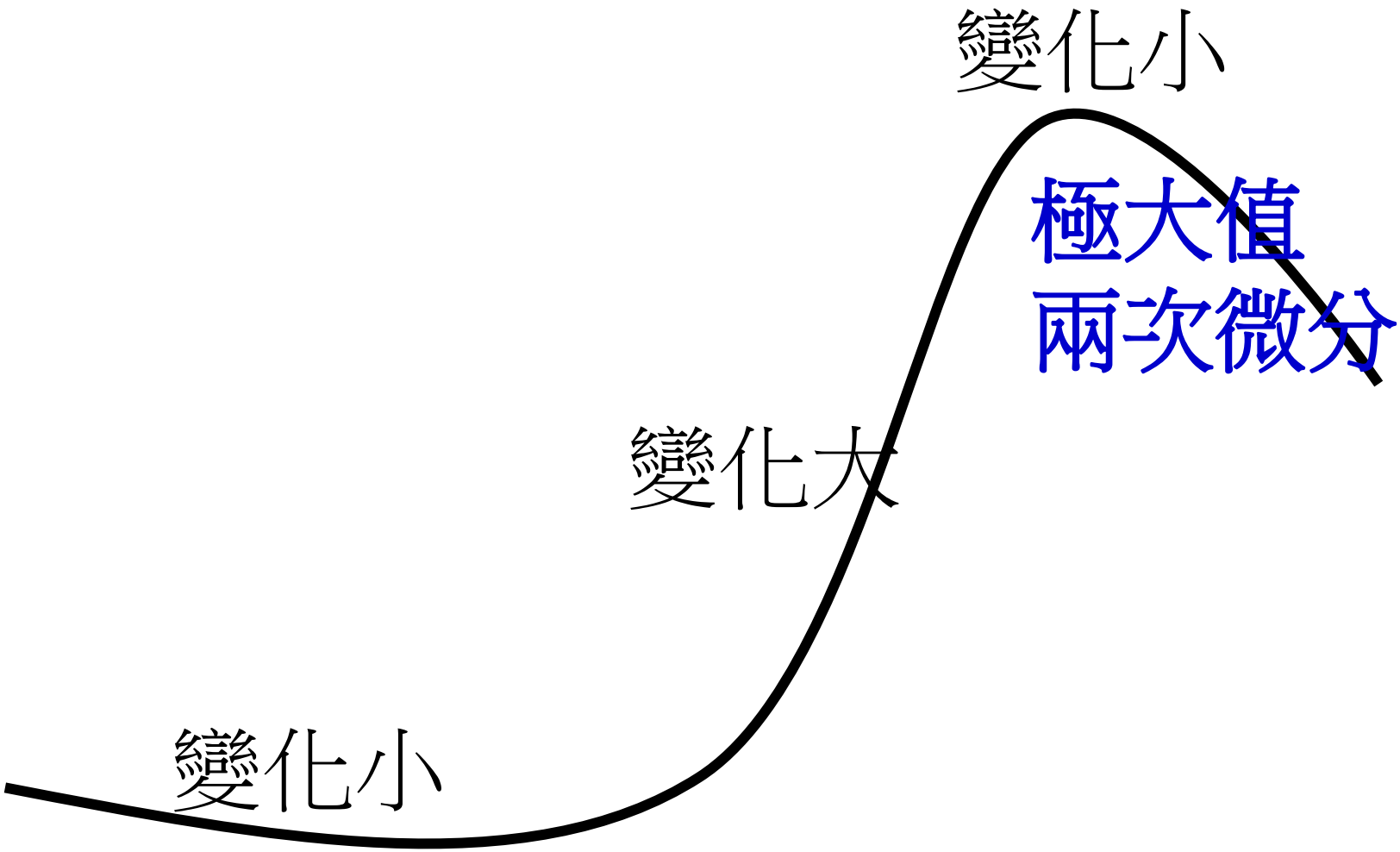
地球歷史 **10^18** sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 **10^4**，

10^10*10^18*10^4 = 10^32

10^32/10^147=10^(-115) ~ 0 機率為零 ，不可能的巧合！

微積分=變化之學

注意變化大 與 變化小的地方；變化小的地方要注意有無極值存在 (二次微分)。
趨勢 與趨勢的變化


變化小
極大值
兩次微分
變化大
變化小

趨勢大師

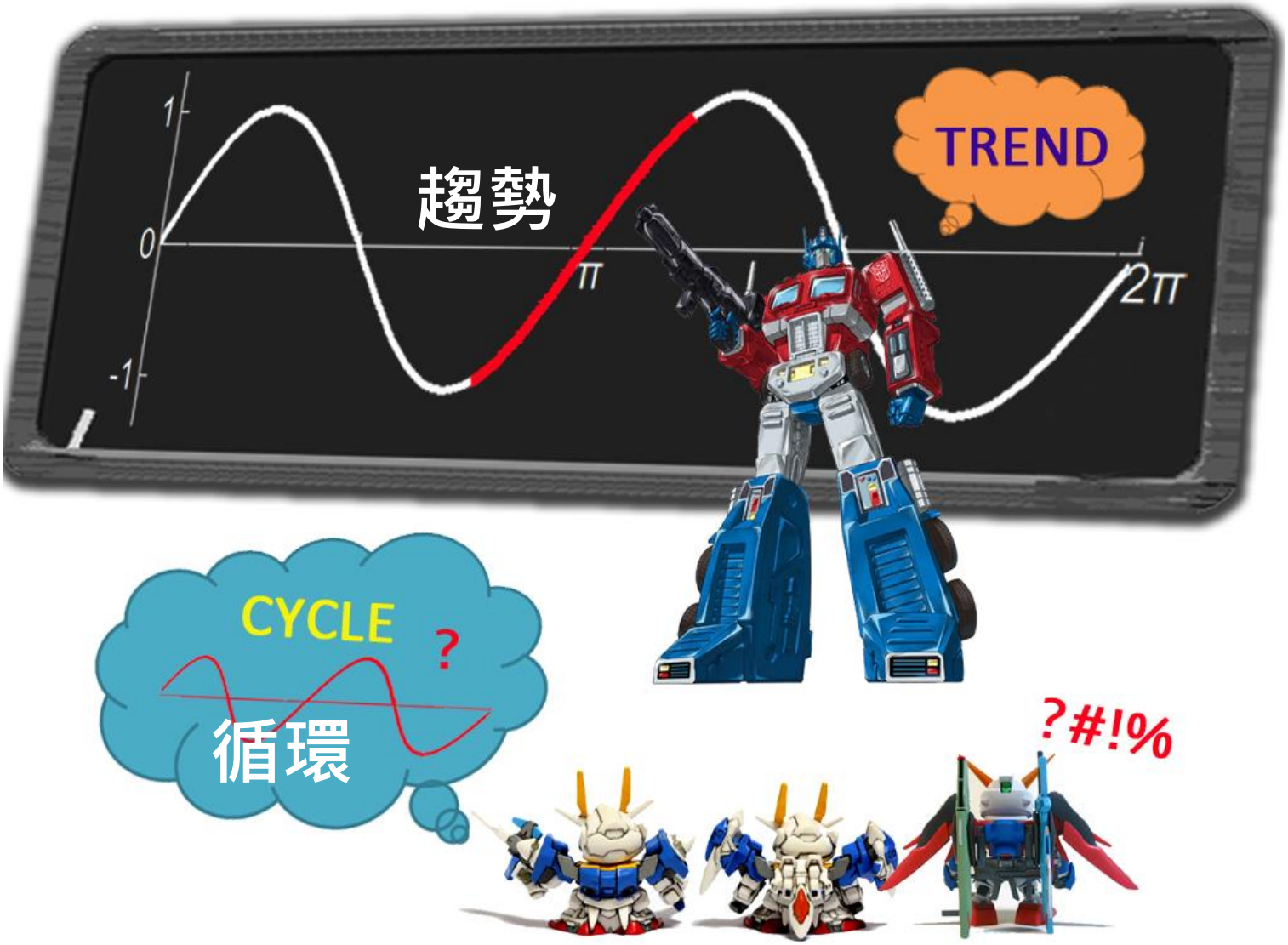
趨勢為何? d/dt

趨勢何時改變?

二次微分

一個經濟學家的趨勢是另一個經濟學家的循環

"One economist's trend is another economist's cycle"

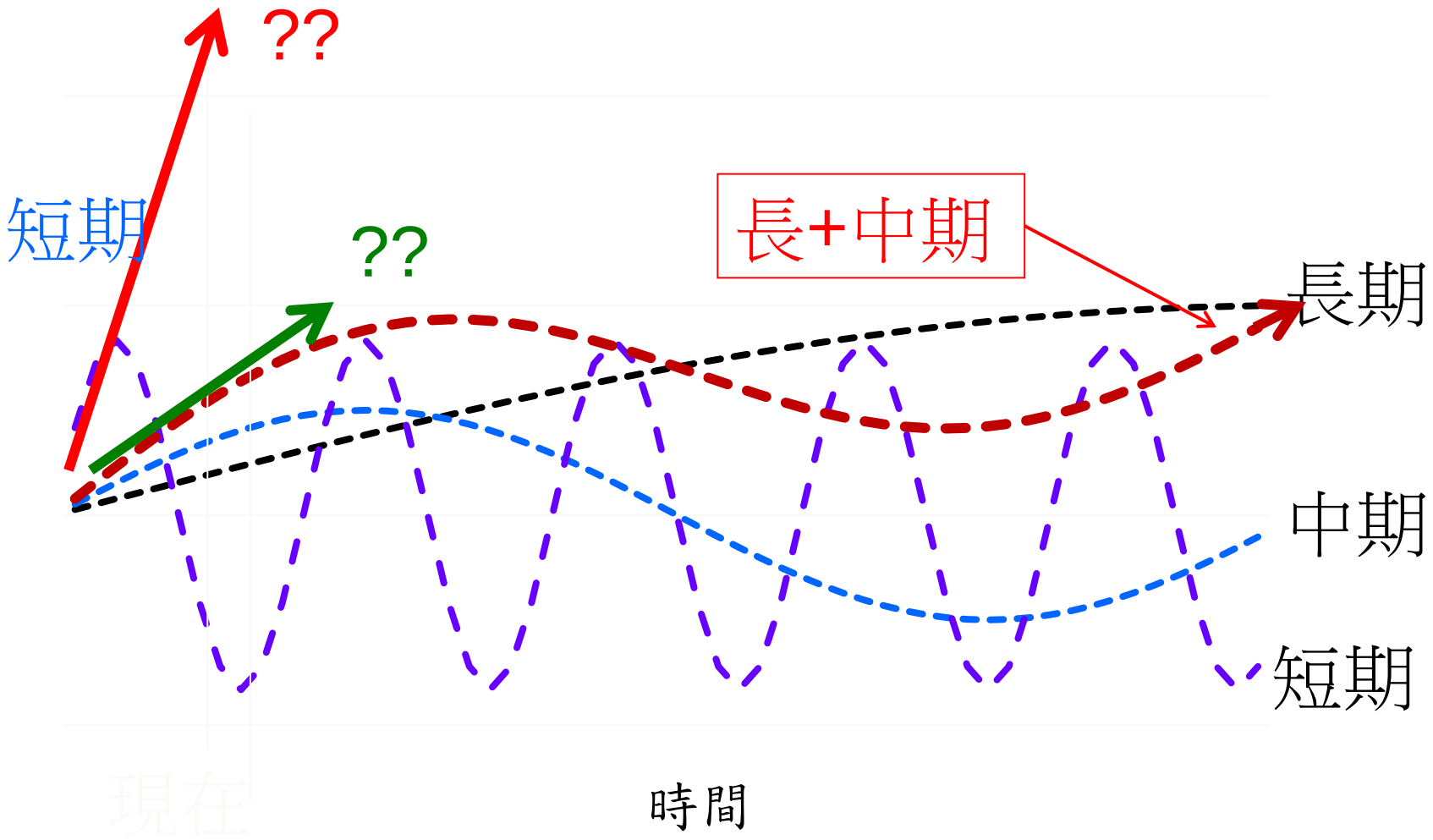


如果目前趨勢不變

Nowcasting 即時預報

僅適合短時間預報

不同時間尺度
不同趨勢與循環



張智北教授 提供

東京台北的愛情故事
20xx友達以上戀人未滿數學版

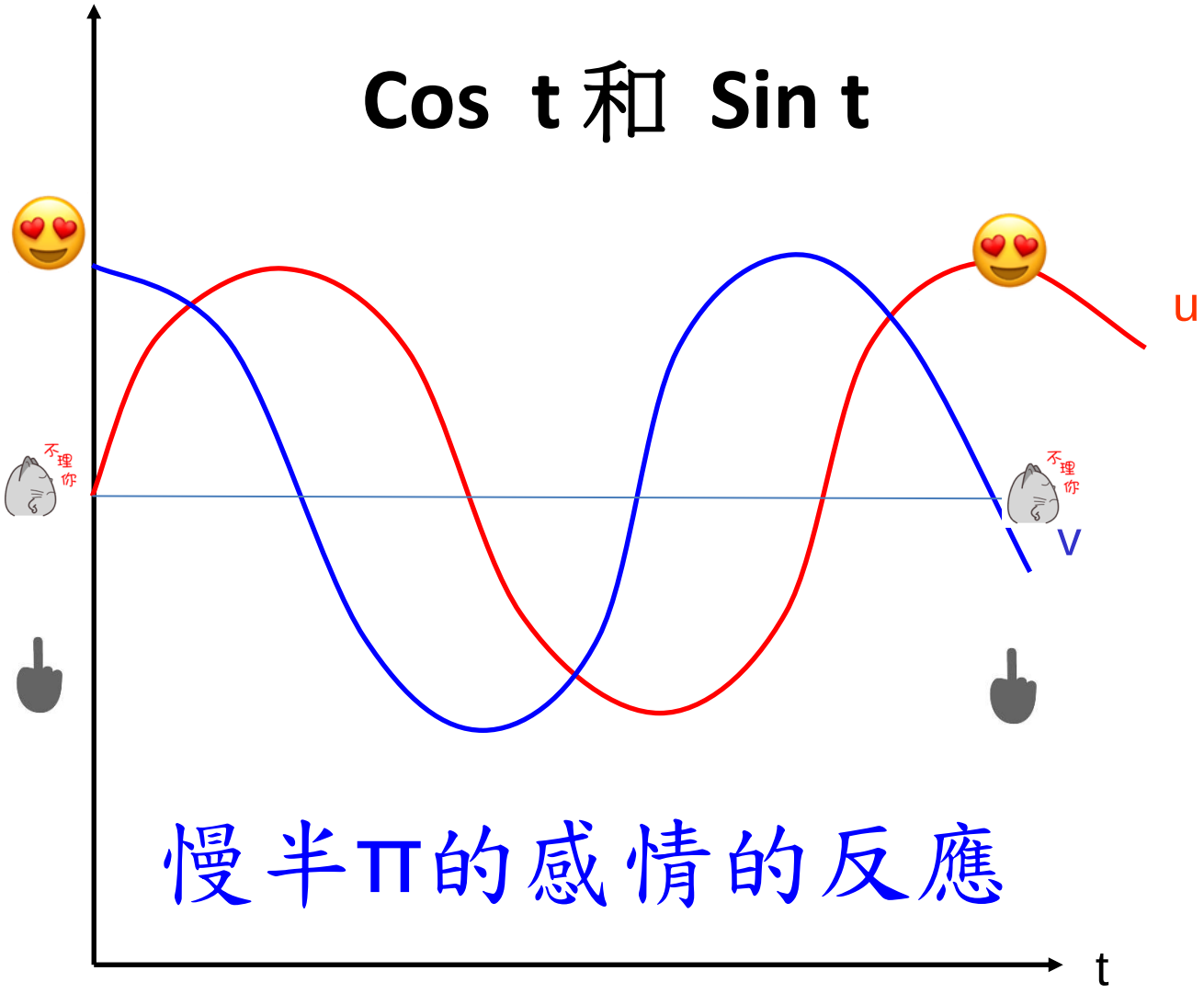
第1場東京告白 A :[😍 喜歡] B: []

第2場台北告白 A :[] B: [😍 喜歡]

第3場東京情緒醞釀中 A :[🖕 討厭] B: []

第4場台北情緒醞釀中 A :[] B: [🖕討厭]

第5場東京再度告白 A :[😍 喜歡] B: []

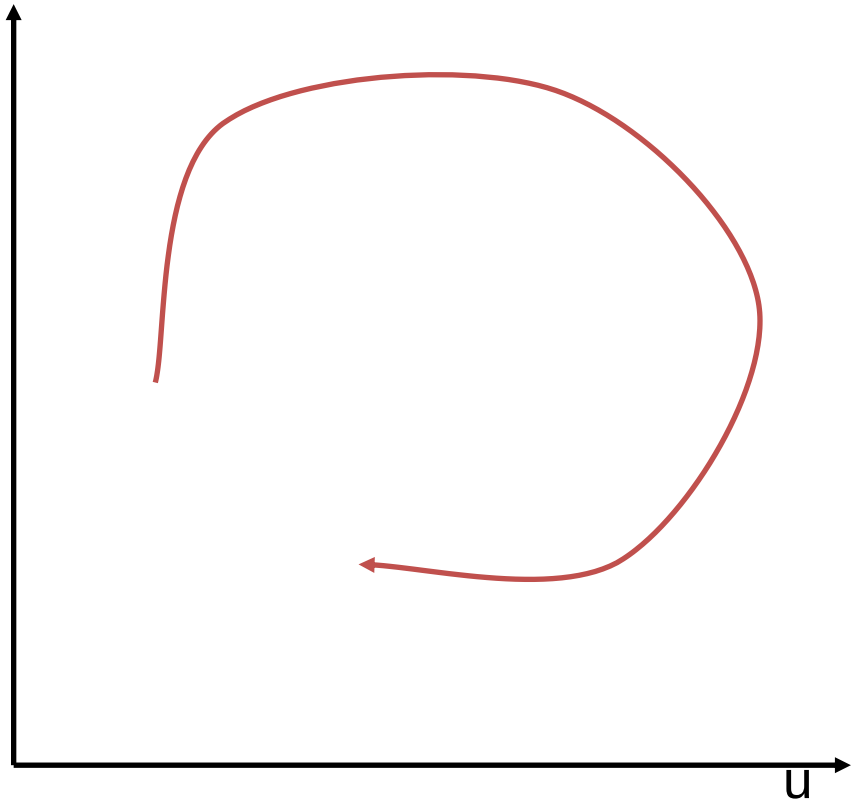


不相關的兩個時間序列

Cos t 和 Sin t 零相關、不來電！

時間的軌跡

相位圖



$$\int_0^{2\pi} \underset{u}{\cos t} \underset{v}{\sin t} dt = 0$$

Romantic Romeo and Fickle Juliet

(Strogatz 1988)

$$\frac{dR}{dt} = J \qquad \frac{dJ}{dt} = -R$$

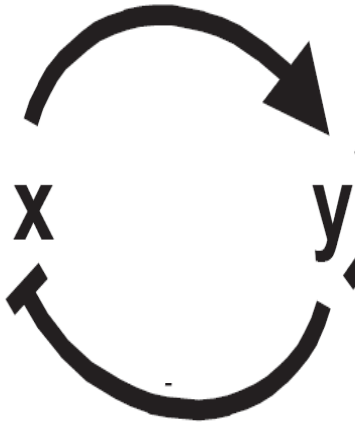
J = Cos t

R = Sin t

慢半π的感情的反應

Negative Feedback Oscillators

X Cost Y Sin t



$$\frac{dy}{dt} = x$$

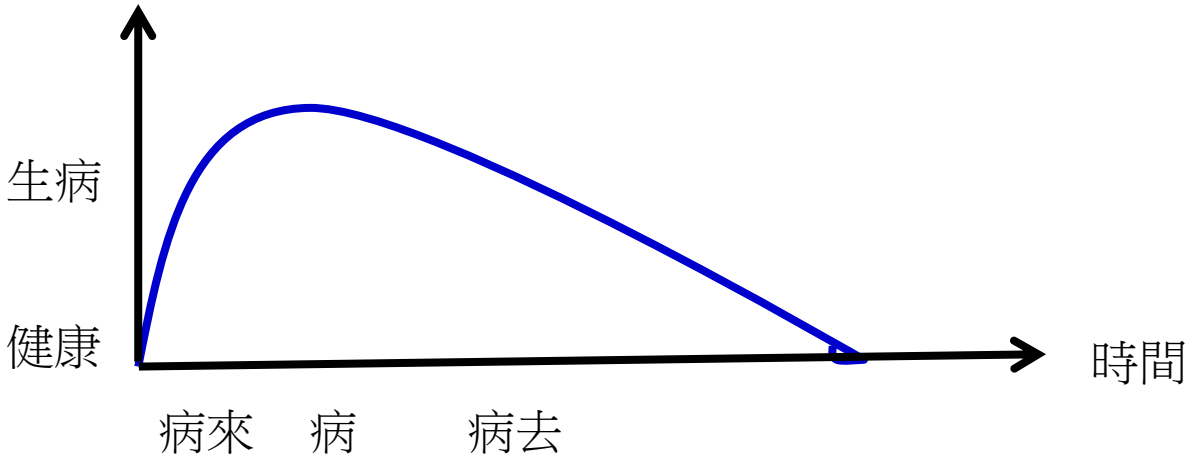
$$\frac{dx}{dt} = -y$$

物廉價美　　顧客增加消費　　高需求價格上揚

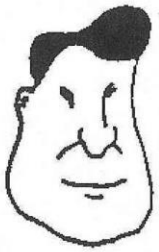
價格上揚　　顧客減少消費　　低需求價格下滑

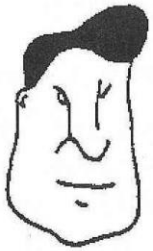
負回饋

病來如山倒
病去如抽絲



來 去 時間之前後，病是時間參考點

1 

2 

3 

4 

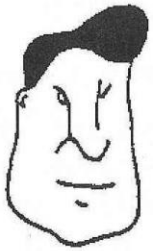
5 

6 

7 

8 

1 

2 

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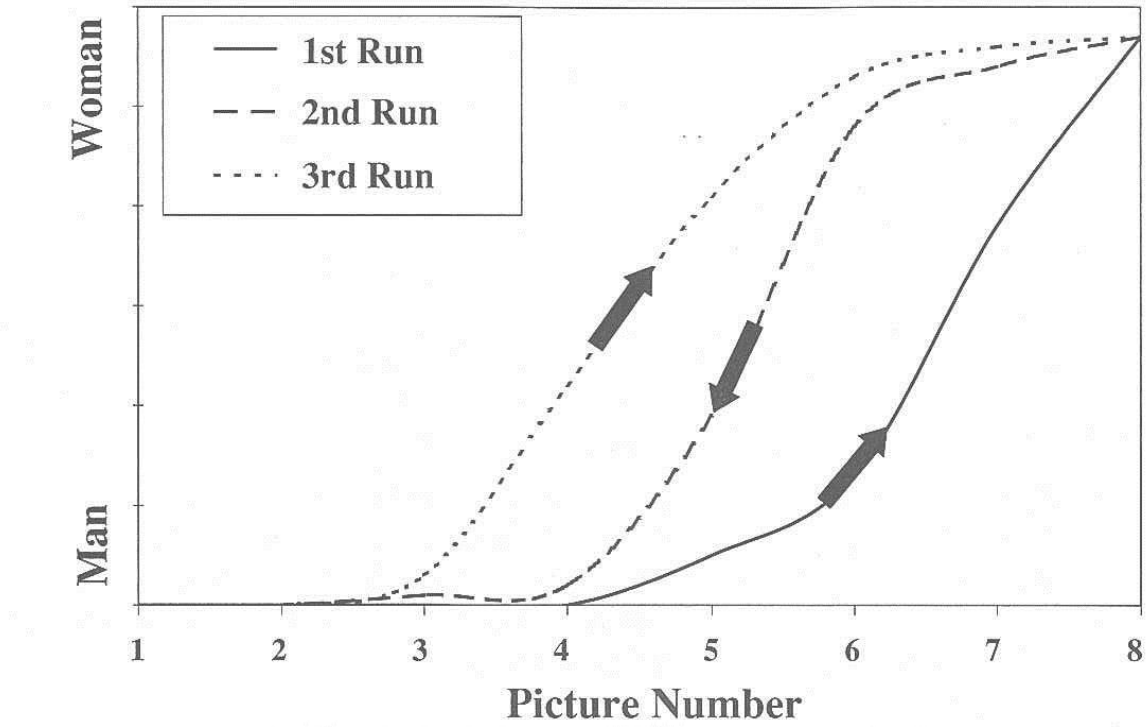


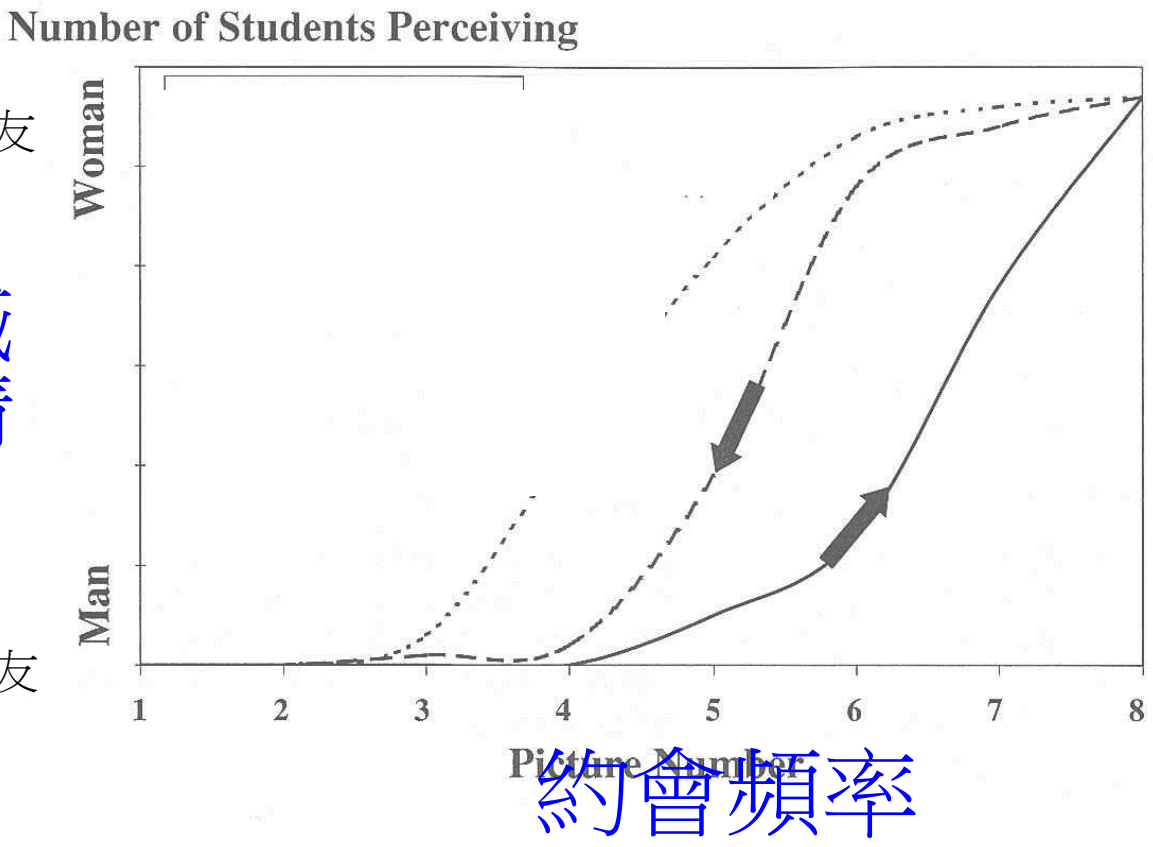
Figure 6.8: Schematic representation of the visual catastrophe based on the data in table 6.1 on three runs (123456787654321234567 8) through the series of pictures in figure 6.7. On the vertical axis is plotted the number of students perceiving a change in their perception. At the level "man," all of the students perceive a man, and at the level "woman," all of the students perceive a woman. On the first run (solid line), all of the students see a man's face until the fifth picture, where a few students suddenly see a woman. The biggest jump occurs at the seventh picture. By the eighth picture, all of the students are seeing a seated woman. On the second run, most of the students continue to see a woman until the fifth and fourth pictures are presented, after which point most of the class is seeing a man's face. A similar pattern appears in the line for the third run, but the shifts in perception do not occur in the same place as they did for the second run. Thus these results are a good illustration of one-time hysteresis.

Hysteresis 遲滯

男女朋友

感情

非男女朋友

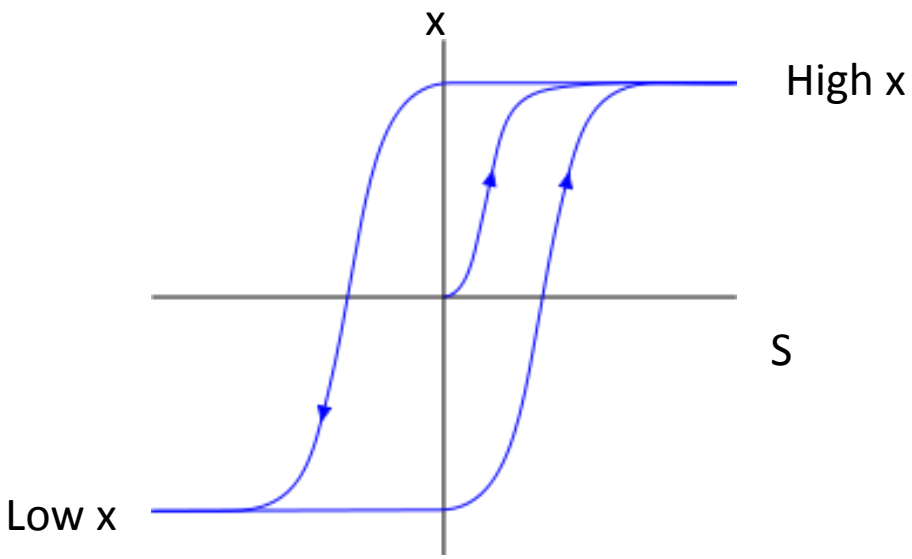


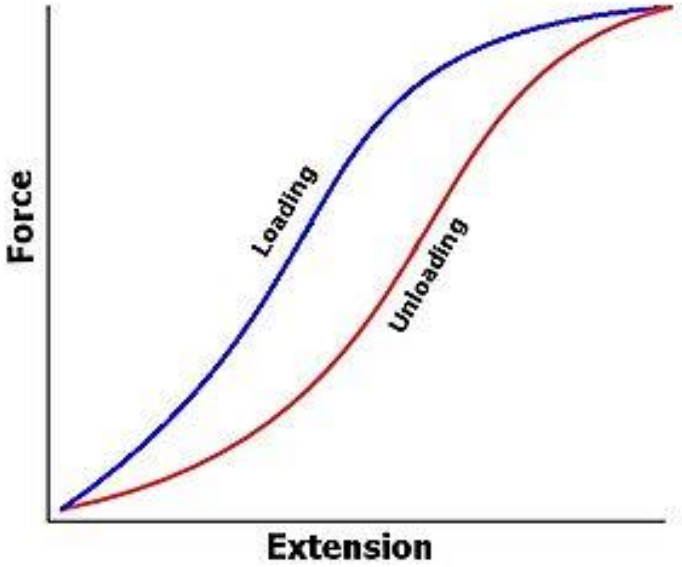
約會頻率

$$\frac{dx}{dt} = -\mu x + \alpha \frac{x^n}{k^n + x^n} + \sigma \frac{S}{\rho + S}$$

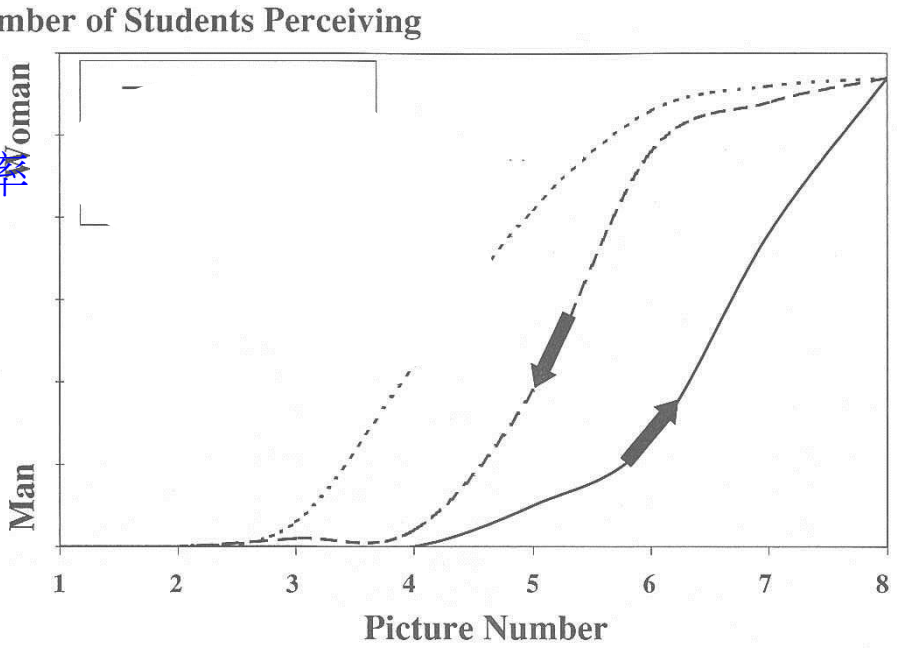
感染人數時間變化　治癒　相互感染　外在公衛健康環境指數

Cytokine Memory　恢復 蛋白質自我正回饋 外在蛋白質驅誘力





In one sense the rubber band was harder to stretch when it was being loaded than when it was being unloaded.



失業率
健康
肥胖
環境
疾病

經濟條件（通貨膨脹）
身體條件
環境條件
公衛條件

Thank you for your attention

Van Gogh

星夜