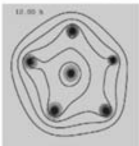
數學可以大不同
輕鬆看數學建模





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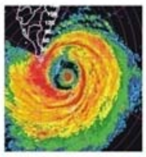
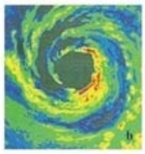
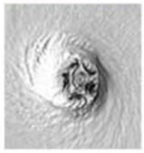
5/31/2010 臺大公衛學院

Politics are for the moment
An equation is for eternity

Math. Model
數學模式

Formulation
微分、差分方程式

Solution/Analysis
分析、解

Interpretation
科學詮釋 科學觀點

數量化、數位化
數學化--模式--動力系統

False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness.

Darwin, **The Origin of Man, chapter 6**

觀點需時時更新　嚴謹驗證資料不容妥協

Function　y = f(x)

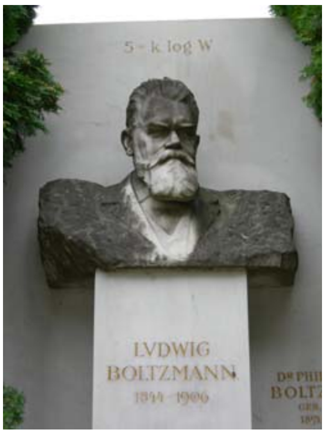
Commonly Occurring Functions

Polynomials: approximate with a high degree of accuracy, almost any existing function

Trigonometric functions　**cos**　**sin**

Exponential functions　**e**

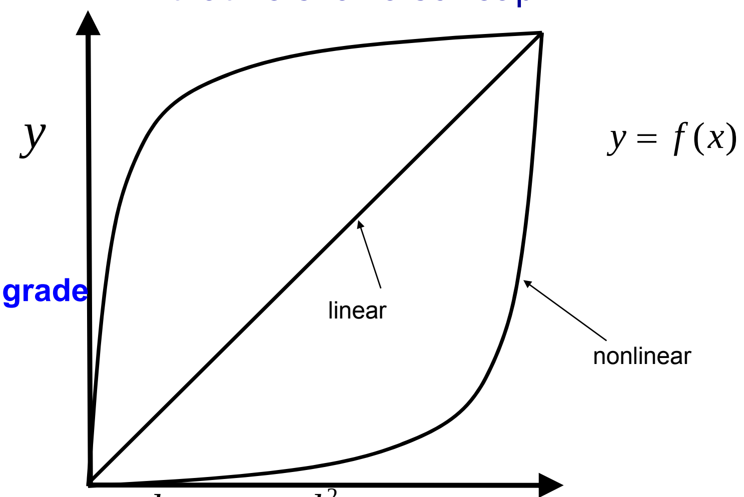
Logarithmic function　**log**　**ln**
s=k log w



"whatsoever a man soweth,
that he shall also reap"

Function

Y=Y(x)



y : exam grade

$$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ?$$

x **x : hard working**

世紀之謎？**1715-2010年**

Bod

0 3

4 7

0.4 0.

Mercury	0.4 (0.39)	Venus	0.7 (0.72)	Earth	1.0 (1.0)
Mars	1.6 (1.52)	Asteroids	2.8 (2.77)	Jupiter	5.2 (5.2)
Saturn	10 (9.54)	Uranus	19.6 (19.19)	Neptune	38.8 (30.07)
Pluto	fails (39.60)				

"**Plutoed**" To demote or devalue someone or something.

American Dialect Society, word of the year 2007

Evangelista Torricelli (1608-1647)

無限面積但有限體積

xy =1 對 x 軸旋轉; 類似聖經故事的Gabriel’s horn

“Torricelli’s paradox funnel”

1672, 英國哲學家Thomas Hobbes (霍布斯）
宣稱只有神經病才會相信 Torricelli的無限面積.

霍布斯的政治原則是「不要傷害」，他的道德黃金律是「己所不欲，勿施於人」

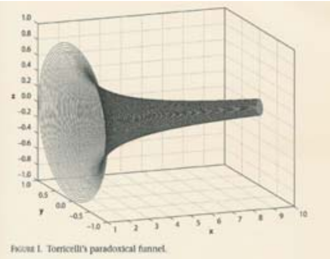


FIGURE 1. Torricelli's paradoxical funnel.

$$\triangle V \approx \pi y^2 \triangle x$$

$$V = \int dV = \pi \int_a^\infty y^2 dx$$

$$V = \pi \int_a^\infty \frac{dx}{x^2} = \frac{\pi}{a}$$

$$A = \int_a^\infty y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$A = \int_a^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx = \int_a^\infty \frac{\sqrt{x^4+1}}{x^3}\,dx > \int_a^\infty \frac{\sqrt{x^4}}{x^3}\,dx = \int_a^\infty \frac{1}{x}\,dx \sim \infty$$

Kelvin wedge   deep water waves

[*Courtesy of Aerofilms Ltd.*]

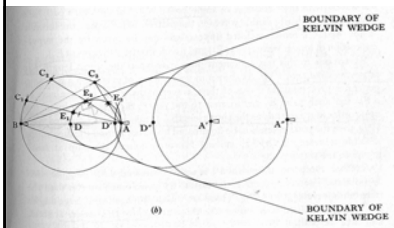
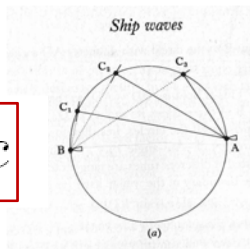


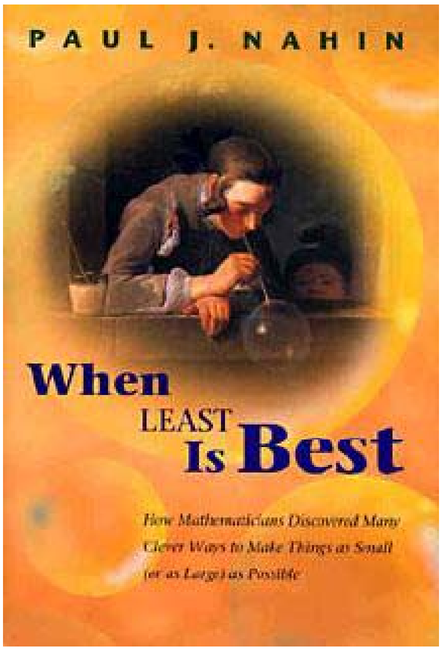
Figure 68. Waves generated in deep water by a ship B. Case (a): positions C_1, C_2, C_3 of any waves generated t_s seconds ago (when the ship was at A) if their energy had travelled a distance ct_s. Case (b): the real positions E_1, E_2, E_3 of the same waves, taking into account that their energy has only travelled a distance $\frac{1}{2}ct_s$. At each, the dependence of wavelength on direction of emission, as inferred from equation (183), is shown. The circle with diameter AD is the locus of all such waves. Other such circles, with diameters A′D′ and A″D″, are where waves generated when the ship was at A′ and A″ are now to be found. All such circles lie within the Kelvin wedge of semi-angle (184).

$$2*\sin^{-1}\left(\tfrac{1}{3}\right) = 2*19.5^{\circ}$$

$$u\cos\phi = c$$

Stationary waves





A universe made by God must be a perfect universe, and consequently should always operate with economy.

上帝創造完美的世界，所以其運作應滿足最小原理。

Shortest path reflection (Hero of Alexandria, 2nd century B.C.) 最短路徑

"Every action done by nature is done in the shortest way."
(Leonardo da Vinci 1452-1519)最短數量

Principle of least time (Fermat 1658)
最短時間
Principle of least action
(Maupertuis 1747)

Hamilton Principle (Sir William Rowan Hamilton 1805-1865)最小能量差

托勒密

Woodcut of Ptolemy World Map (1482)

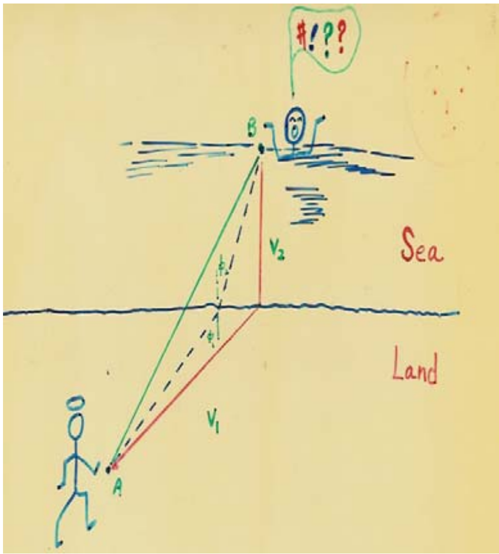
Claudius Ptolemy AD 90-168

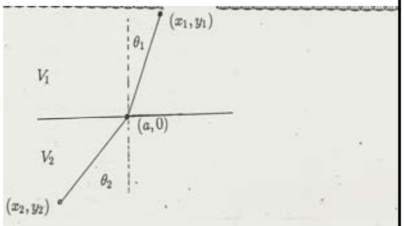
10	8
20	15.3
30	22.3
40	29
50	35
60	40.3
70	45.3
80	50

千年數字之謎??

入射角與折射角 ！！

Principle of Least Time (Fermat's principle)
光在兩點之間行進走最短時間路徑





$$T(a)=\frac{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}{V_1}+\frac{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}{V_2}$$

$$\frac{\partial T}{\partial a}=0$$

$$\frac{-(x_1-a)}{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}\frac{1}{V_1}+\frac{(a-x_2)}{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}\frac{1}{V_2}=0$$

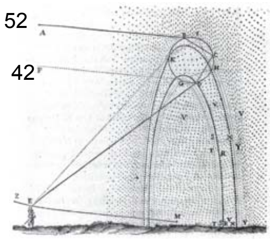
$$\boxed{\frac{\sin\theta_1}{\sin\theta_2}=\frac{V_1}{V_2}}$$

This is the Snell's Law.

一樣觀魚多樣情！

光學如此多嬌，
引得無數英雄折腰！

(1) 魚快樂嗎？
(2) 姜太公釣魚，願者上鉤。
(3) 張無忌冰火山刺魚。
(4) 熱血沸騰，立志革命！
(5) 折射定律，最小原理
彩虹原理。



René Descartes　笛卡爾

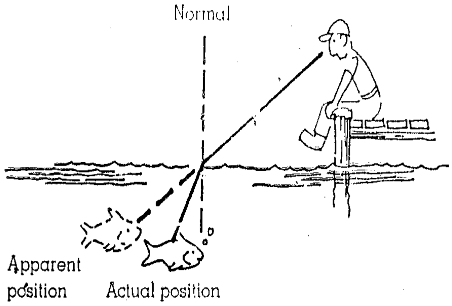


FIGURE 5.13 The refraction of light as it passes fro m the water into the less-dense air causes a fish to appear closer to the s urface than it actually is.

"Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum." Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，
師古在乎隨時。必參古今之宜，
窮終始之要，始可以度其古，中可以行於今。通典

共49個字，假設中文常用字為1000字，機率為10^-147

地球歷史 10^18 sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 10^4，

10^10*10^18*10^4 = 10^32

10^32*10^(-147)=10^(-115) ~ 0 機率為零 ，不可能的巧合！

研究學問是苦心孤詣的事業！ 不要人云亦云！

微積分數學

the Mathematics of Change 變化率

莊子：一尺之錘、日取其半、萬世不絕。

逼近於零不等於零

$\triangle x \to 0$

Rate of Change

$$\lim_{\triangle \to 0} \frac{f(\triangle)}{g(\triangle)} = \frac{df}{dg}$$

$$\lim_{\epsilon \to 0} \frac{\sin \epsilon}{\epsilon} = ?$$

$$u = u(x, y)$$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

偏微分

$$\frac{\partial x^2 y}{\partial y} = x^2$$

只對y變數微分，不改變x變數

你快樂嗎？一個簡單的生涯規劃動力系統

u：快樂指數

x：考試作業量

y：玩魔獸的時間

$$\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{dy}{dt}$$

天縱英明的資優生　<0　>0　<0　<0　>0

考試越少越不快樂，
玩魔獸的時間越多越不快樂

$\frac{\partial u}{\partial x}$>0 考試越多越快樂

$\frac{\partial u}{\partial y}$<0 電動越玩越不快樂

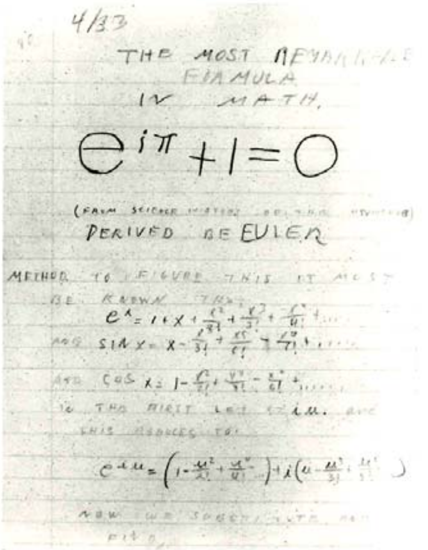
→ 人的個性

$\frac{dx}{dt}$, $\frac{dy}{dt}$ → 人的境遇

個性+境遇=人生

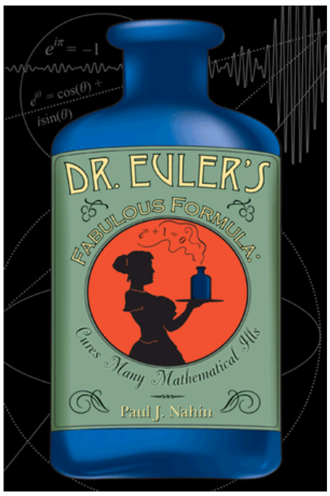
相形不如論心
論心不如則術
形不勝心
心不勝術　荀子非相

Note of Feynman at age of 15
費曼15歲的筆記

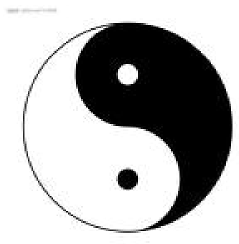


歐拉治療數學疑難雜症的神奇藥方

$$e^{i\theta}=\cos\theta+i\sin\theta$$



正反　　　陰陽　乾坤



一陰一陽之謂道

反者道之動

正回饋

負回饋

Feedback　　　回饋

Positive Feedback
正回饋

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = x$$

e^{-t}　　e^{t}

Negative Feedback
負回饋

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

其勢不可久

回也，其心三月不違仁，
其餘則日月至焉而已矣。

不常

反者道之動
一陰一陽之謂道
天之道其猶張弓　損有餘　　補不足
坤　直方大
天道好還週而復始
復　　　常

Exponential functions are both man's best friend and worst enemy.

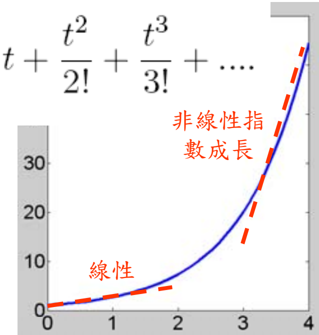
正回饋

$$\frac{dx}{dt} = -x$$

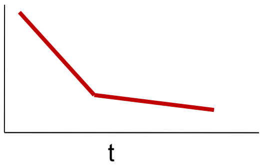
$$x(t) = x(0)e^{-t}$$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + ...$$

$$e^{t} = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...$$



指數遞減
e-folding time
（類似半衰期）



$$\lim_{t\to\infty} \frac{t^n}{e^t} = 0$$

Exponential Growth
指數成長超過任何多項式

Thomas Robert Malthus

(1766～1834)

English demographer and political economist

人口學家
政治經濟學家

$$\frac{dp}{dt} = \alpha p$$

$$p = p_0 e^{\alpha t}$$

人口成長由線性變成指數
指數成長無以倫比
環境、社會災難性

$$\frac{dN}{dt} = -kN$$

$$N(t) = N(t_0)e^{-k(t-t_0)}$$

4個變數

$N(t)$ $N(t_0)$ k t_0

Exponential Decay 指數遞減

Dating 同位素定年

Half life 半衰期

Finding *k* from two observations

由2個時間的N求取k

$$N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$N(t_2) = N(t_0)e^{-k(t_2-t_0)}$$

$$p = \frac{N(t_1)}{N(t_2)} = e^{-k(t_1-t_2)}$$

$$k = \frac{\ln p}{t_2 - t_1}$$

Radiocarbon Dating

Libby Nobel Prize for Chemistry in 1960

C14 half life ~5700 years:
Ratio of C14 to C12 is constant for living matter,
Begin to decay when dead.

$$k = \frac{\ln 2}{5700} \approx 1.216 \times 10^{-4}$$

$$N_0 e^{-kt_{1/2}} = \frac{1}{2} N_0$$

$$N(t_1) = pN(t_0)$$

Assumed to be the same as present

$$pN(t_0) = N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$t_0 = t_1 + \frac{\ln p}{k}$$

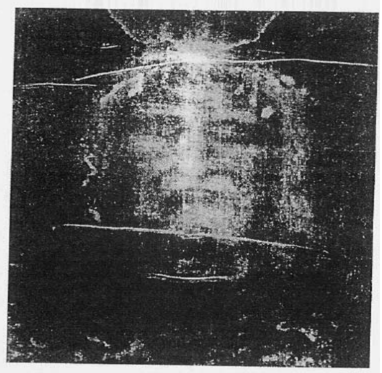
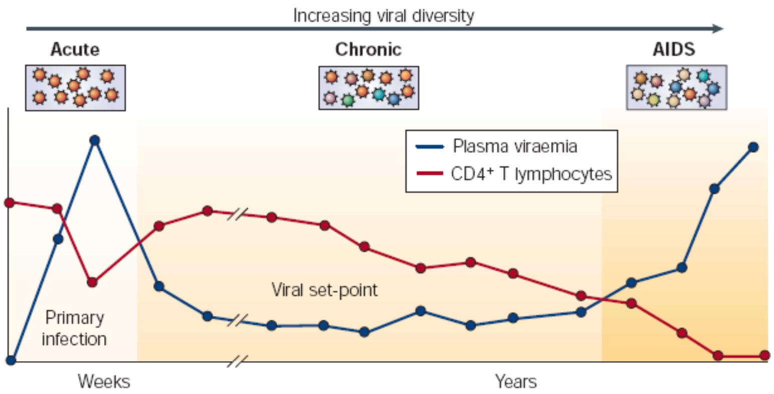


Figure 3.9 Shroud of Turin

Lascaux cave painting



Dormant Phase
潛伏期

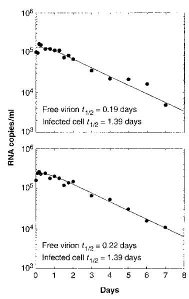
HIV Modeling

Perelson and Nelson (1999)

$$\frac{dV}{dt} = P - cV,$$ 藥物治療

$$\frac{dT}{dt} = kT_0V - \alpha T,$$

$$P = N\alpha T.$$



觀察病人服藥後反應決定**C**

Figure 4.5. Log of plasma concentrations (copies per mL) of HIV-1 RNA (circles) for two representative patients (upper panel, patient 104; lower panel, patient 107) after *ritonavir* treatment was begun on day 0. The solid line is a nonlinear least square fit to the data. HIV-1 RNA level is an easier measure of HIV virions since each HIV virion contains two RNA molecules. (See exercise 5 for more details.) (From Perelson et al. [1996], used by permission of Alan S. Perelson.)

$$P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \quad (1/(day \cdot ml))$$

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
  : non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

何大一雞尾酒療法

羅曼蒂克的羅蜜歐 與善變的茱莉葉

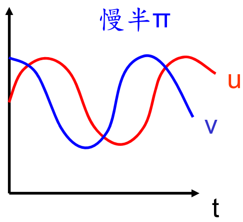
Romantic Romeo and Fickle Juliet (Strogatz 1988)

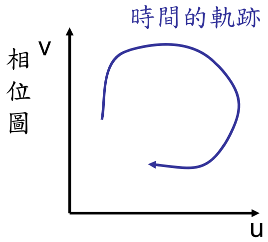
$$\frac{dR}{dt} = J \qquad \frac{dJ}{dt} = -R$$

$$\int_0^{2\pi} \cos t \sin t dt = 0$$

$u \quad v$

$$\overline{uv} = 0$$





Cos 和 Sin 零相關、不來電！

過程可以很熱鬧

瓊瑤小說？

相位圖

傅立葉

The profound study of nature is the most fertile source of mathematical discoveries.

自然研究是數學發展最肥沃的土壤

Fourier, Jean Baptiste Joseph

1768-1830

$$f(x) = \sum \hat{f}_k e^{ikx} \qquad \text{傅立葉級數}$$

$$\hat{f}_k = \frac{1}{2\pi} \int_0^{2\pi} f(x) e^{-ikx} dx \qquad \text{傅立葉轉換}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}_k e^{ikx} dx$$

$$\hat{f}_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

地球若從太陽取的能量，也必須散熱不然溫度會一直上升。

溫室效應
他的計算顯示地表溫度太低
（溫室效應低估）

1807年，傅立葉 39歲; 因為以cosine 和 sin級數表達三角形狀波動，計算熱傳導，而被偉大的數學家如 Lagrange，Laplace，Cauchy所責備與攻擊，罵他是「騙子」。

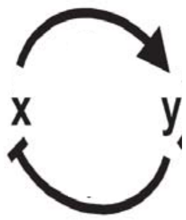
f(x) does not have to be analytical;
f(x) does not have to be periodic.

Periodic phenomena are actually everywhere in the biological world.

What else can you think of?

Negative Feedback Oscillators

X Cost Y Sin t



$$\frac{dy}{dt} = x$$

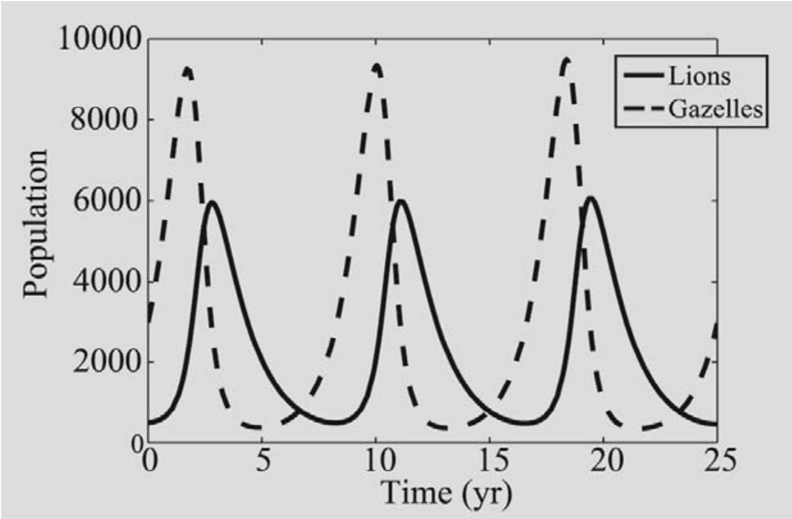
$$\frac{dx}{dt} = -y$$

物廉價美 顧客增加消費 價格上揚

價格上揚 顧客減少消費 價格下滑

負回饋

獅子與羚羊



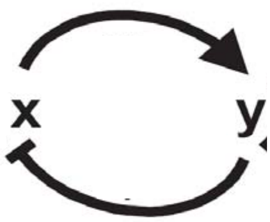
Negative Feedback Oscillators

負回饋　　恩將仇報　　以德報怨

反者道之動　　常

天之道其猶張弓　損有餘　　補不足

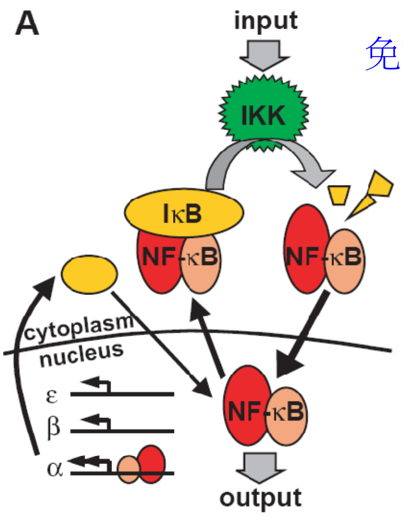
$$\frac{dy}{dt} = x$$



$$\frac{dx}{dt} = -y$$

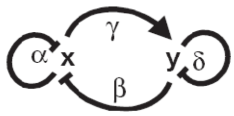
What can X and Y be? **X Cost　　Y Sin t**

NF-κB and IκB Model



Science **298:** 1241-1245.

免疫生命科學運作的穩定機制



X: nucleus NF-κB

Y: IκB

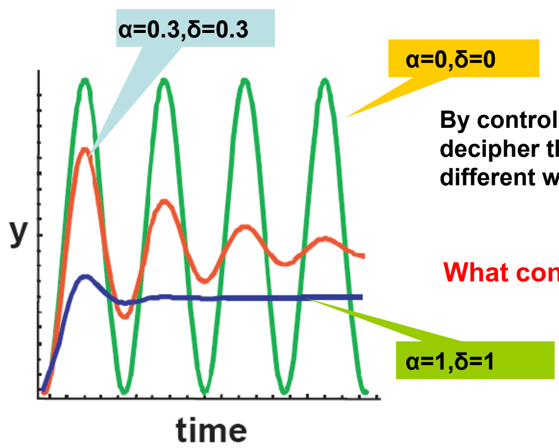
自我恢復

$$\frac{d}{dt}x = S - \alpha x - \beta y$$

$$\frac{d}{dt}y = \gamma x - \delta y$$

負回饋

Summary of NF-κB and IκB Model



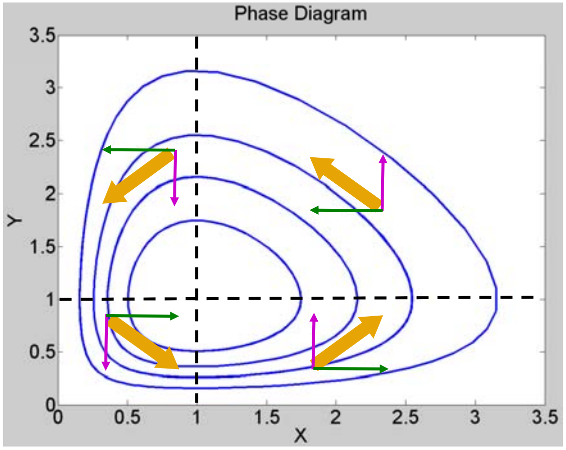
By controlling α and δ, a cell can decipher the same stimulus in different ways.

What controls α and δ?

自我恢復 彼此負回饋 外在刺激
產生不同系統反應

Science **298:** 1241-1245.

動力系統



dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

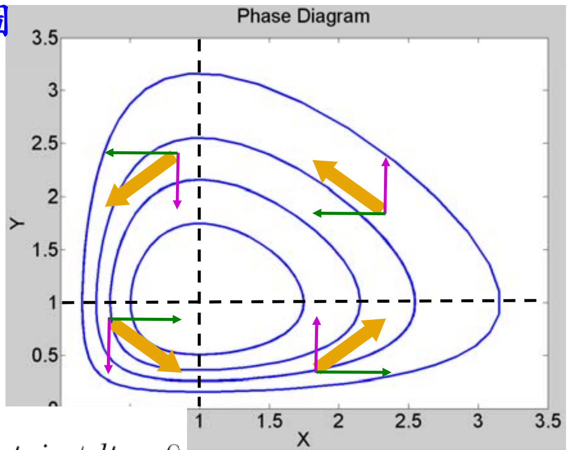
$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

相位圖

愛情動力系統

相位圖



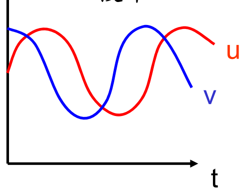
dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

慢半π



$$\int_0^{2\pi} \underset{u}{\cos t} \underset{v}{\sin t} dt = 0$$

Cos 和 **Sin** 零相關、不來電！

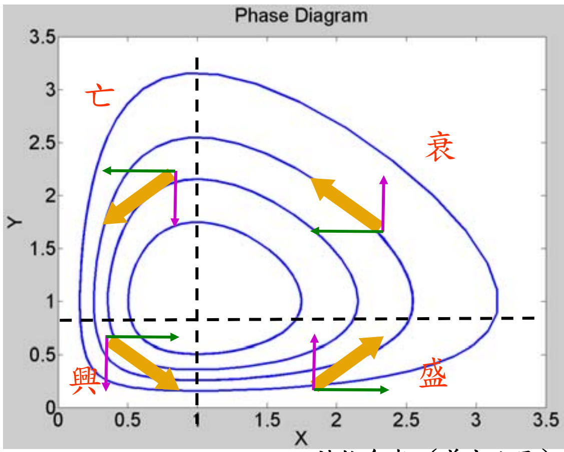
動力系統

鴛鴦蝴蝶夢，一段愛情故事；

千古興盛衰亡。

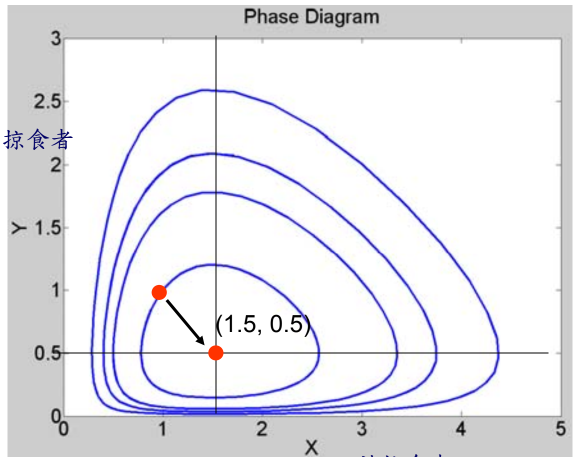
$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$



$$\begin{cases} \frac{dx}{dt} = x - xy - hx \\ \frac{dy}{dt} = -y + xy - hy \end{cases}$$

h=0.5



Harvest effect
對掠食者與被掠食者同時進行獵取殺害，可以使生態往對被掠食者有利方向移動。

1960 DDT 事件

無間道警察
水滸傳哲學

Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)

The "***Black Death***" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

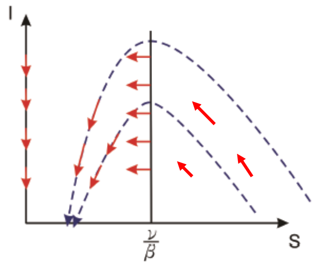
$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu)I$$

$$\frac{dR}{dt} = -\nu I$$

ν Recovery Rate 治癒率

β Infection Rate 感染率

No Death in the model



$S = \frac{\nu}{\beta}$ 有 null cline

都會區容易大流行
提升治癒率降低感染率
感染人數多不一定大流行

Forecast and control of epidemics in a globalized world **PNAS vol.101 no.42**
Hufnagel†, Brockmann, and Geisel

演講者: 陳怡文 日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

S I S 模式　連續感染 感染不產生免疫

$$\frac{dS}{dt} = -\alpha SI + \mu I$$

$$\frac{dI}{dt} = \alpha SI - \mu I$$

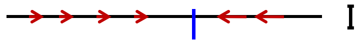
感染　治癒

$$\frac{dS}{dt} + \frac{dI}{dt} = 0$$

$$S + I = 1$$

$$\frac{dI}{dt} = \alpha(1-I)I - \mu I$$

$$\frac{dI}{dt} = \alpha(1-\frac{\mu}{\alpha}-I)I$$



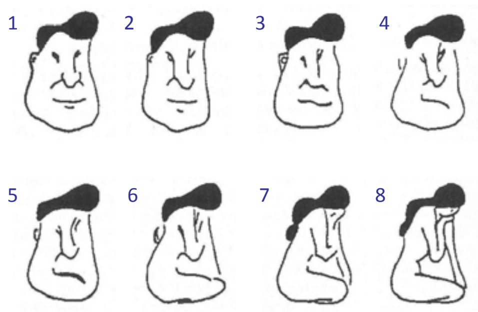
$$1-\frac{\mu}{\alpha}$$

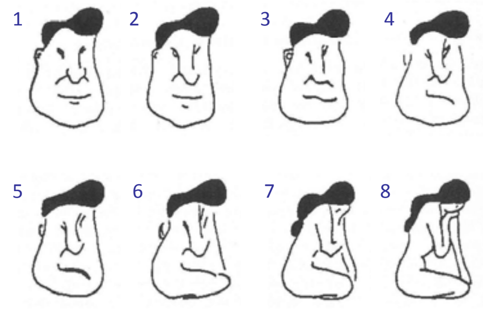
$$\frac{dI}{dt} = \alpha(1-\frac{\mu}{\alpha}-I)I$$

永遠有一部份人得病

疾病絕跡不在於零感染

若治癒率超過感染率則可以使病絕跡





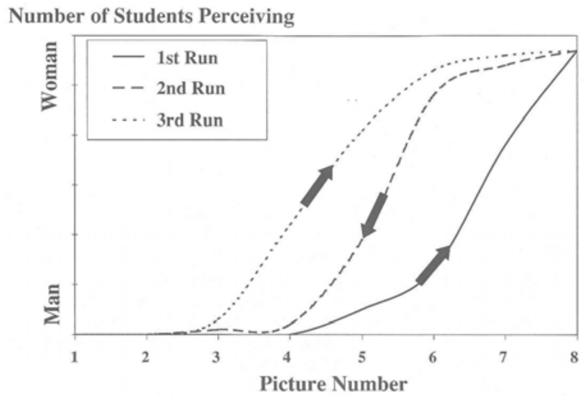
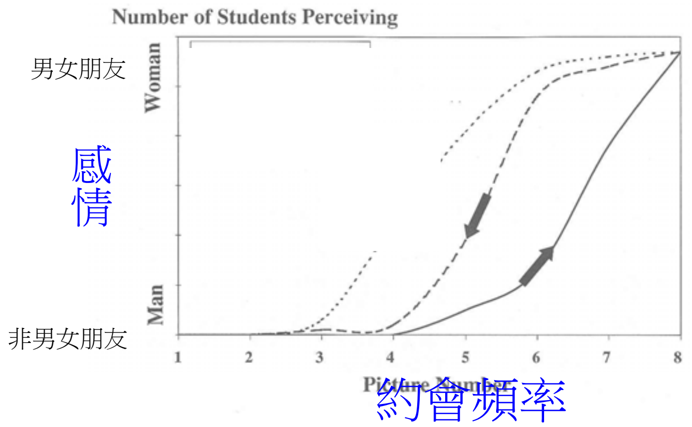


Figure 6.8: Schematic representation of the visual catastrophe based on the data in table 6.1 on three runs (123456787654321234 5678) through the series of pictures in figure 6.7. On the vertical axis is plotted the number of students perceiving a change in their perception. At the level "man," all of the students perceive a man, and at the level "woman," all of the students perceive a woman. On the first run (solid line), all of the students see a man's face until the fifth picture, where a few students suddenly see a woman. The biggest jump occurs at the seventh picture. By the eighth picture, all of the students are seeing a seated woman. On the second run, most of the students continue to see a woman until the fifth and fourth pictures are presented, after which point most of the class is seeing a man's face. A similar pattern appears in the line for the third run, but the shifts in perception do not occur in the same place as they did for the second run. Thus these results are a good illustration of one-time hysteresis.

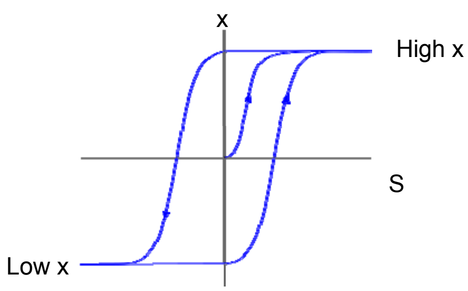
Hysteresis　遲滯

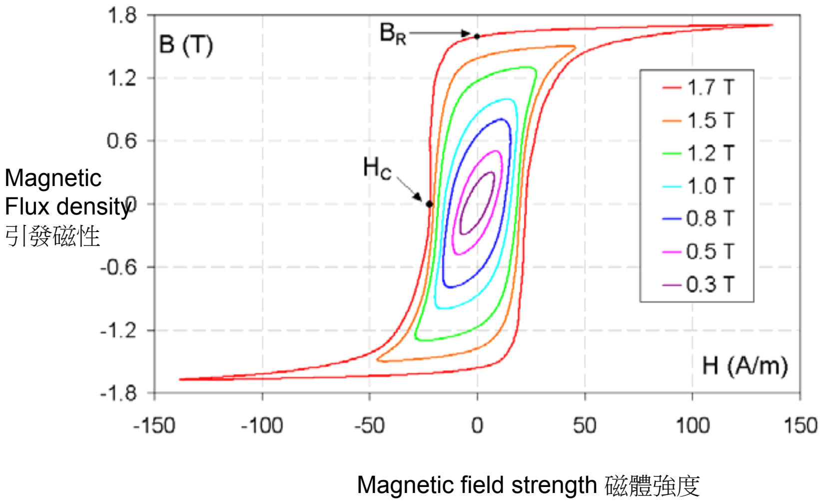


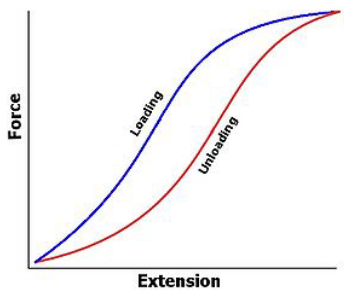
$$\frac{dx}{dt} = -\mu x + \alpha \frac{x^n}{k^n + x^n} + \sigma \frac{S}{\rho + S}$$

感染人數時間變化　治癒　相互感染　外在公衛健康環境指數

Cytokine Memory　恢復　蛋白質自我正回饋　外在蛋白質驅誘力

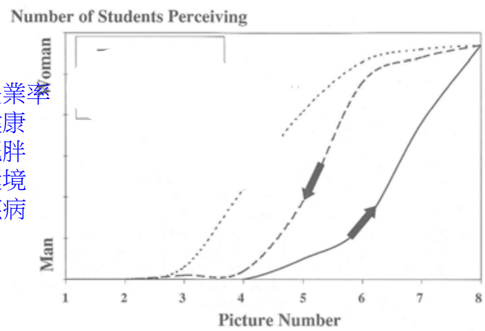






In one sense the rubber band was harder to stretch when it was being loaded than when it was being unloaded.

失業率
健康
肥胖
環境
疾病



經濟條件 （通貨膨脹）
身體條件
環境條件
公衛條件

Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

His 'forecast' was a catastrophic failure:

$\Delta p = 145$ hPa in 6 hours

His method was unimpeachable.

So, *what went wrong?*

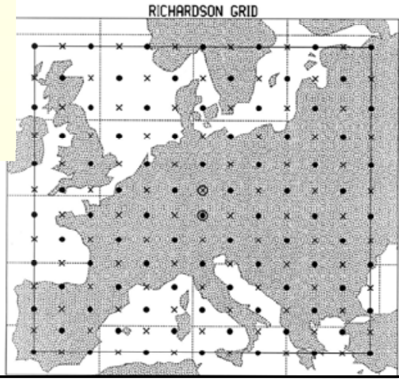
$$\frac{dQ}{dt} \to \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

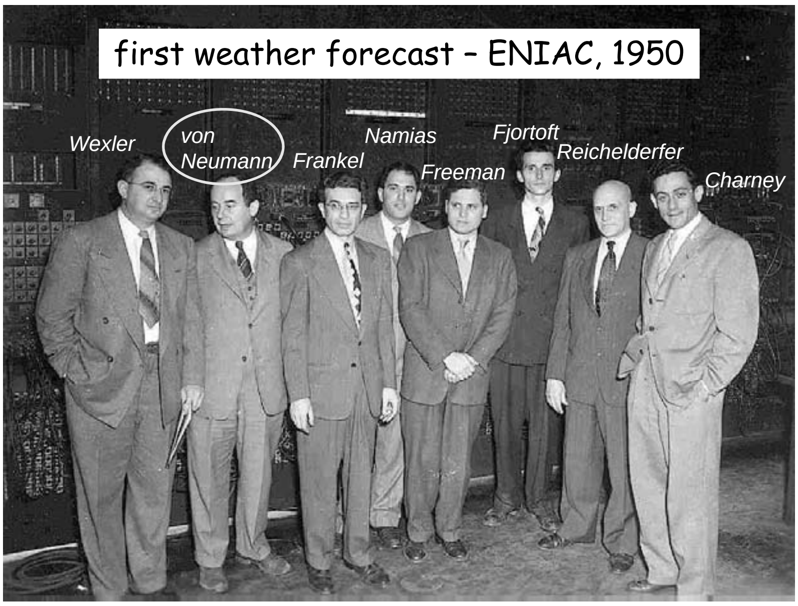
$$\frac{df}{dx} \to \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}.$$

和平非戰主義者
一次大戰自願至西線開救護車
同時發了兩年計算未來天氣

對於機槍、鐵絲網與碉堡產生的人類最血腥戰爭，以及氣象幫助毒氣施放深惡痛絕。

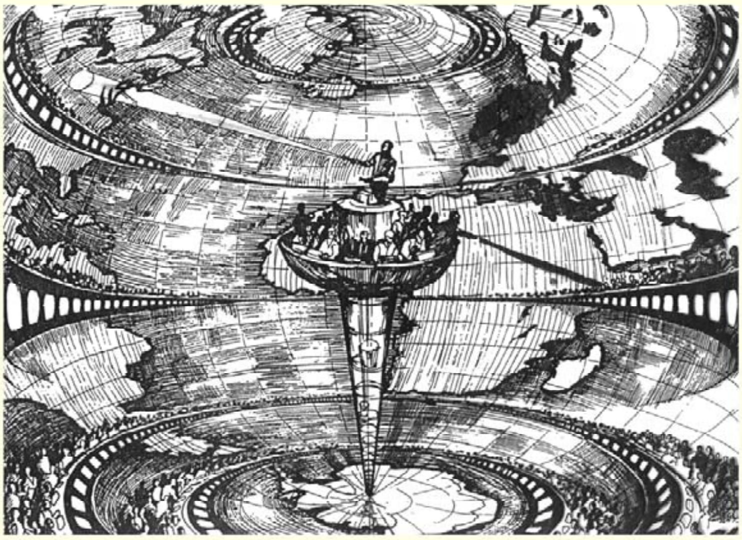
13×13=169個ODE
169 自由度





In front of the Eniac, Aberdeen Proving Ground, April 4, 1950, on the occasion of the first numerical weather computations carried out with the aid of a high-speed computer.

Richardson’s Dream



Richardson’s Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor

Richardson’s Game Theory 賽局理論

短文「軍備競賽可否不產生戰爭？」
此文在Richardson逝世後，
1953年受到重視。

Richardson認為讓雙方加強軍備競賽，
但因雙方都無必勝把握暫時不開戰，
最後沒效率的國家因軍備競賽而經濟破產。

Arm Races

$$\frac{dx}{dt} = -a(x - x_0) + by$$

指數歸零降溫

$$\frac{dy}{dt} = -a(y - y_0) + bx$$

正回饋
（彼此越看越不順眼）

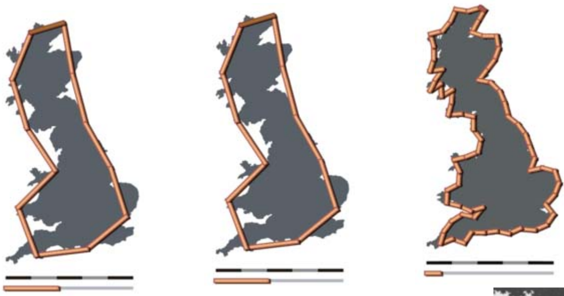
x, y war potential

1/a < 1/b   Truce

(Peace?)

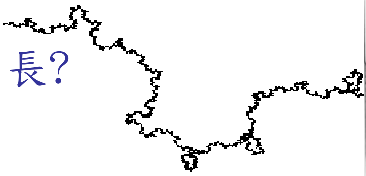
Could an arms-race end without fighting?
“Yes, without a shot being fired,”
if one side outspent the other on armaments
and the weaker nation bankrupted itself.

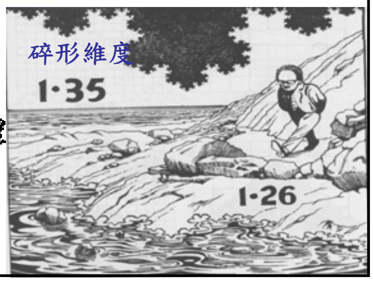
A relation between the probability of two countries going to war and the length of their common border.
戰爭可能性和共同邊界長短成比例。



相同邊界 不同長度

海岸線有多長?





碎形源起

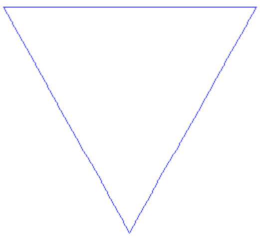
自然的形狀近似於碎形

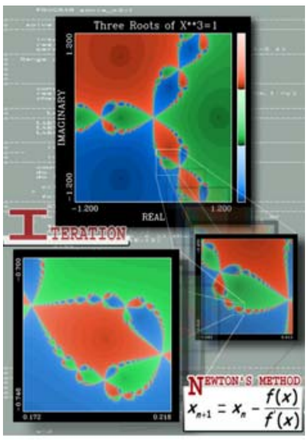
Fractal 碎形

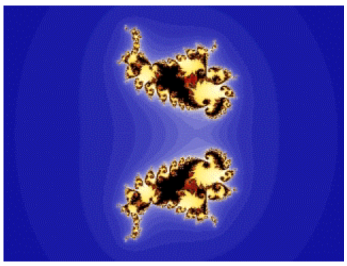
自我相似性

奇異吸子

碎形維度

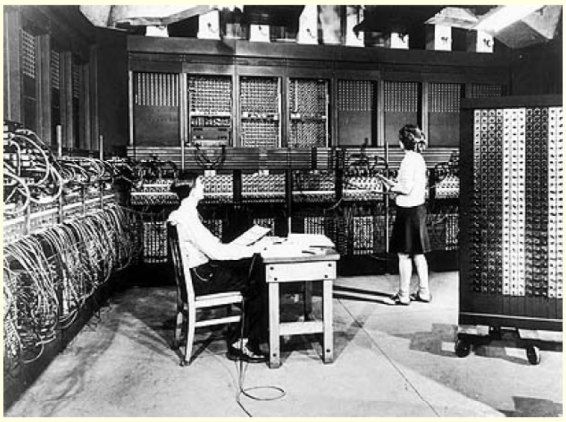






Richardson 據說是一個很無趣的演講者，
沒得過任何大獎，但其思考超越他的時代。

The ENIAC
Electronic Numerical Integrator and Computer



18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

No high-level language
Assembly language

500 Flops
Function Table 0.001 s

3,700,000,000 times slower than current day large computer

第一部電腦　　氣象預報

To see a world in a grain of sand
And heaven in a wild flower
Hold infinity in the palm of your hand
And eternity in an hour.
William Blake

一沙一世界
一花一天堂
手中掌握無限
剎那即是永恆

Scale free

0.01s

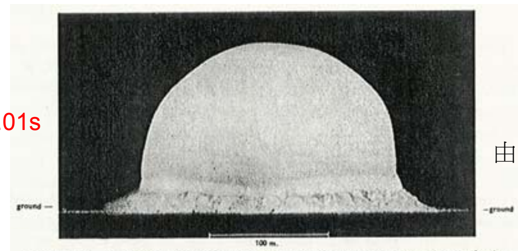


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963

G.I. Taylor 1950

由照片0.01s猜出爆炸威力 10^{14} J

$$\frac{E}{\rho r^5 t^{-2}} = \Pi$$

$$\rho = 1kgm^{-3}, \quad r \approx 100m, \quad t \approx 0.01s$$

$$E \approx \rho r^5 t^{-2} \approx 10^{14} J$$

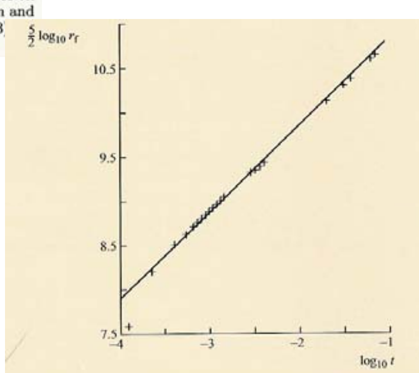


Figure 0.3. Logarithmic plot of the fireball radius, showing that $r_f^{5/2}$ is proportional to the time t (Taylor 1950b, 1963).

廣島原子彈能量～ 10^{14} J

Metabolic rate vs size

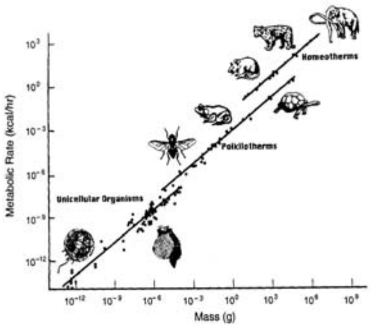


FIGURE 2 Metabolic rate (in kcal/hr) for a series of organisms ranging from the smallest microbes to the largest mammals as a function of mass (in g), exemplifying the persistence of the 3/4-power scaling law (the solid lines) over 20 orders of magnitude (Hemmingsen [12]).

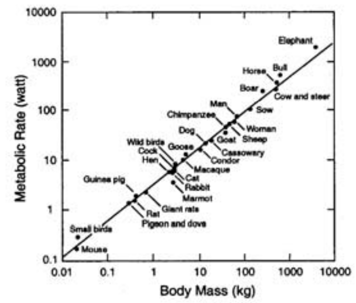


FIGURE 1 Metabolic rate (in watts) for a series of mammals and birds as a fur of mass (in kg); the scale is logarithmic and exemplifies the 3/4-power scalin discovered by Kleiber [2, 22, 27, 29].

$$I = I_0 M^{3/4}$$

Hemmingsen (1960) *Reports of the Steno Memorial Hospital and Nordisk Insulin Laboratorium* **9**, 6-110

Kleiber (1932) Body size and metabolism. *Hilgardia* **6**, 315-353.

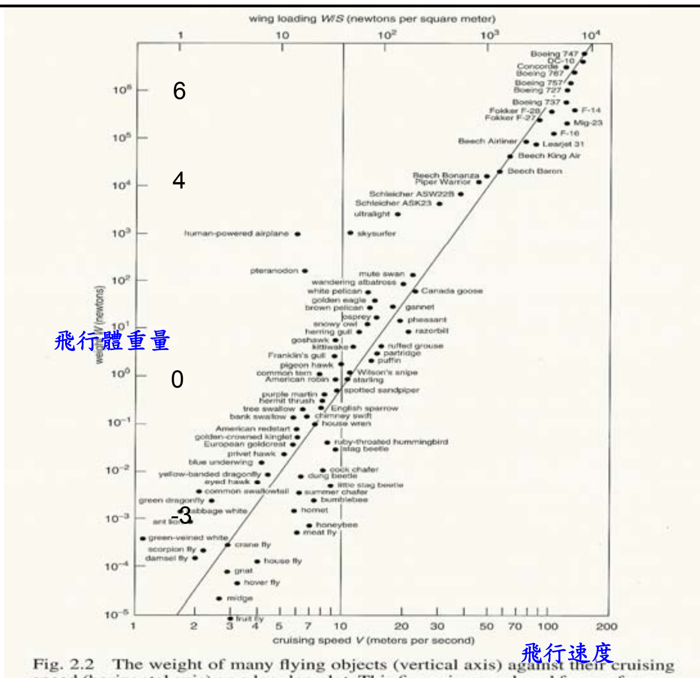


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.

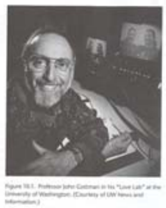
A beautiful woman !

不同尺度不同科學

Anything different ?

Lincoln !!

The Mathematics of Marriage,
MIT Press 2002
Gottman, Murray, Swanson, Tyson, Swanson



H unhappy
W happy

H happy
W happy

H unhappy
W unhappy

H happy
W unhappy

問世間婚姻是何物？

To derive the equations ..... or
not to derive the equations
that is a question !!

- First Application of Mathematical Modeling in Social-Sciences.
- Problem: The divorce rate for second marriages is even higher! Why? Don't they become wiser from their first experience?
- Based on mismatches in the couples personality or modes of communication seems NOT be too successful.

- "Men are from Mars, women are from Venus"— a lack of understanding of gender differences in communication styles is at the root of marital problems.

- Another way approaching.

- Self-Interaction (uninfluenced steady state) 本性

$$\frac{dx}{dt} = r_1(x_0 - x), \qquad u(t) = u(0)e^{-r_1 t}$$

$$\frac{dy}{dt} = r_2(y_0 - y). \qquad v(t) = v(0)e^{-r_2 t}$$

- Marital Interactions (influenced function) 影響對方

$$\frac{dx}{dt} = r_1(x_0 - x) + I_1(y),$$

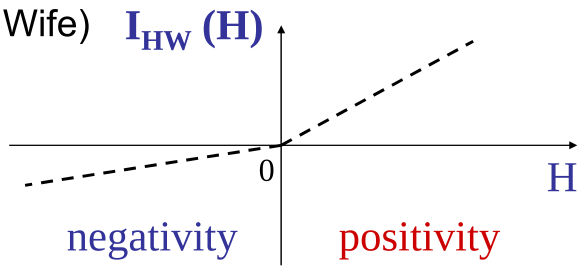
$$\frac{dy}{dt} = r_2(y_0 - y) + I_2(x).$$

$$I_i(z) = \begin{cases} a_i z & if\ z > 0 \\ b_i z & if\ z < 0 \end{cases}$$

Introduction ■ **Modeling** Example Solve & Explanation Conclusion

- **Five marriage types** – different interaction styles – related to interaction functions (I) in the model
- Basic model fits empirical data to a two-slope linear interaction function I (one each for the Husband and Wife)

-	+	I validating
0	+	II avoider
-	0	III volatile



Friday Applied Mathematic Seminar, National Taiwan University, Department of Mathematics

- Characteristics (stable and unstable types):

Volatile (S) – romantic, passionate, have heated arguments with cycles of fights and sex.

Validating (S) – calmer, intimate, value companionate marriage, shared experience rather than individuality.

Avoiders (S) – avoid confrontation and conflict, interact only in positive range of their emotions.

Hostile (U) – (mixed) conflict-avoiding wife, validating husband.

Hostile-Detached (U) – (mixed) volatile wife, validating husband

An example of a Validating Couple

- For a validating couple, take for simplicity, we have

$$\frac{dx}{dt} = r_1(x_0 - x) + a_1 y,$$
$$\frac{dy}{dt} = r_2(y_0 - y) + a_2 x.$$

Let (x^*, y^*) denote the equilibrium solution, then

An example of a Validating Couple

$$r_1(x_0 - x^*) + a_1 y^* = 0,$$
$$r_2(y_0 - y^*) + a_2 x^* = 0.$$

The solution is

$$x^* = [x_0 + \frac{a_1}{y_1} y_0]/[1 - \frac{a_1 a_2}{r_1 r_2}],$$
$$y^* = [y_0 + \frac{a_2}{y_2} x_o]/[1 - \frac{a_1 a_2}{r_1 r_2}].$$

$$x^* > x_0, \quad and \quad y^* > y_0$$

Stable Marriage

吾生也有涯，吾知也無涯，已有涯逐無涯，殆矣。莊子
有限時空之觀察或有限資料去推導無限時空的科學定律。

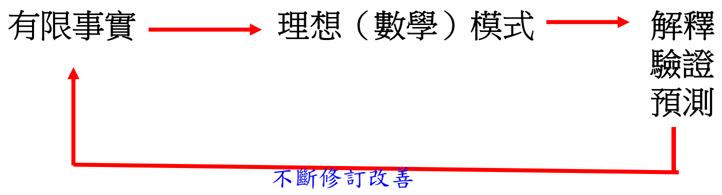
凡所有相皆是虛妄，若見諸相非相，即見如來。金剛經
人類建構的知識與真實世界的差異很大。

科學需形而上學的假設，也需嚴謹科學方法。科學觀點需要不斷更新修訂。

預測=外插，外插是絕對不穩定（數學）。

複雜系統變數太多，找相同條件下成立因果律不易。

以特殊事實為憑藉，逐漸推廣引伸， 成立概念式定律的系統，以便籠罩更複雜耿廣泛的對象，科學家依據事實為前提來證明普遍的結論。 方東美



科學方法

Now we only see models,
like reflections in a mirror;
but then we shall see face to face.
Now I only know partially;
but then I shall know as fully as
I am myself known.

St. Paul, 1st letter to the Corinthians, 13:12

Models、經典、聖哲就如鏡子，讓我們看到自己，讓我們瞭解自己的侷限，更進而體會完整的人性。

「數學科學模式」幫助我們由片面觀察的自然界，統會瞭解共通完整的科學定律。寄王汎森院士

動力系統

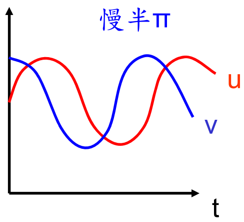
$$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}, \gamma_i)$$

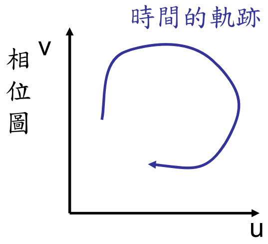
時間變化謂之動力　　變數　　許多外在及內在控制參數

$$\int_0^{2\pi} \cos t \sin t dt = 0$$

u　　v

$$\overline{uv} = 0$$





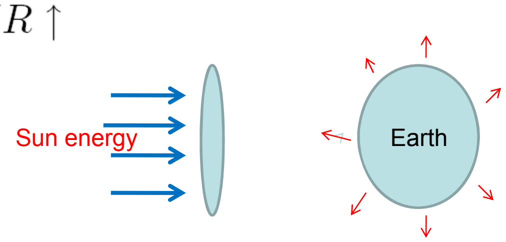
Cos 和 Sin 零相關、不來電！

相位圖

推背圖：前知三百年，後知三百年
可以解釋〝已知〞，可以預測〝未來〞

$$C\frac{dT}{dt} = S\downarrow - IR\uparrow$$

比熱
specific
heat



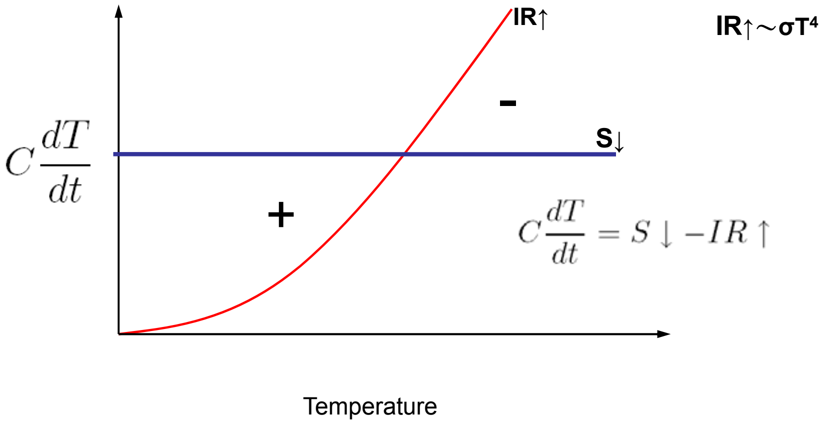
$$S\downarrow = \pi a^2\, s(1-\alpha) \qquad IR\uparrow = 4\pi a^2\, \epsilon\sigma T^4$$

反照率 albedo

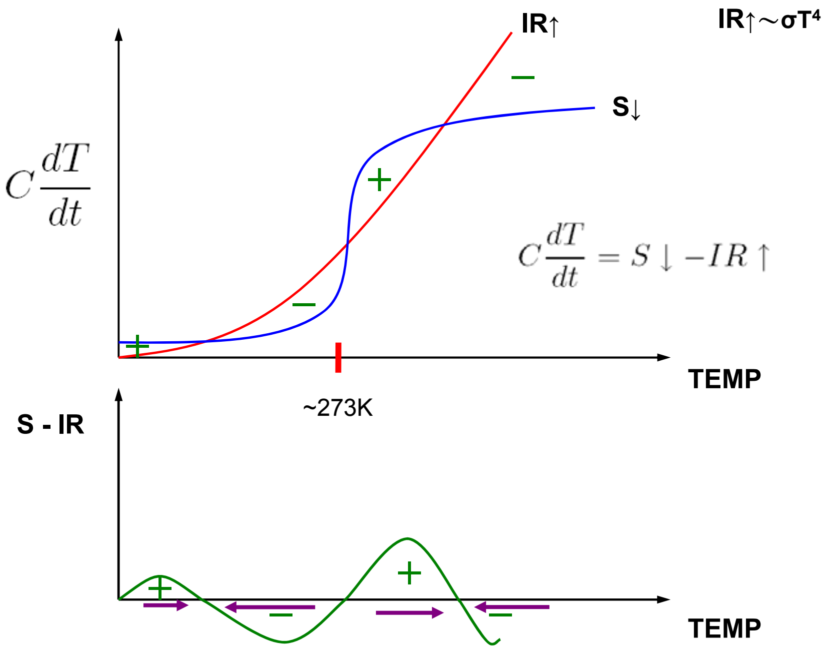
比熱　　海水　深層海水

反照率　　冰雪　　雲　(IPCC沒討論的因素，氣象最大的挑戰)

太陽常數　　天文因素　　太陽物理



Don’t Worry! Be Happy!



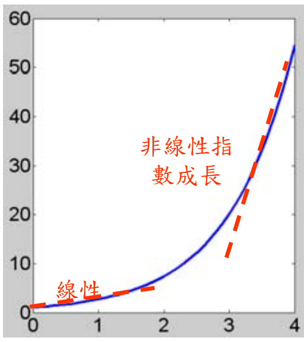
2 stable multiple equilibria

Malthusian Model

Population Growth

$$\lim_{\Delta \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\frac{dp}{dt} = \alpha p \quad \Rightarrow \quad p = p_0 e^{\alpha t}$$



線性　　非線性

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Logistic Effect

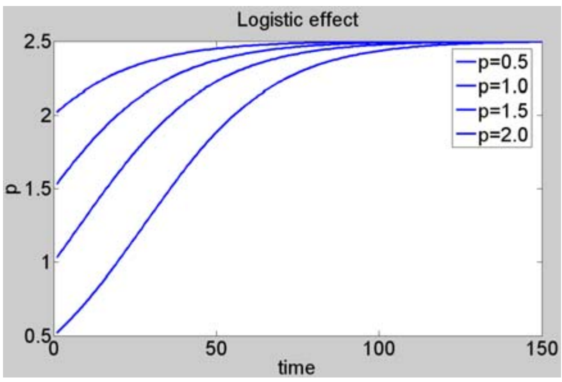
$$\frac{dx}{dt} = x(1-x)$$

Time Series (時間序列)

$$\frac{dp}{dt} = \alpha p - \beta p^2 = \alpha p\left(1 - \frac{p}{\frac{\alpha}{\beta}}\right)$$

其中 $\frac{\alpha}{\beta}$

→ 環境之承載效應



Lotka-Volterra Model

A Predator-Prey Model

Humberto D'Ancona
1926

World War One 掠食者比例變大

Port	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
Fiume	12%	21%	22%	21%	36%	27%	16%	16%	15%	11%
Trieste	14%	7%	16%	15%	-	18%	15%	13%	11%	10%

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

Vito Volterra

Wikipedia

(1860～1940)

Italian
Mathematician & Physicist

Contribution:

1) Mathematical biology

2) Volterra-Lotka equations

$$\frac{dx}{dt} = x(1-x) - h$$

h: constant rate of population harvested.

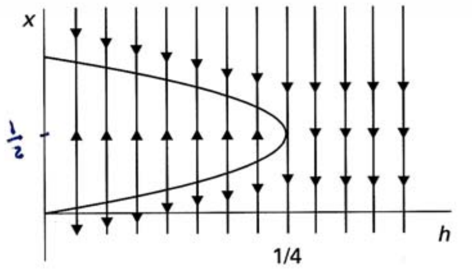


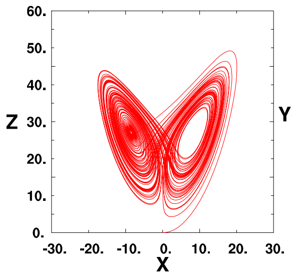
Figure 1.7 The bifurcation diagram for $f_h(x) = x(1-x) - h$.

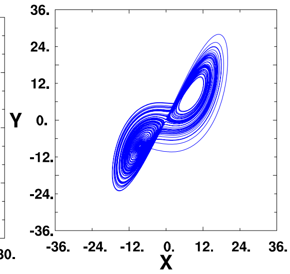
Small changes in harvesting rate can lead to disastrous changes in population has been observed many times in real situations on earth.

- New language for describing marital interaction and social influence and rationale for the marital experiments
- Concept that marriages can be classified into one of 5 types of marriage depending on the couple's interaction style:
- Stable marriages have matched interaction styles. Unstable marriages have mismatched interaction styles
- Couple's interaction data suggest specific therapy

Edward Norton Lorenz
(1917～2008)

American
Mathematician & Meterologist

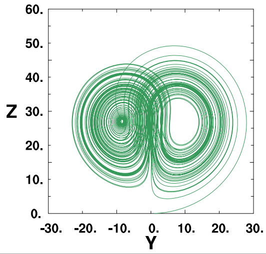




$$\frac{dX}{dt} = -\sigma X + \sigma Y$$

$$\frac{dY}{dt} = -XZ + rX - Y$$

$$\frac{dZ}{dt} = XY - bZ$$



$$\sigma = 10, \ r = 28, \ b = \frac{8}{3}$$

蝴蝶效應 Butterfly Effect Chaos 混沌

混沌 非線性 $y = x^{30}$ 精確度有限 非線性

$$0.02 \begin{cases} x = 0.99 \\ x = 1.01 \end{cases} \qquad 0.61 \begin{cases} y \approx 0.74 \\ y \approx 1.35 \end{cases}$$

預報能力的喪失！！

"Sensitivity dependence on initial condition."

H Poincare

London Millennium Bridge

London's Millennium Bridge is the first pedestrian river crossing over the Thames in central London for more than a century.

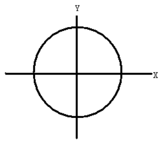
It is a 325m steel bridge linking the City of London at St. Paul's Cathedral with the Tate Modern Gallery at Bankside.

“Nice” lateral vibrations (20 cm S shape wobble, 1cycle per second) like on Tacoma Bridge developed on the day (June 12, 2000) of the opening……

Negative Feedback

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

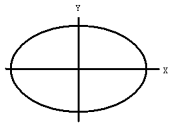


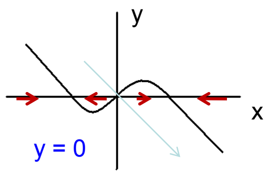
x = sint, y = cos t

NF + Fast and Slow

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -\epsilon x$$

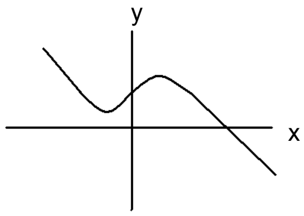


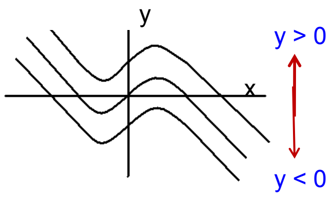


NF + FS + Multiple equilibriums

$$\frac{dx}{dt} = x - x^3 + y$$

$$\frac{dy}{dt} = -\epsilon x$$





A, G constant

$$F \sim \rho v^2 l^2$$

$$P \sim \rho v^3 l^2$$

$$G \sim \frac{l^3}{n}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$v \sim n^{1/9}$$

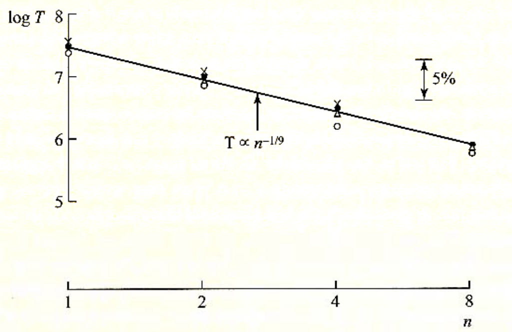
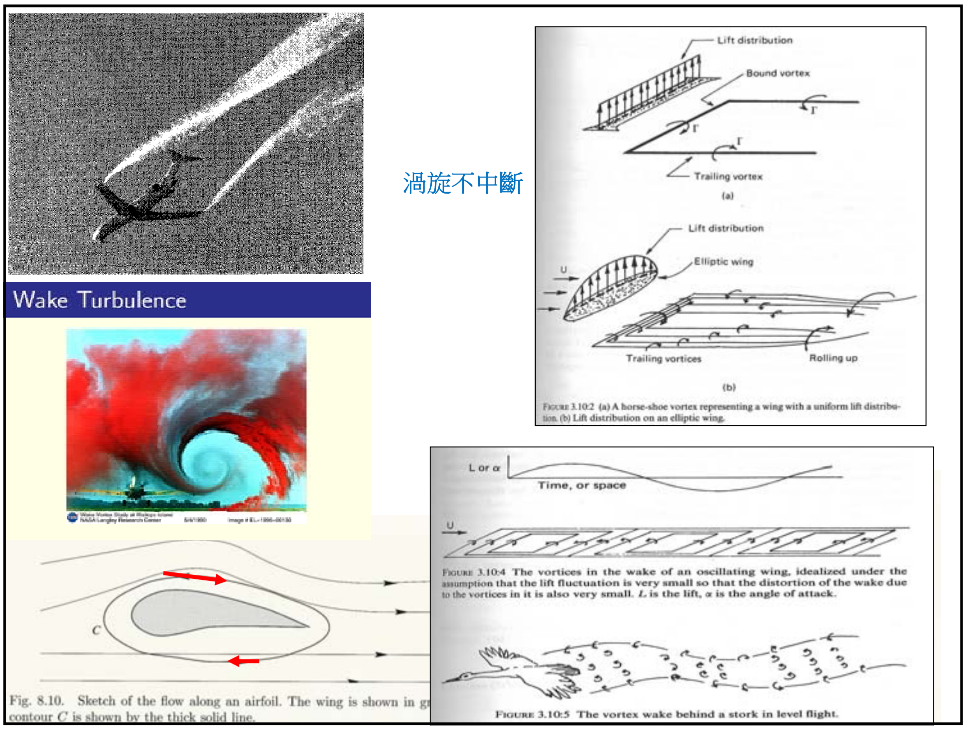
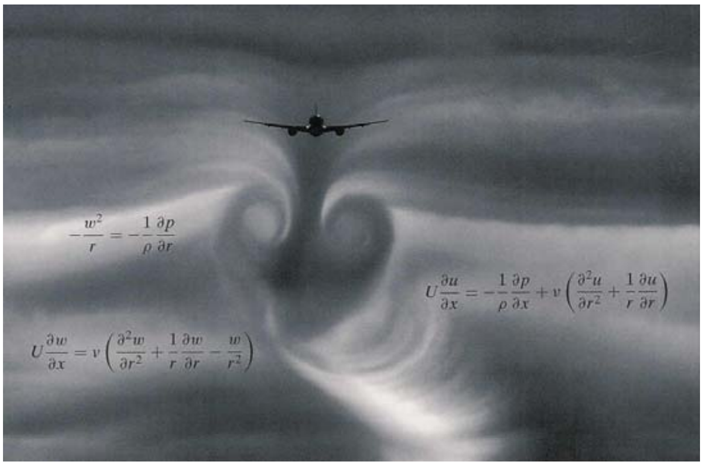


Figure 1.13. The −1/9 power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; ×, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).





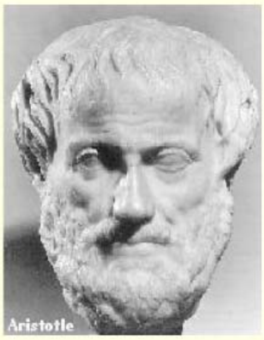
A British Airways Boeing 777-200 aircraft is approaching to land at Gatwick Airport traveling at 170 kts at approximately 1800 ft. The cloud base Is 2200 ft, RH = 83%, T = 16.8, Td = 14.5, p = 1022.2 hPa, wind = 6.4 km/h.

亞里斯多德

Aristotle's *Meteorologia*

氣象：空氣裡的事情

He wrote the first book on Meteorology, the Μετεωρολογια (μετεωρον: Something in the air)

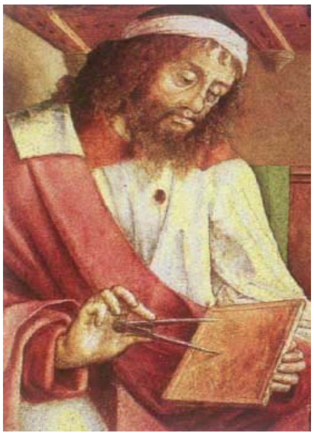


擅長於懷疑與發問

他的觀察讓他相信先聽到打雷再看到閃電

讀、算、寫

形而上學：憑藉第一原因，一切事物方能知曉，但其本身是自明的。



閱讀使人豐富
寫作使人精確

讀寫讓知識穿越不同時空

讀 算 寫

幾何
代數
微積分
電腦計算繪圖
數學建模/科學計算
Mathematical Modeling
Scientific Computing

+ -	x /
加、減	乘、除
線性	**非線性**
大題大作	小題大作