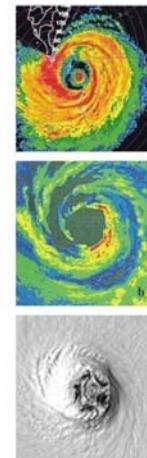


Vortex Interactions and Barotropic Aspects of Concentric Eyewall Formation



Hung-Chi Kuo

National Chair Professor
Department of Atmospheric Sciences
National Taiwan University
Taipei, Taiwan

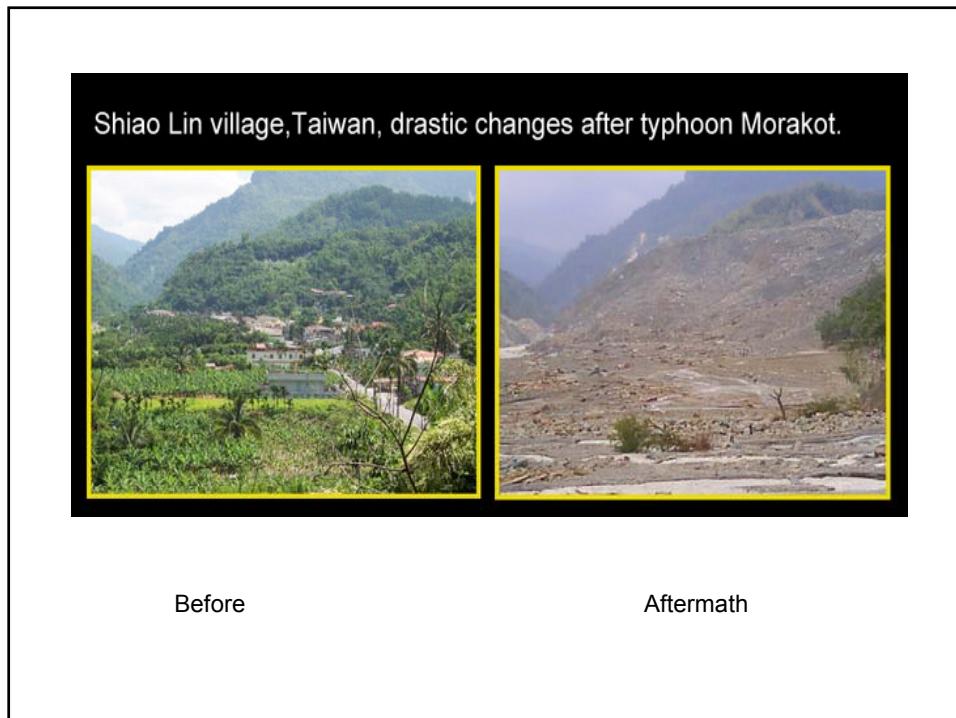


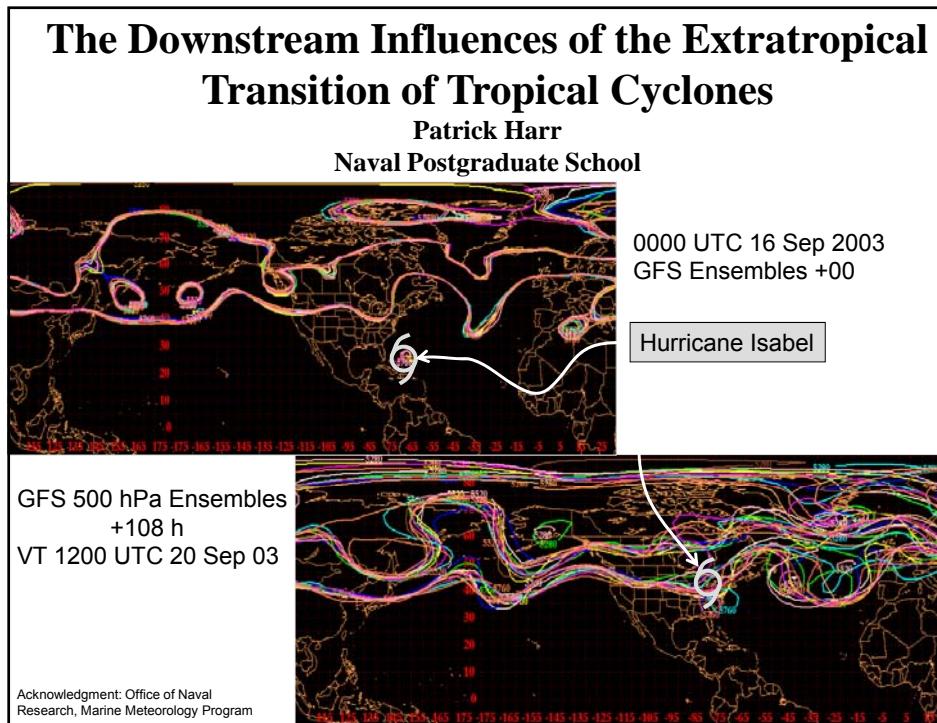
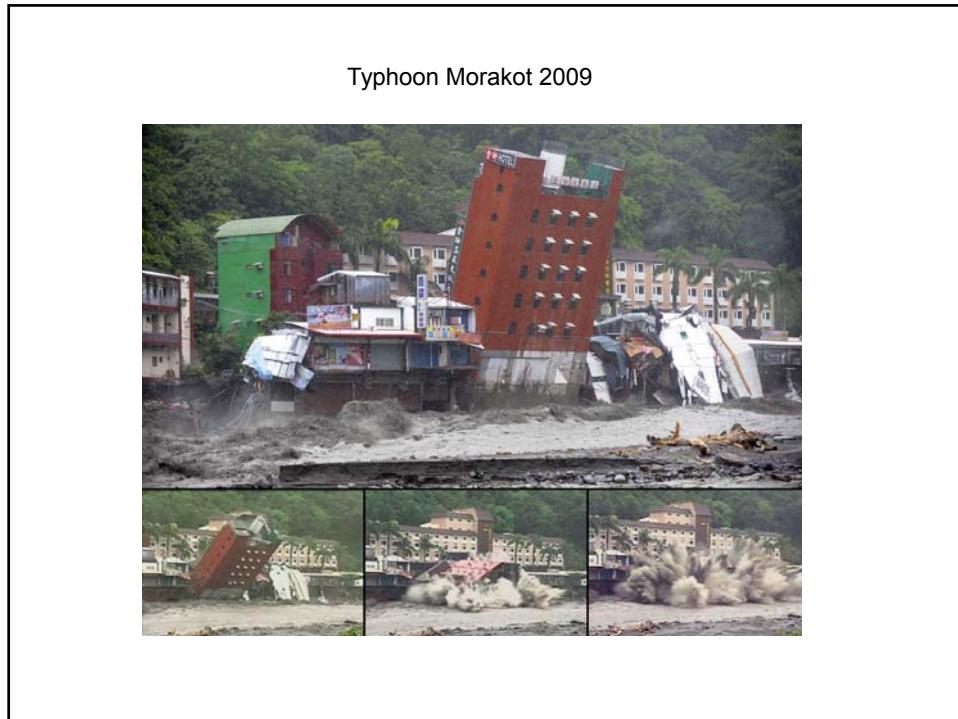
University of California Davis

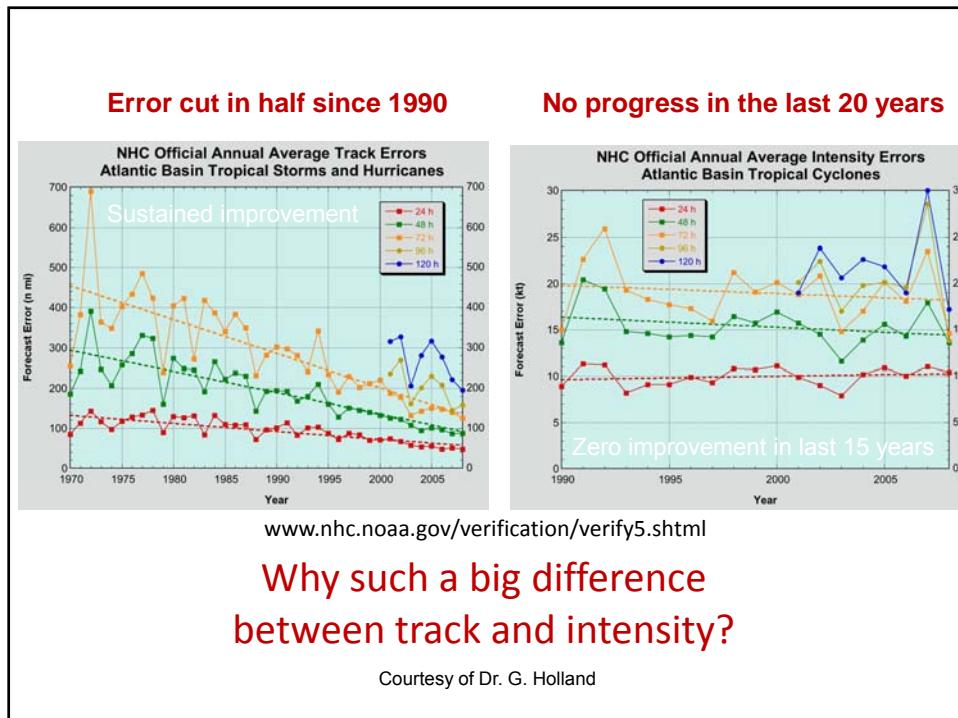
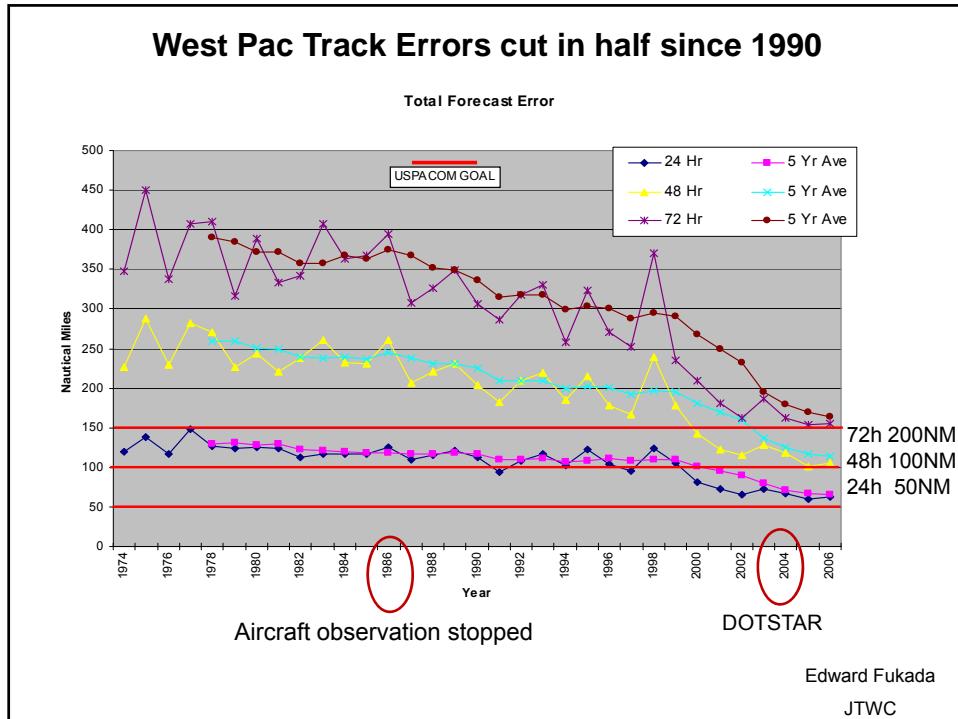
May 7 2010

象神颱風侵襲後的台北市 Taipei city aftermath of typhoon









Environmental Factors

Typhoon weakens
over region of cold water or low ocean heat content,
over land or region of decreased humidity,
over region of strong vertical wind shear.

However, the variance of typhoon intensity change from climatology is **not** explained well by the synoptic-scale environmental conditions.

It is fairly typical for typhoons to strengthen or weaken rapidly without any clear commensurate changes in the environment.

Internal meso-scale processes matter!

Latent Heat during typhoon made landfall

Precipitation ~1600mm

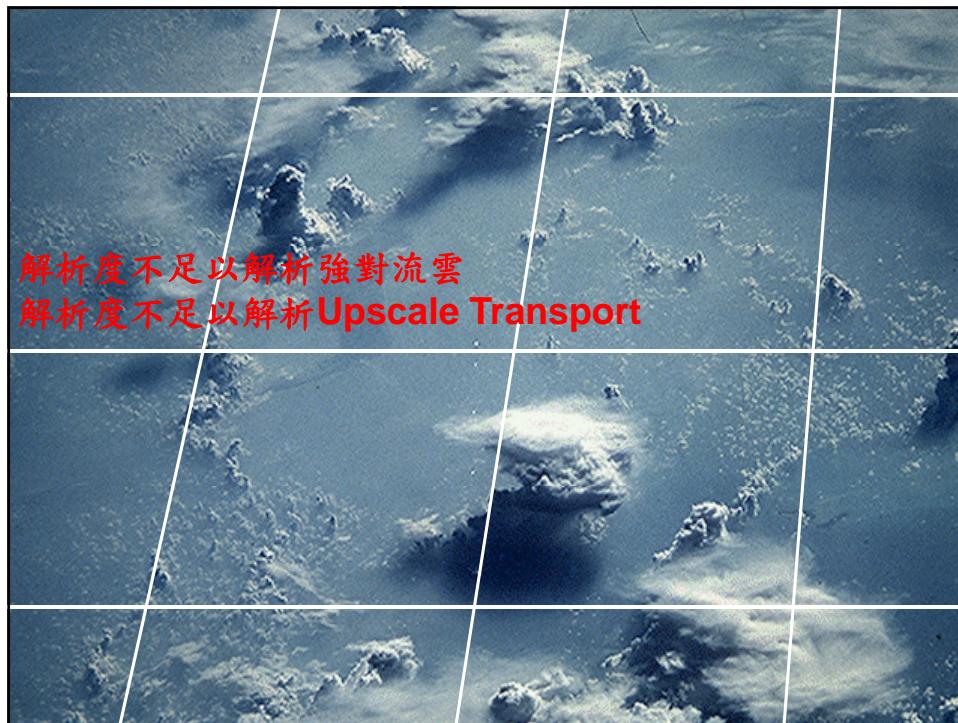
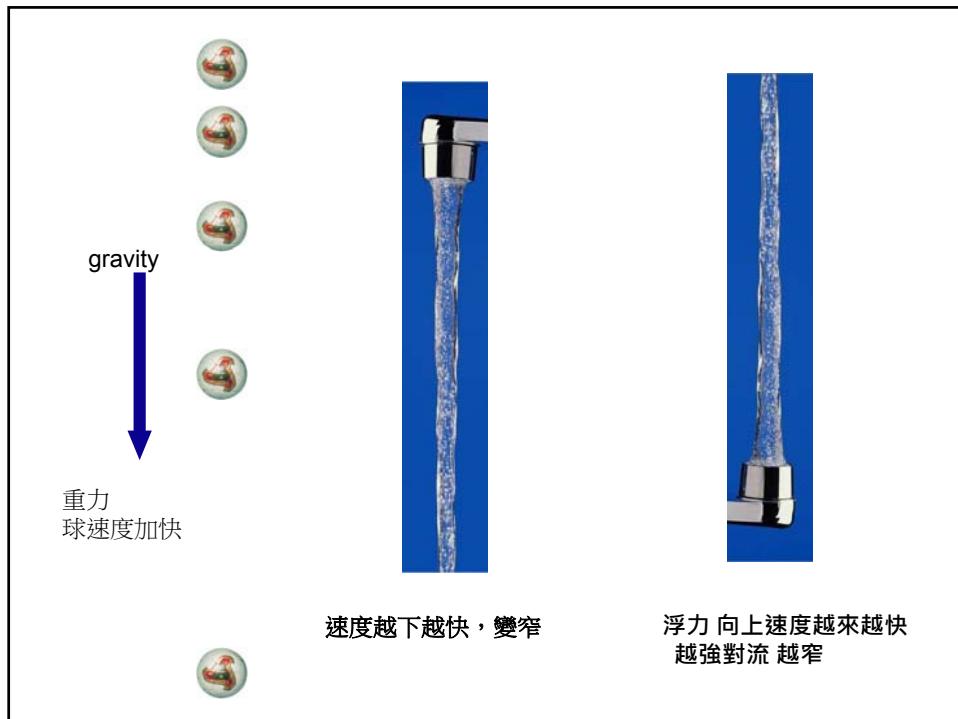
$$1600 \text{ mm} = 1.6 \text{ m}$$

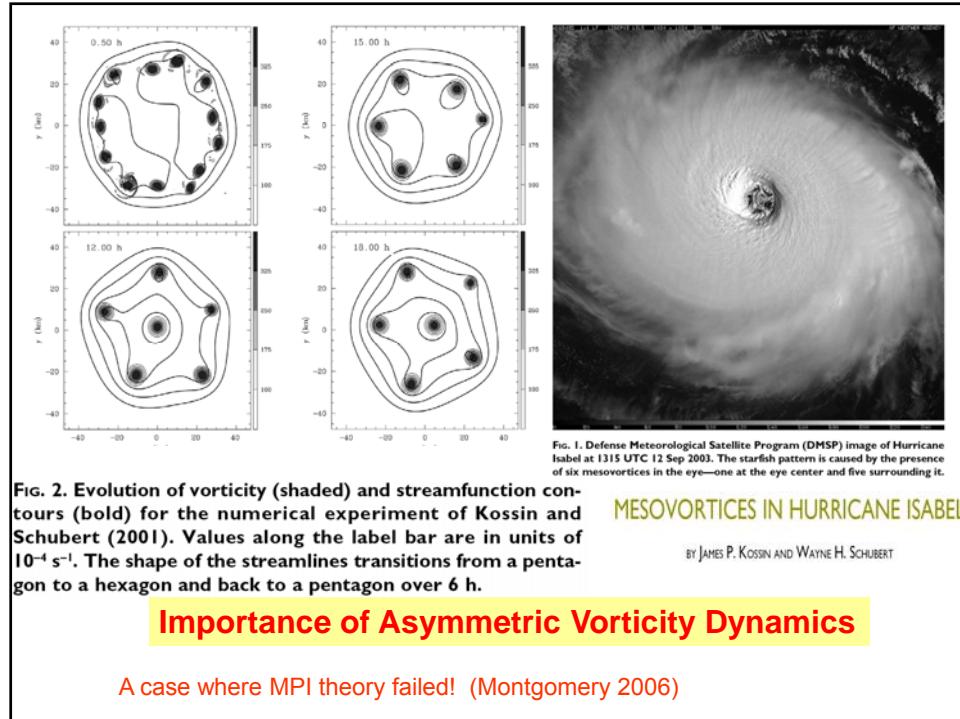
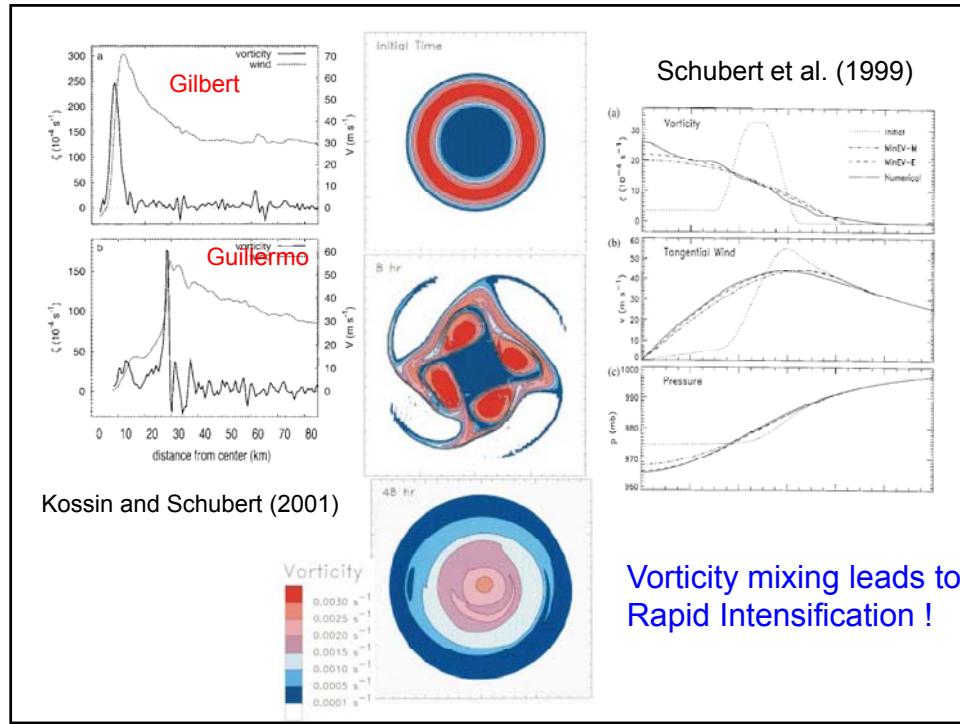
$$1.6 \text{ m} * 1000 \text{ kg m}^{-3} * 2.5 \times 10^6 \text{ J kg}^{-1}$$

$$= 4 \times 10^9 \text{ J m}^2$$

$$4 \times 10^9 \text{ J m}^2 * 2.5 \times 10^9 \text{ m}^2 \sim 10^{19} \text{ J}$$

Annual energy usage in Taiwan 10^{17} J





熱力學 + 流體力學

Euler 1755

$$\frac{d}{dt} \int_{v_m} \rho \vec{v} dv = - \int_{\partial v_m} p d\vec{s}$$

$$\int_{v_m} \rho \frac{d\vec{v}}{dt} dv = - \int_{v_m} \nabla p dv$$

$$\rho \frac{d\vec{v}}{dt} = -\nabla p$$

電磁學

Lorentz Force Law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\mathbf{F} = q(-\nabla V + \mathbf{v} \times \mathbf{B})$$

Lagrange 1781

$$\frac{\partial \vec{u}}{\partial t} + \vec{\zeta} \times \vec{u} = -\frac{1}{\rho} \nabla p - \nabla K - \nabla \Phi$$

Rotation , Vortex

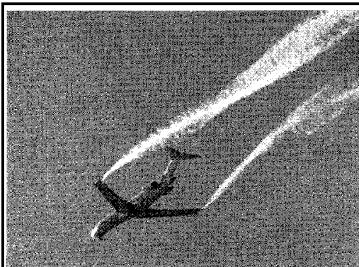
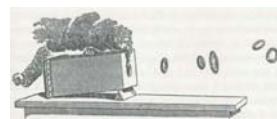
Helmholtz

1858

$$\frac{\partial \vec{\zeta}}{\partial t} + \vec{v} \cdot \nabla \vec{\zeta} + \vec{\zeta} \cdot \nabla \cdot \vec{v} = \vec{\zeta} \cdot \nabla \vec{v} + \vec{B}$$

$$\vec{B} = \nabla \times \left(-\frac{1}{\rho} \nabla p \right)$$

Vortex cannot terminated in the fluid interior.



Wake Turbulence



Fig. 8.10. Sketch of the flow along an airfoil. The wing is shown in gray; contour C is shown by the thick solid line.

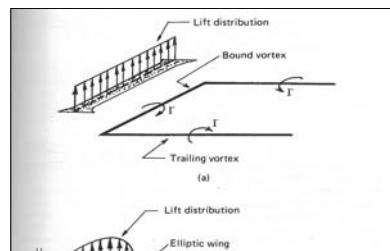


FIGURE 3.10.2 (a) A horse-shoe vortex representing a wing with a uniform lift distribution. (b) Lift distribution on an elliptic wing.

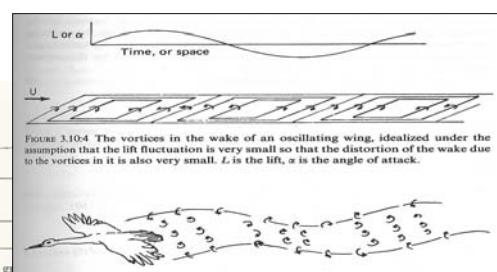
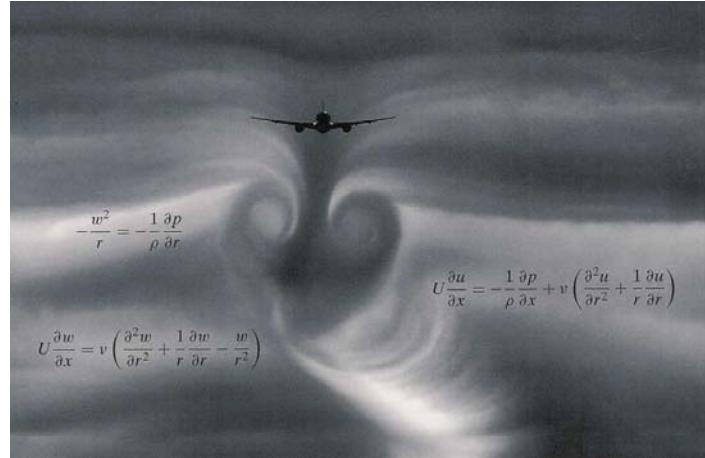


FIGURE 3.10.4 The vortices in the wake of an oscillating wing, idealized under the assumption that the lift fluctuation is very small so that the distortion of the wake due to the vortices in it is also very small. L is the lift, α is the angle of attack.



FIGURE 3.10.5 The vortex wake behind a stork in level flight.



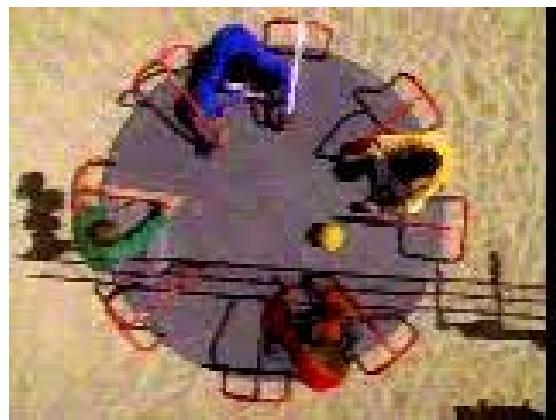
A British Airways Boeing 777-200 aircraft is approaching to land at Gatwick Airport traveling at 170 kts at approximately 1800 ft. The cloud base is 2200 ft, RH = 83%, T = 16.8, Td = 14.5, p = 1022.2 hPa, wind = 6.4 km/h.

Jean Leon Foucault 1851



$$\frac{d^2 \mathbf{x}}{dt^2} = \mathbf{g} + \frac{\mathbf{T}}{m} - 2\boldsymbol{\Omega} \times \frac{d\mathbf{x}}{dt}$$

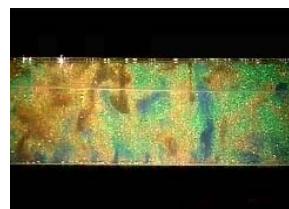
$$\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho} \nabla p - 2\boldsymbol{\Omega} \times \mathbf{V} + \mathbf{g} + \nu \nabla^2 \mathbf{V}$$



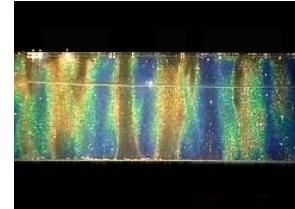
Coriolis Force

Non-inertial Frame

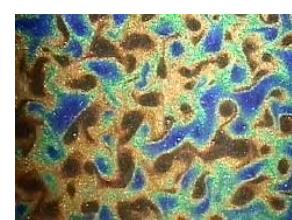
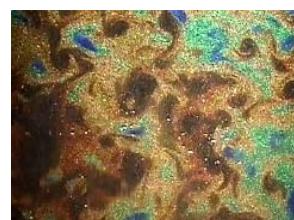
3D



2D (strong rotation)

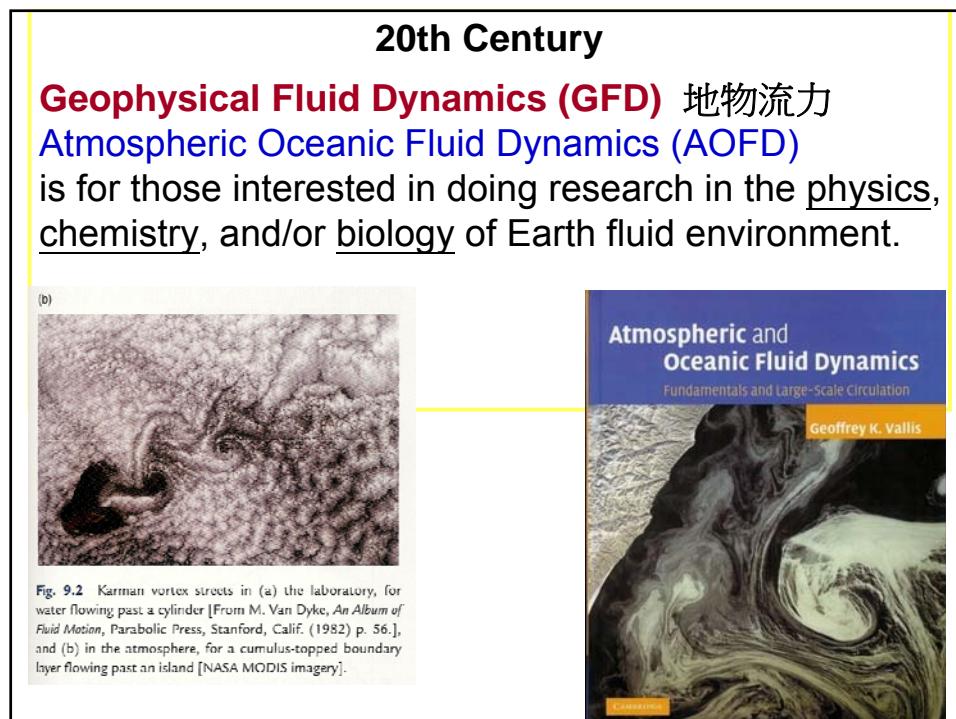
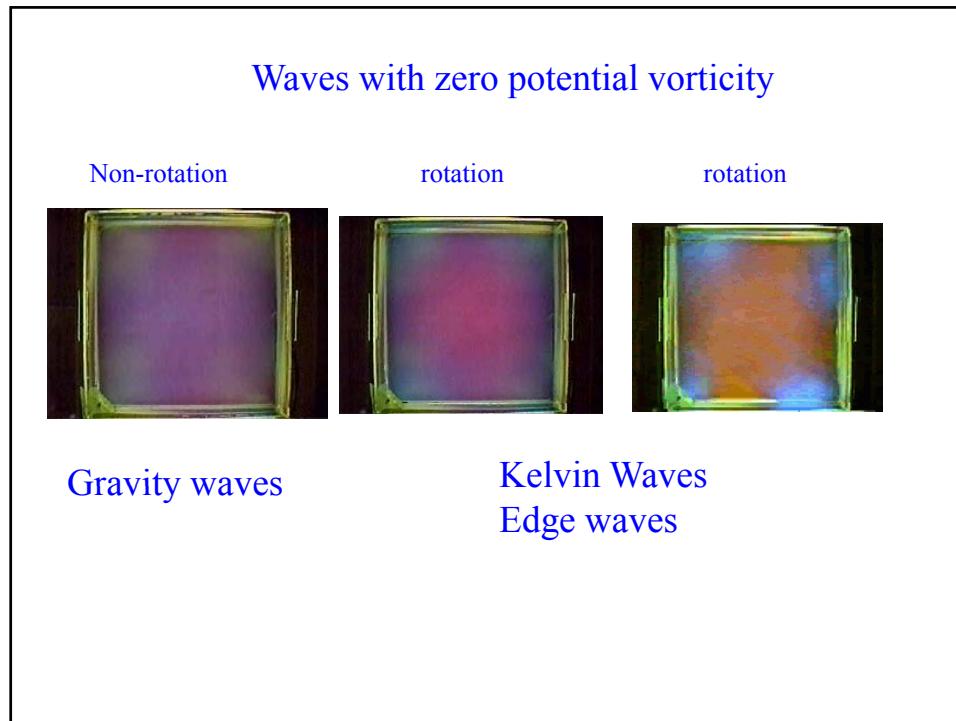


Taylor columns Vortex Tubes



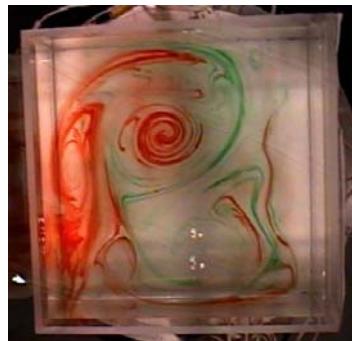
Vortices with sharp edge

Kyoto Univ. GFD group



2D Turbulence

Stratification and/or Rotation Vortex Waves Turbulence

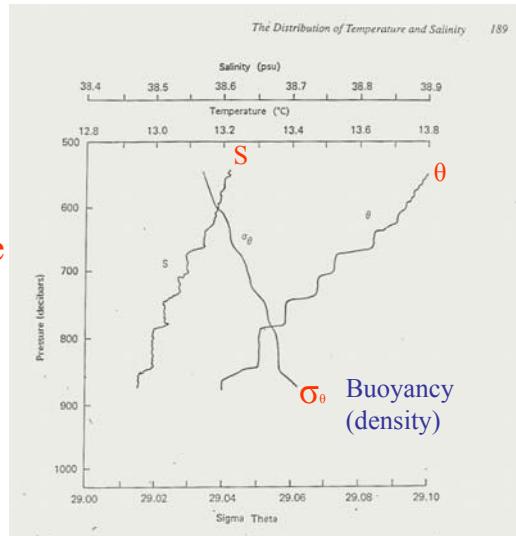


$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} = \nu \nabla^2 \zeta$$

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}.$$

$$\boxed{\frac{\partial \zeta}{\partial t} + \frac{\partial(\psi, \zeta)}{\partial(x, y)} = \nu \nabla^2 \zeta}$$

Ocean Spice



Stratification
層化

Figure 8.19. Temperature steps are usually concurrent with steps in the salinity gradient. The two tend to offset one another so that the density gradient is relatively smooth. (After Mörard and Williams, *Ms. Soc. Roy. des Sci. de Liège*, 7, 1973.)

Some of the horizontal layering implied by discontinuities such as in Figure 8.19 can be explained on the basis of stirring (see "stirring and mixing" in Chapter 4). Because stirring in the horizontal plane is several orders of magnitude larger than that in the vertical, as different types of water mix horizontally, sharp vertical gradients can be expected to occur occasionally. A variety of additional mechanisms have been suggested for generating such fine structure, including turbulence generated by both bottom topography and surface waves as well as the breaking of small-scale internal waves (Figure 8.20).

Multiple Scale Interactions in Vortex



Wave mean flow interaction in **stable stratified** fluid
Turbulent feed back to the vortex mean flow

2D turbulence

Kyoto Univ. GFD group

$$\frac{d}{dt} \int E(k) dk = 0, \quad \frac{d}{dt} \left(\int k^2 E(k) dk \right) = \frac{d}{dt} \int Z(k) dk = 0,$$

$$\frac{d}{dt} \left(\int (k - k_1)^2 E(k) dk \right) > 0$$

$$\frac{d}{dt} \left(\int k^2 E(k) dk + k_1^2 \int E(k) dk - 2k_1 \int k E(k) dk \right) > 0$$

$$\frac{d}{dt} \left(\frac{\int k E(k) dk}{\int E(k) dk} \right) < 0,$$

Kinetic energy moves toward large scales

$$\frac{d}{dt} \left(\int (k^2 - k_1^2)^2 E(k) dk \right) > 0$$

$$\frac{d}{dt} \left(\int k^2 Z(k) dk + k_1^4 \int E(k) dk - 2k_1^2 \int k^2 E(k) dk \right) > 0$$

$$\frac{d}{dt} \left(\frac{\int k^2 Z(k) dk}{\int Z(k) dk} \right) > 0,$$

Enstrophy moves toward small scales

Non-divergent barotropic model (Nearly Inviscid Fluid)

$$\frac{\partial}{\partial t} \zeta + J(\psi, \zeta) = v \nabla^2 \zeta \quad \boxed{\nabla^2 \psi = \zeta}$$

The energy and enstrophy relations

$$\begin{aligned} \frac{d\mathcal{E}}{dt} &= -2v\mathcal{Z} & \mathcal{E} &= \iint \frac{1}{2}(u^2 + v^2) dx dy \quad \text{kinetic energy} \\ \frac{d\mathcal{Z}}{dt} &= -2v\mathcal{P} & \mathcal{Z} &= \iint \frac{1}{2}\zeta^2 dx dy \quad \text{enstrophy} \\ && \mathcal{P} &= \iint \frac{1}{2}\nabla\zeta \cdot \nabla\zeta dx dy \quad \text{palinstrophy} \end{aligned}$$

Batchelor 1969

$E \sim p'^2 / L^2$ (KE) geostrophy

$Z \sim p'^2 / L^4$ (Enstrophy)

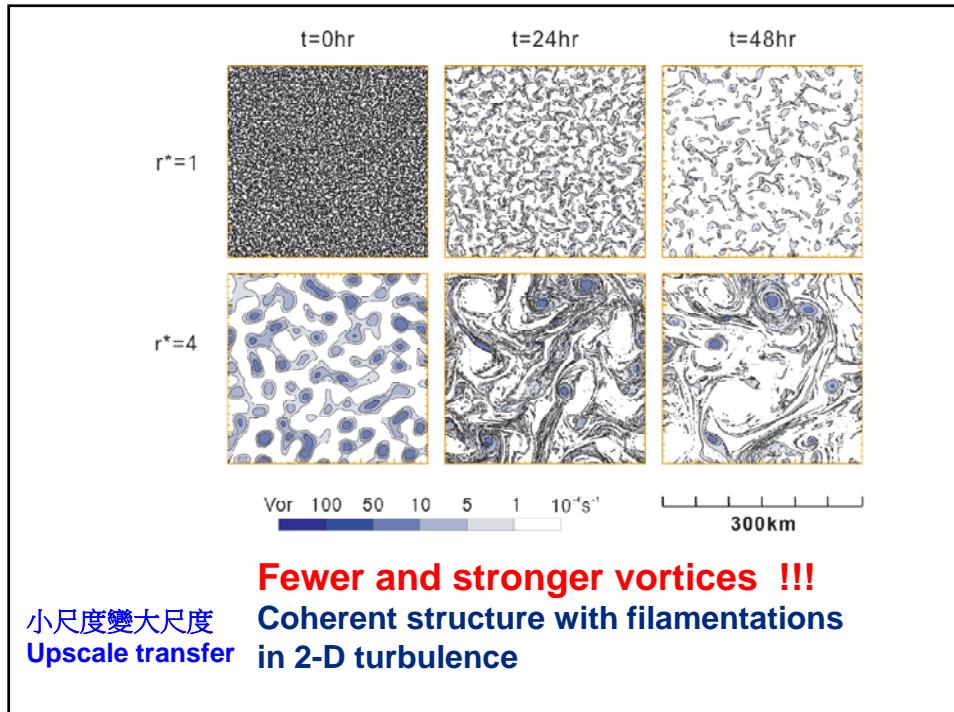
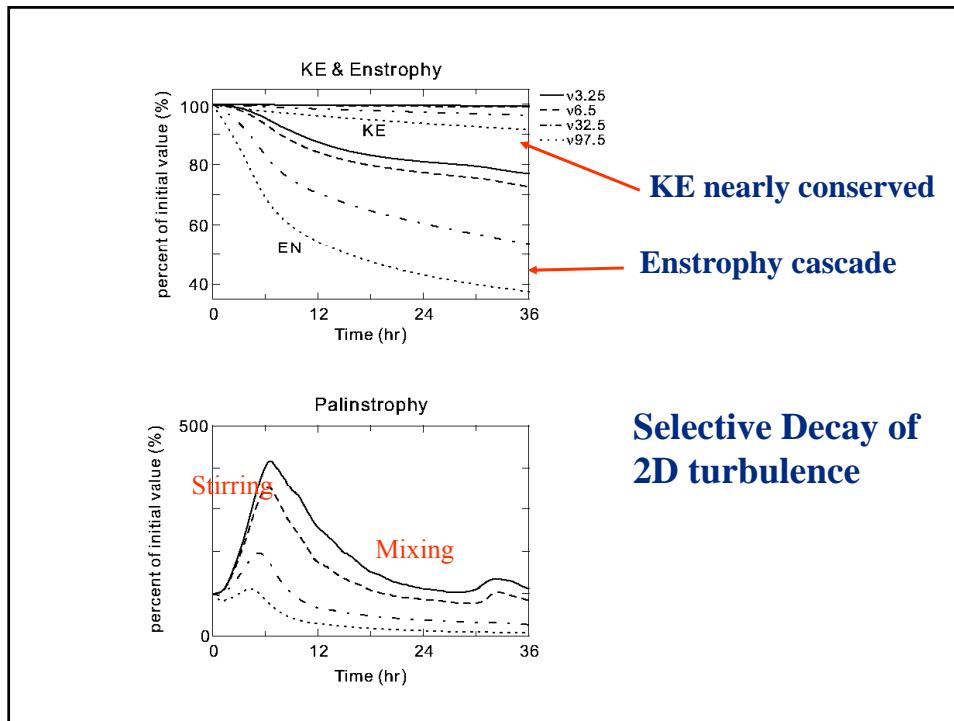
KE nearly conserved $L \sim p'$

Enstrophy cascade $L \uparrow$ (L increase Z decrease)

Selective Decay of 2D turbulence

The vortices become, on the average, larger, stronger, and fewer.

Merger and Axisymmetrization Dynamics



Weiss(1981,1991), Rozoff et al. (2004)

$$\frac{D}{Dt}(\nabla \zeta) = -J(\nabla \psi, \zeta)$$

$$\rightarrow \nabla \zeta(t) \propto \exp(\lambda t) \quad \lambda = \pm \frac{1}{2} \sqrt{Q} = \pm \frac{1}{2} \sqrt{S_1^2 + S_2^2 - \zeta^2}$$

$$S_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \text{ (stretch deformation)}$$

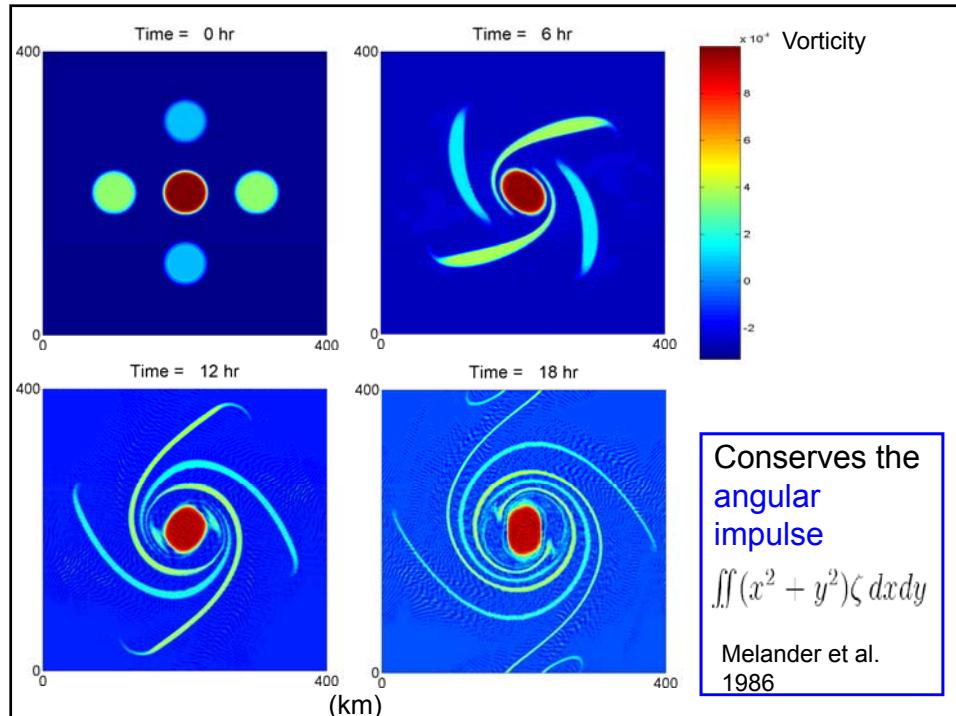
$$S_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \text{ (shear deformation)}$$

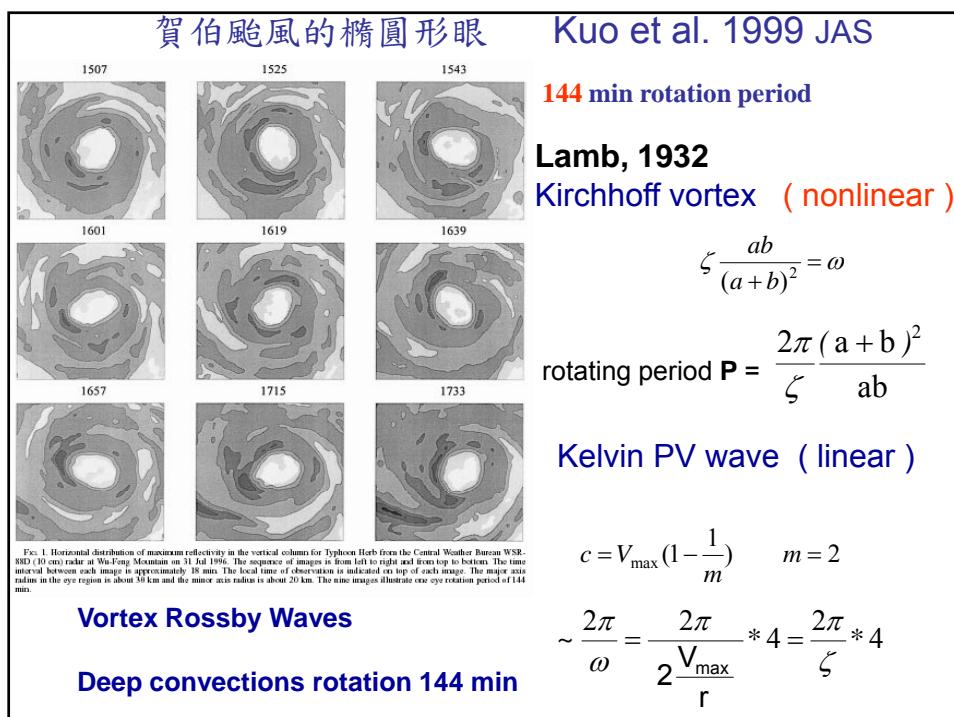
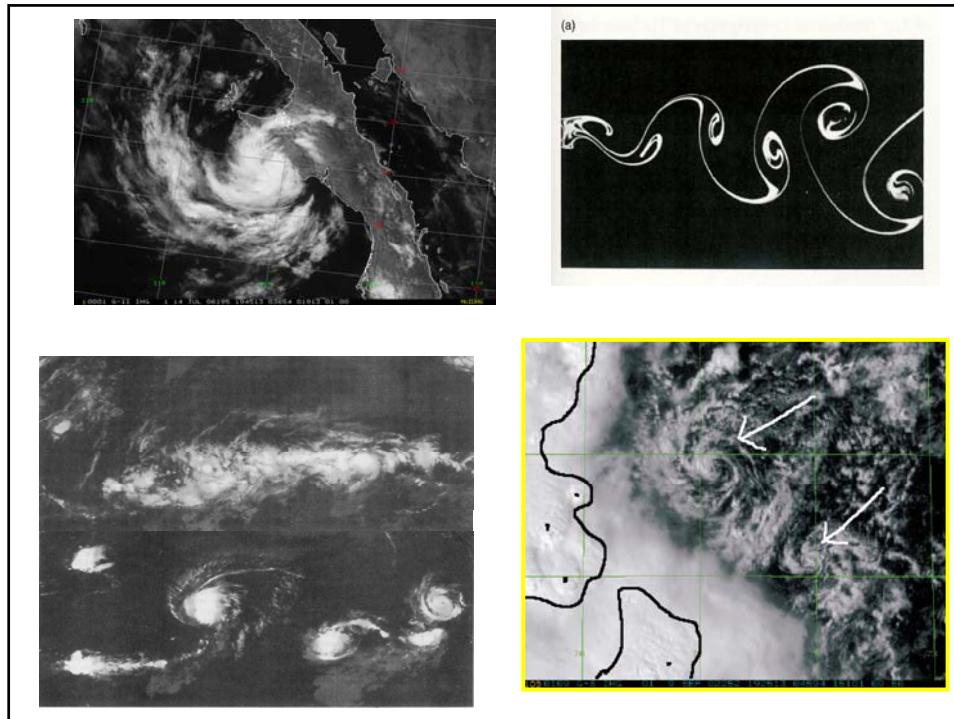
$Q > 0$ (strain dominates)

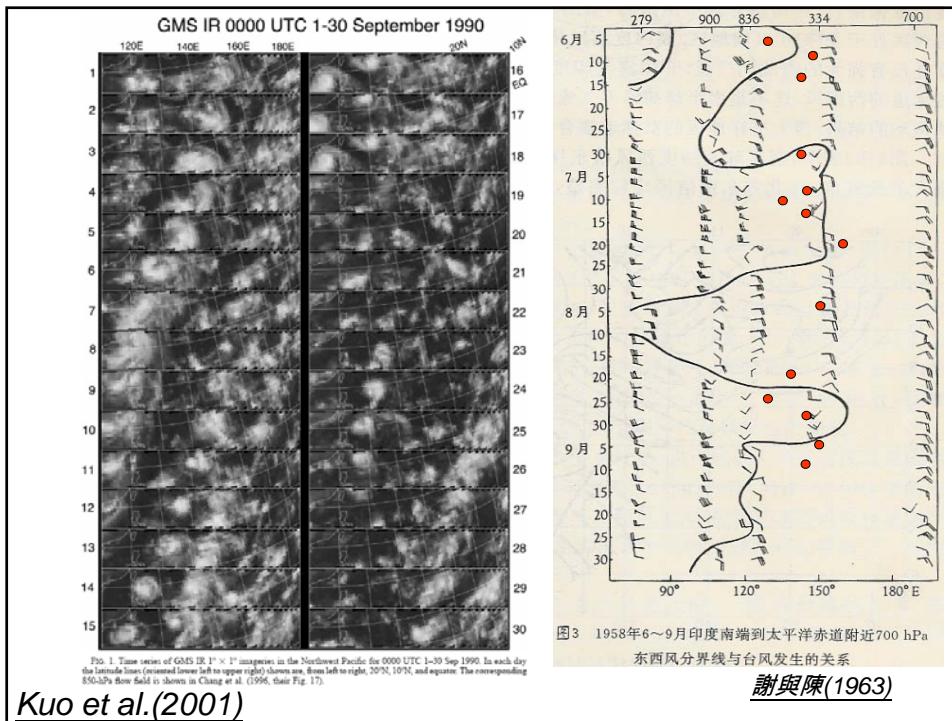
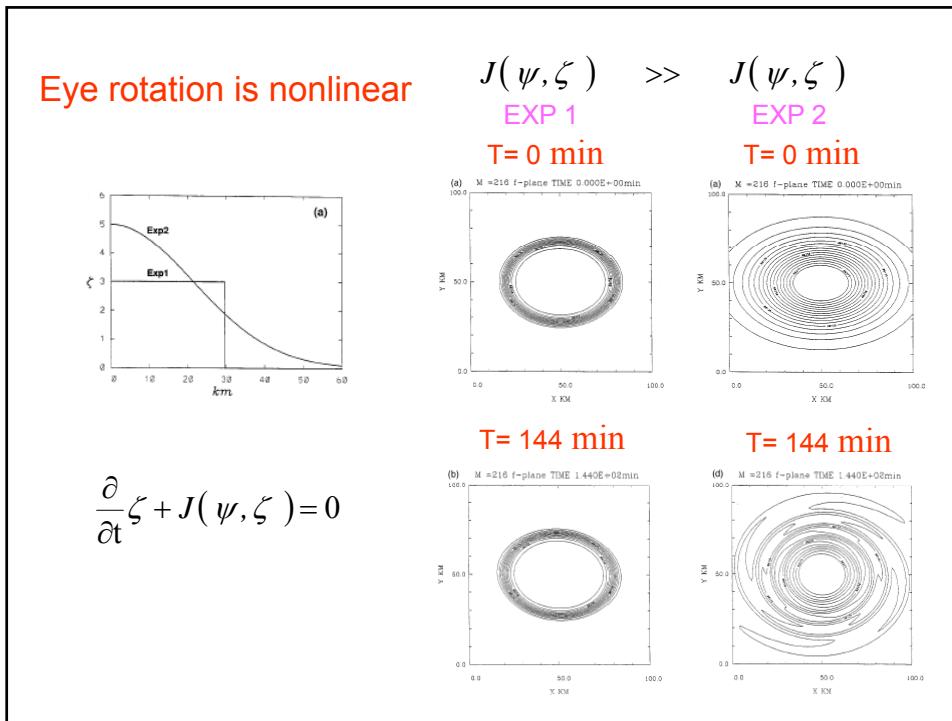
→ vorticity gradient will be stretched

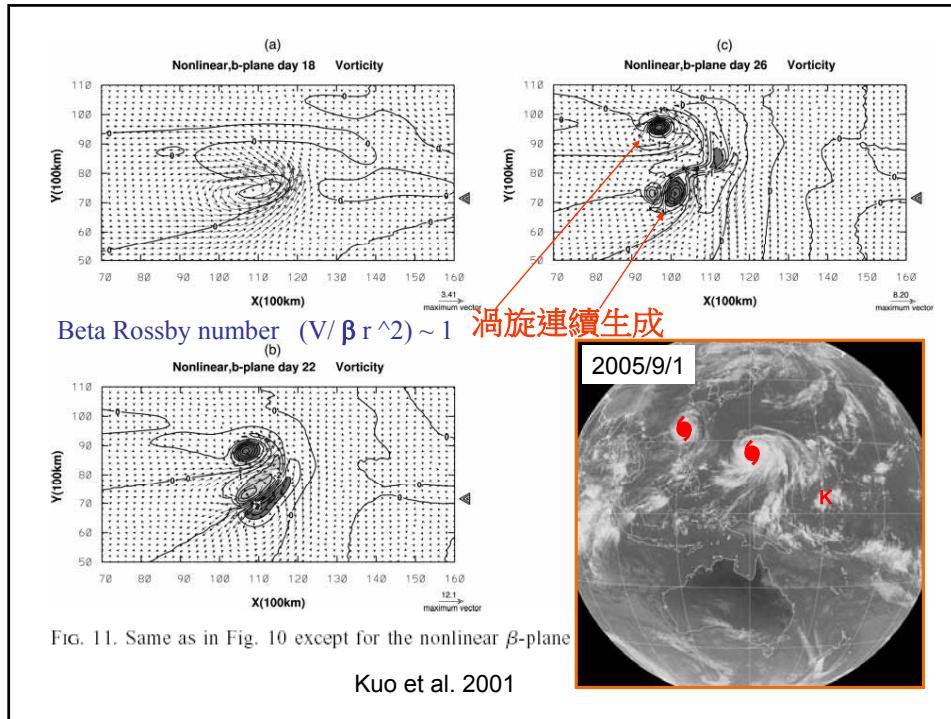
$Q < 0$ (vorticity dominates)

→ vortex is stable (survival of eyewall meso-vortices)

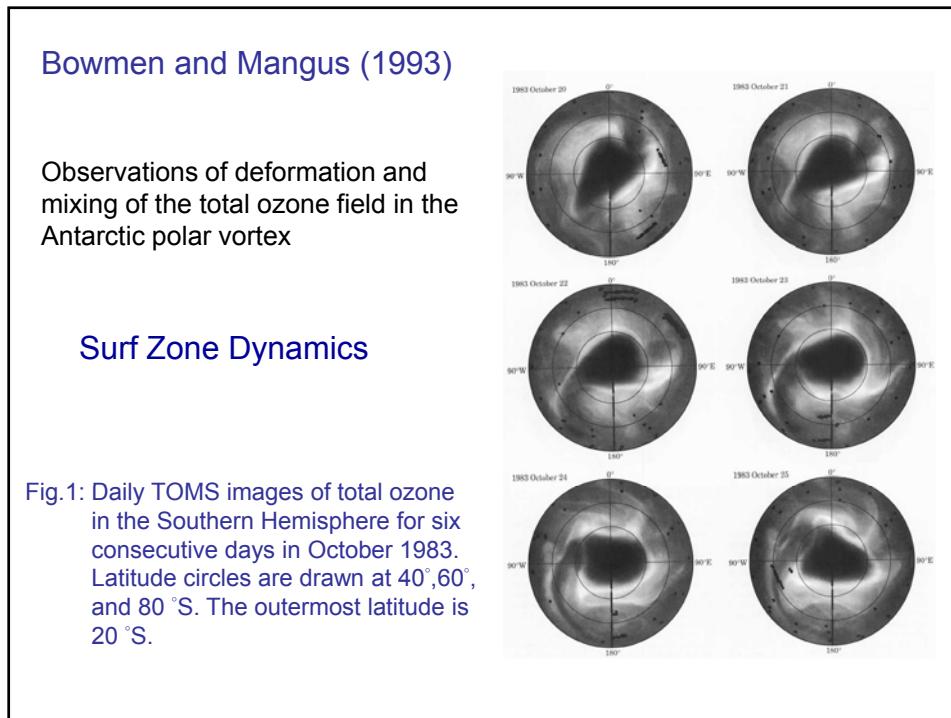


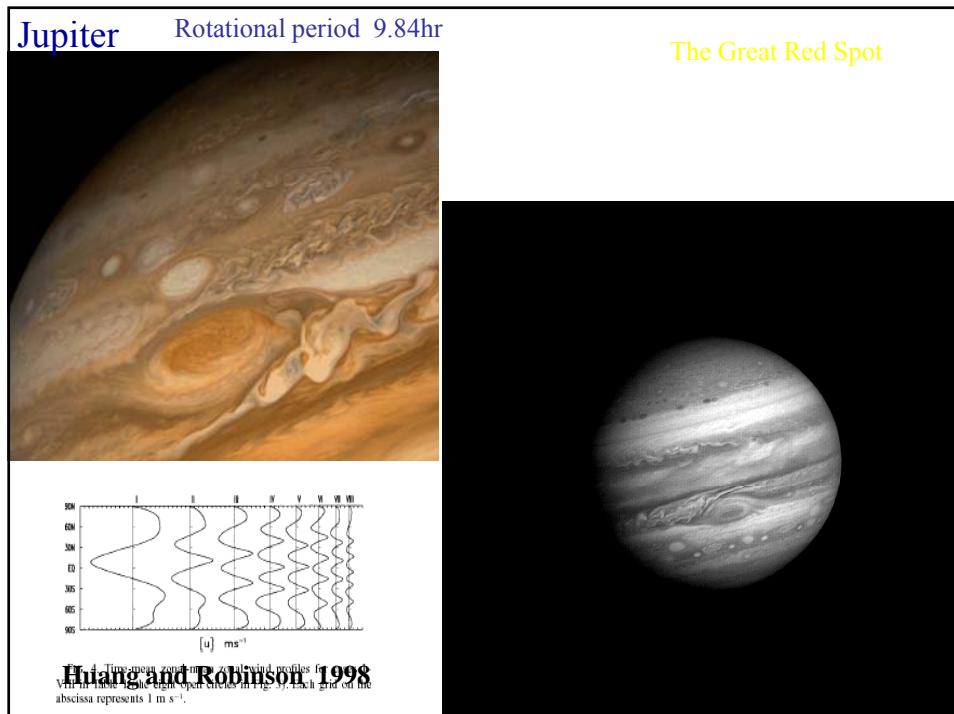
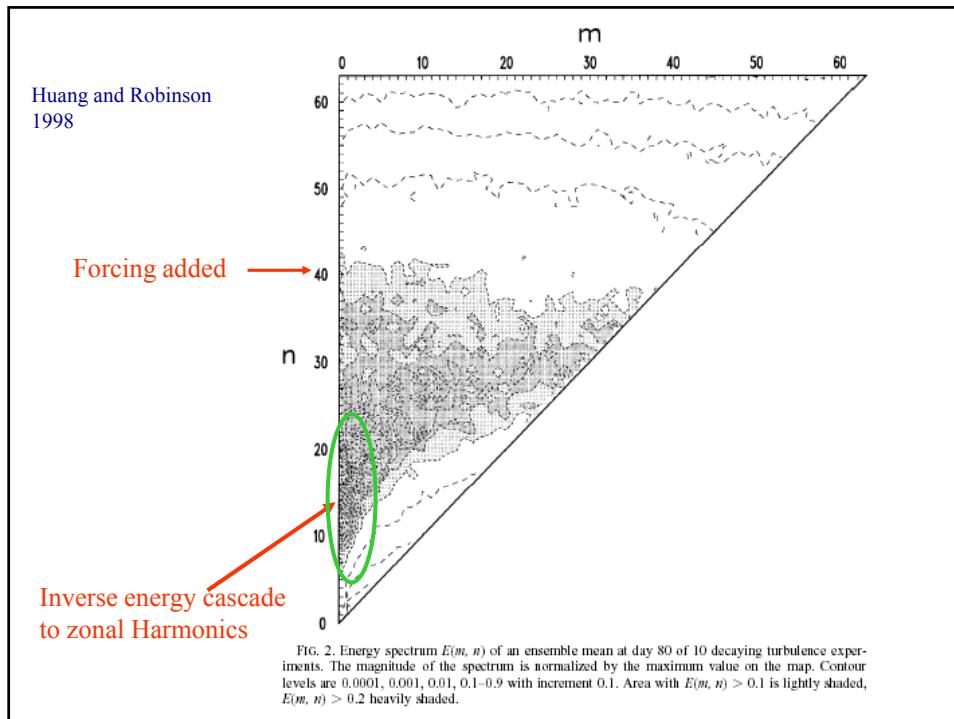




FIG. 11. Same as in Fig. 10 except for the nonlinear β -plane

Kuo et al. 2001



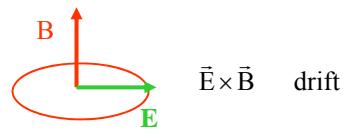


Electron density redistribution in experimental plasma physics

single sign charge
+
axial magnetic field
confinement

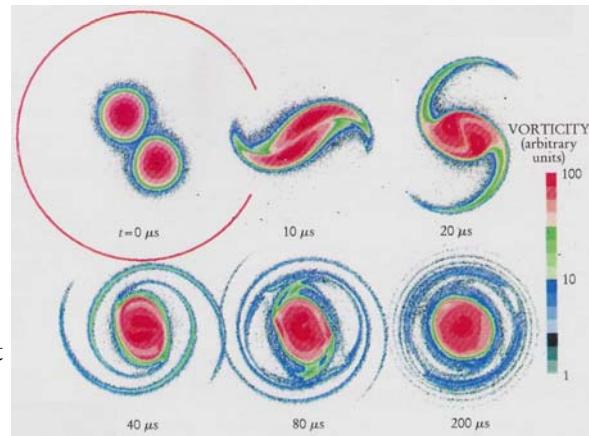
$$\mathbf{E} = -\nabla\psi$$

$$\nabla \cdot \mathbf{E} = -\nabla^2\psi = \frac{\rho}{\epsilon}$$

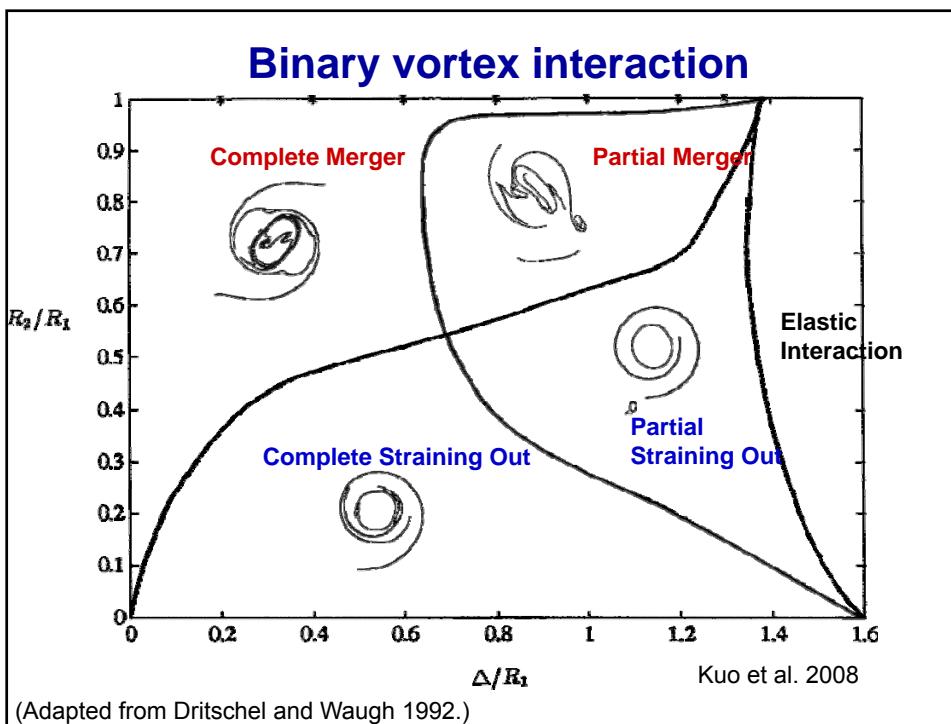


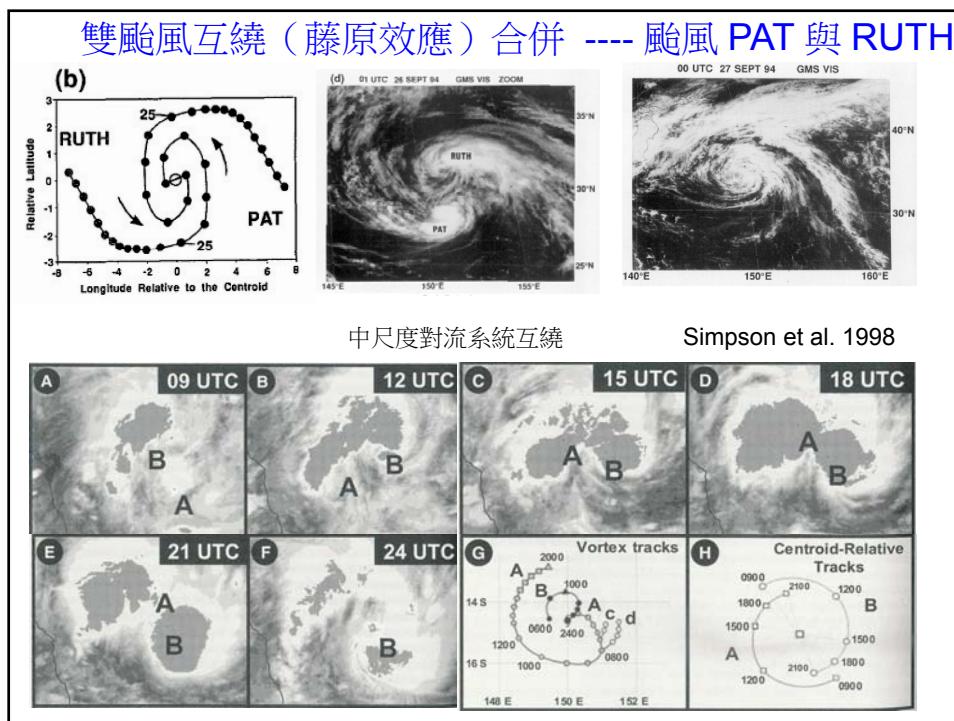
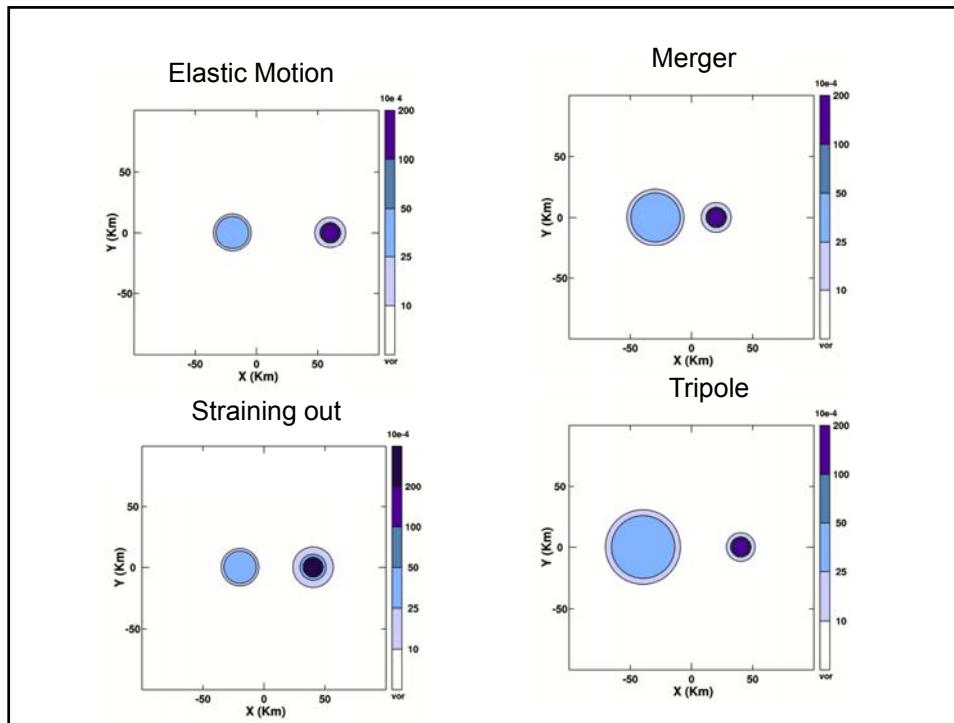
Coriolis force

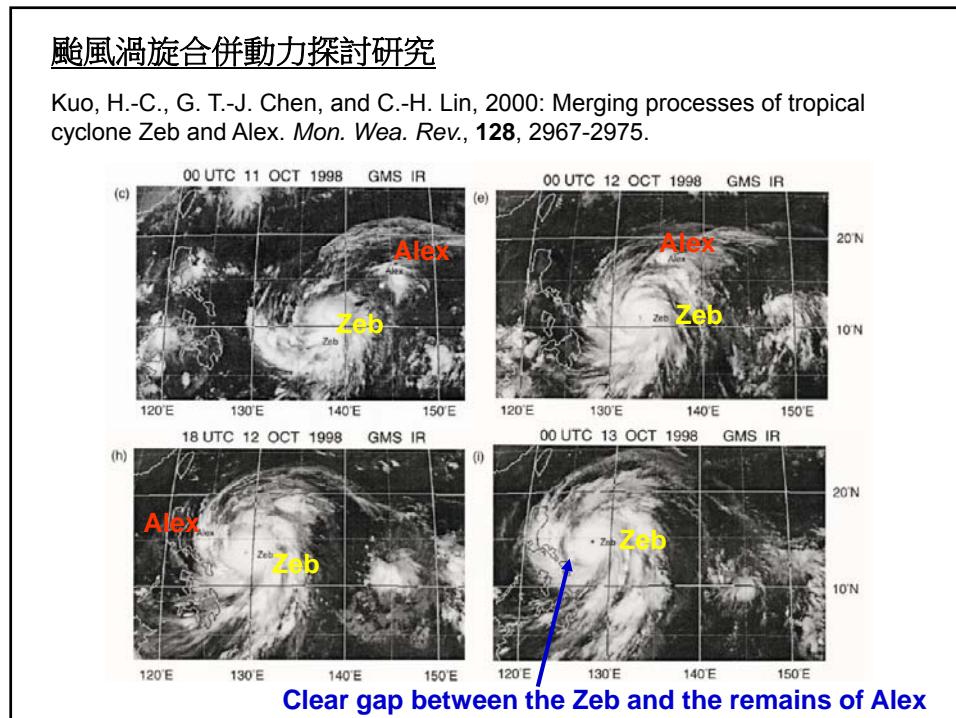
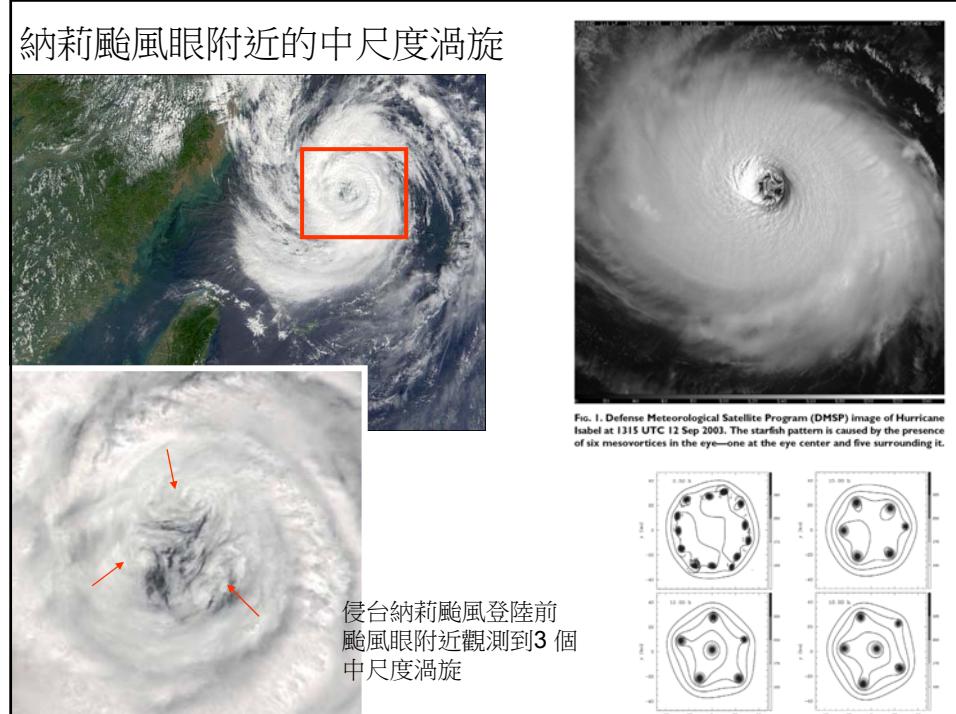
Axisymmetrization 軸對稱化

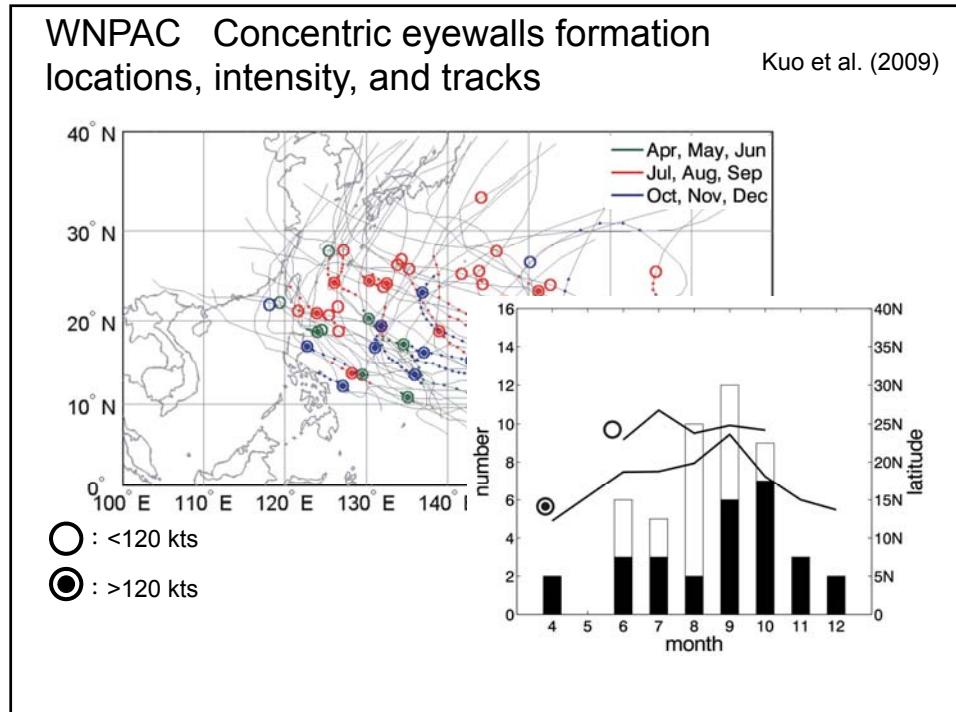
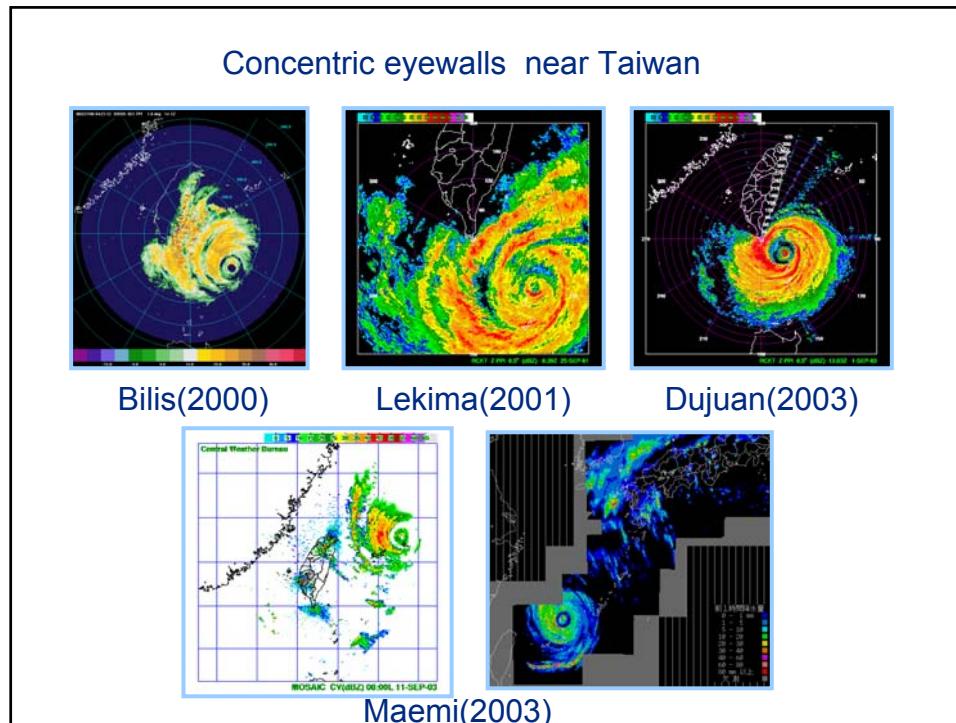


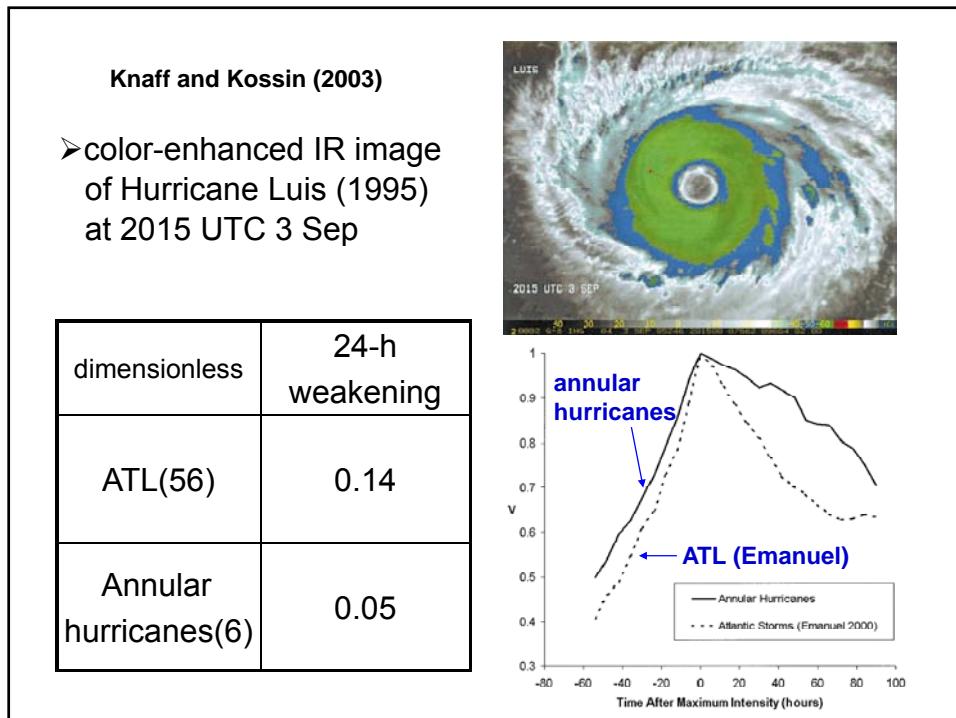
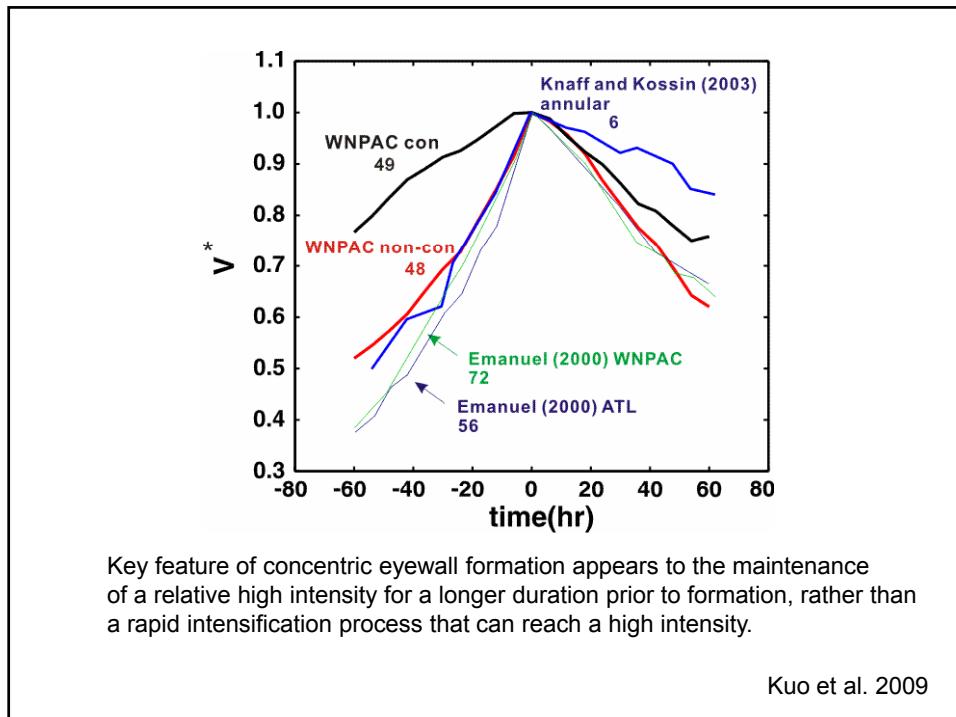
Core is protected, thin filaments from edges

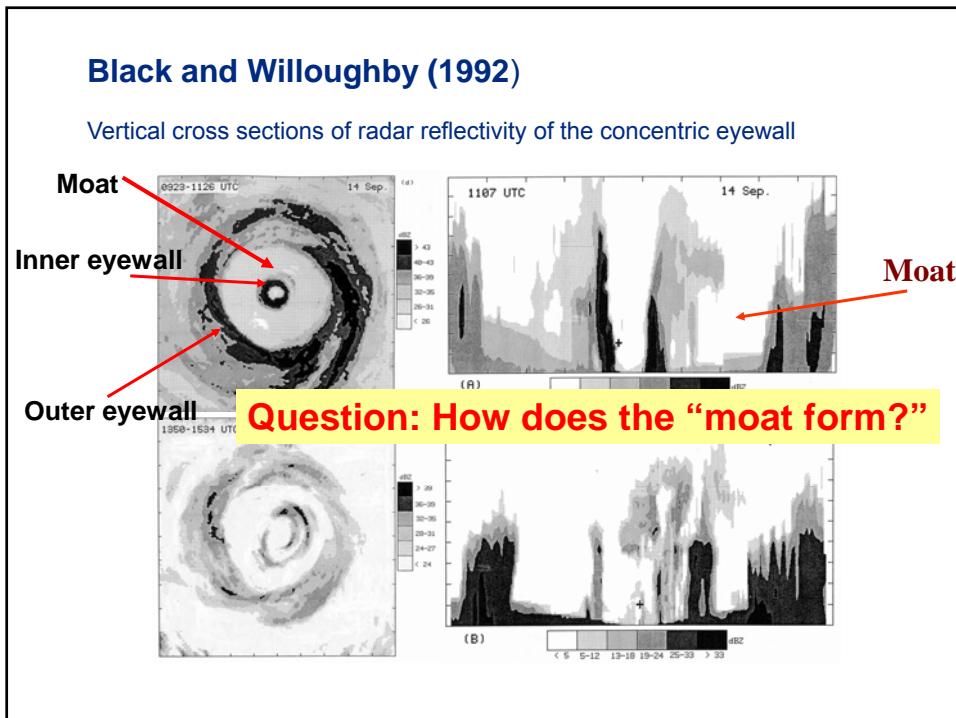
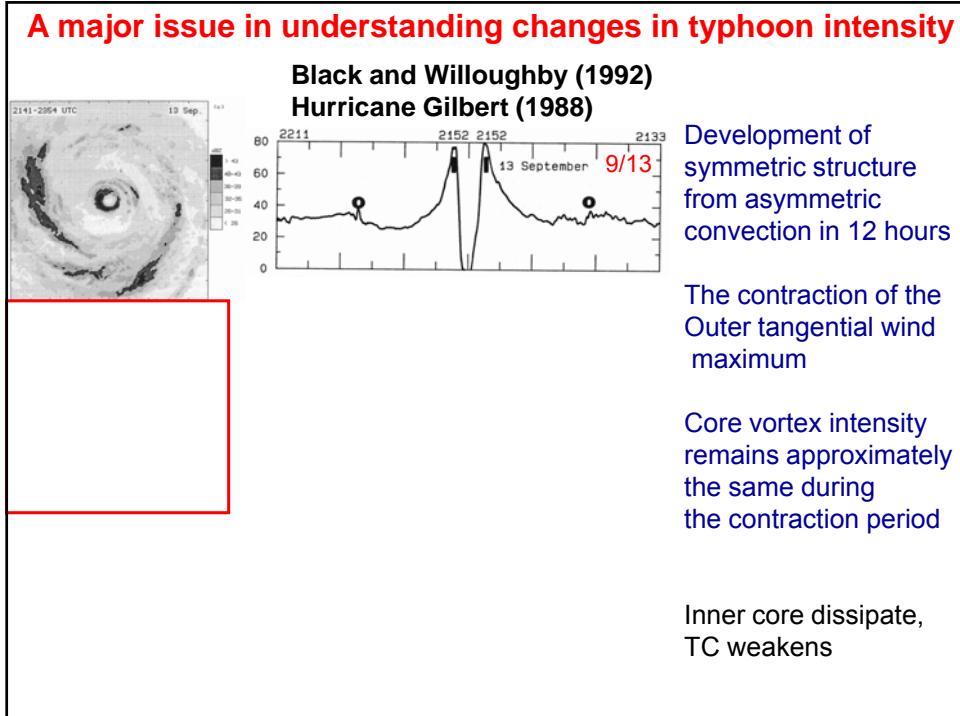








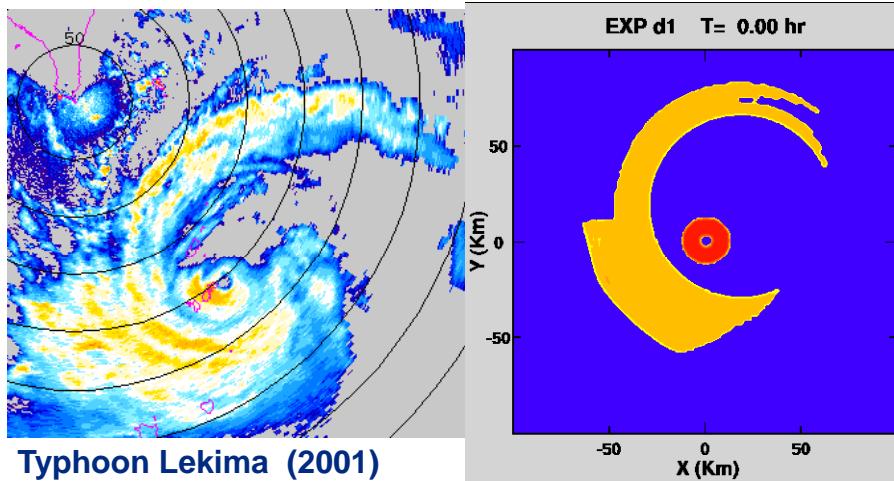




Concentric Eyewall formation

Kuo, H.-C., L.-Y. Lin, C.-P. Chang, and R. T. Williams, 2004: The formation of concentric vorticity structure in typhoons. *J. Atmos. Sci.*, **61**, 2722–2734.
 Kuo, H.-C., W. H. Schubert, C.-L. Tsai, and Y.-F. Kuo, 2008: Vortex interactions and barotropic aspects of concentric eyewall formation. *Mon. Wea. Rev.*, **136**, 5183–5198.

0935-1935 LST



Binary vortex interaction

Kuo et al. (2004,2008)

【Variables】

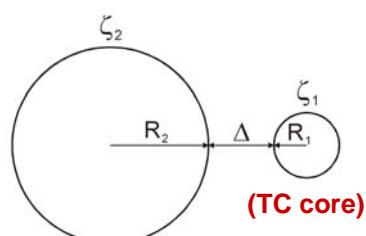
$$R_1, R_2; \Delta; \zeta_1, \zeta_2 \quad \text{Beta-skirt}$$

【Parameters】

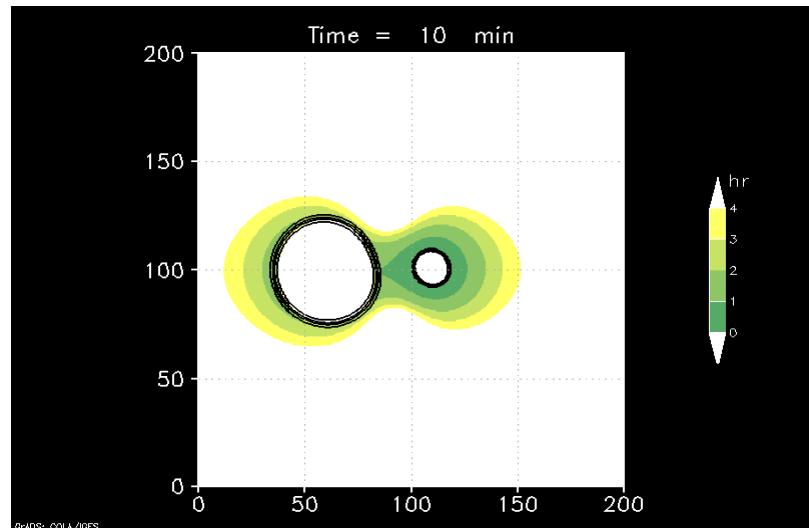
- Vortex radius ratio (r) = $\frac{R_1}{R_2}$

- Dimensionless gap ($\frac{\Delta}{R_1}$)

- Vortex strength ratio (γ) = $\frac{\zeta_1}{\zeta_2}$



- An extension of Dritschel and Waugh's (1992) work.
- In addition to the radii ratio and the normalized distance between the two vortices, the vorticity ratio is added as a third external parameters.



Thoughts from the 80's and 90's

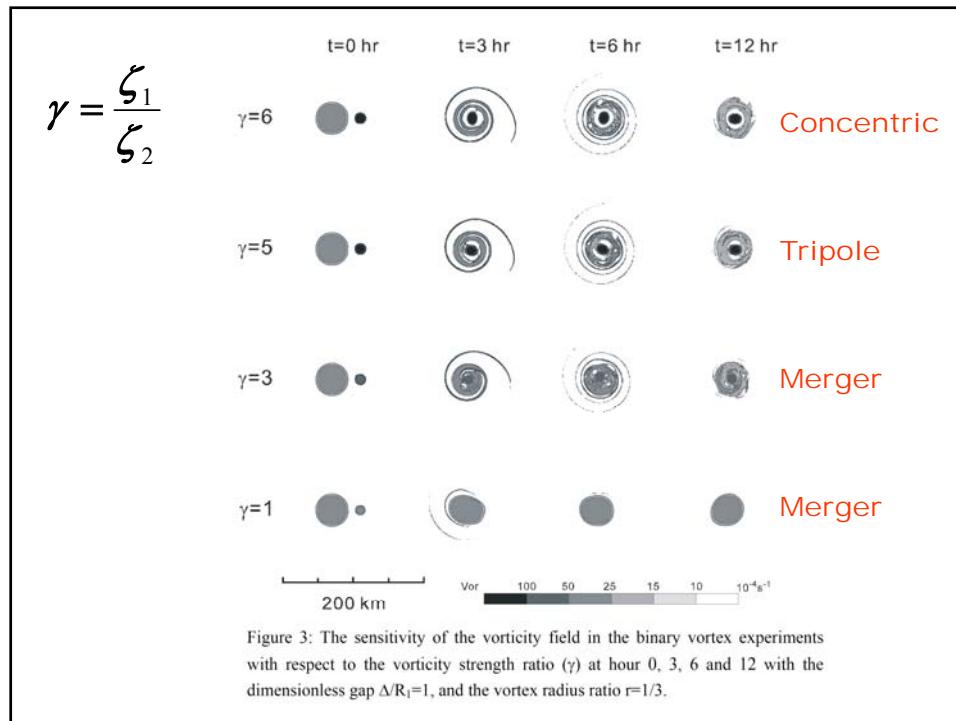
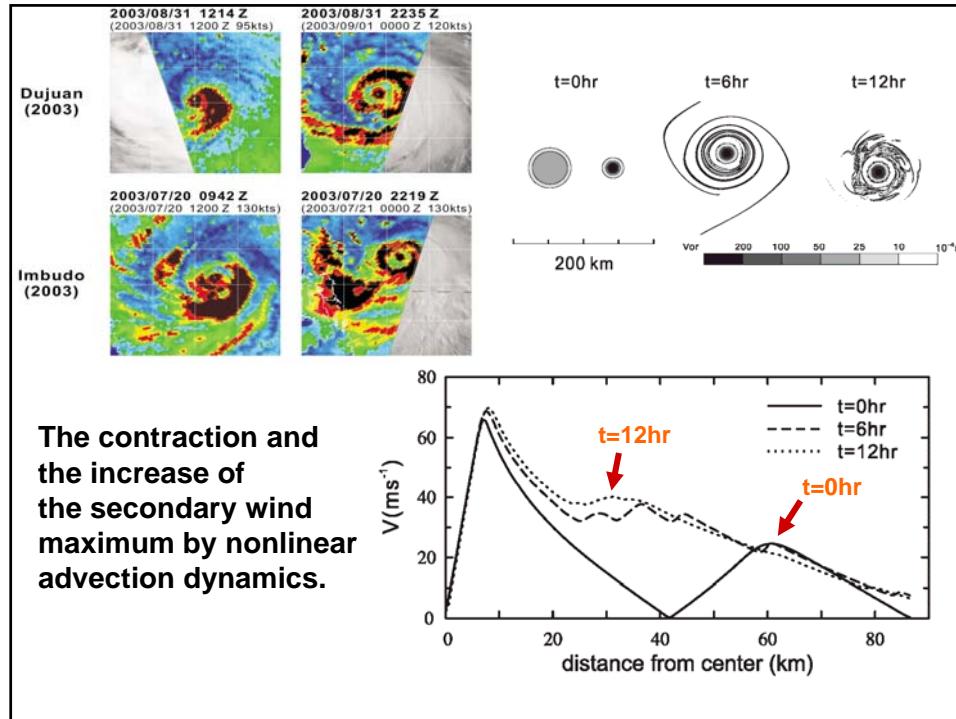
Shapiro and Willoughby (1982) and Schubert and Hack (1982) proposed that heating-vorticity interaction can lead to convective-ring contraction.

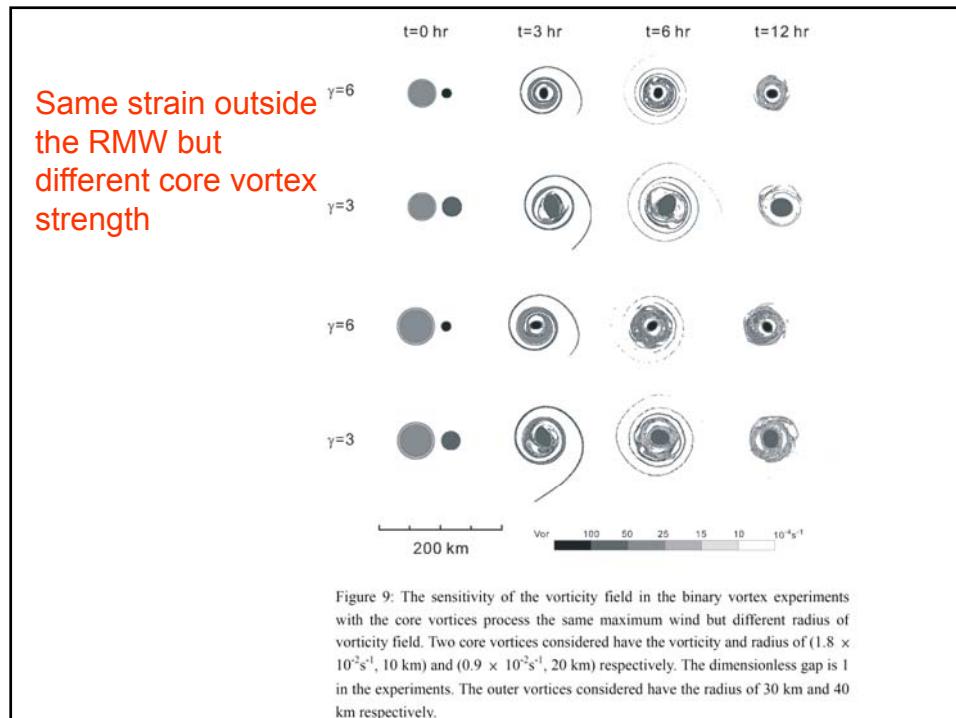
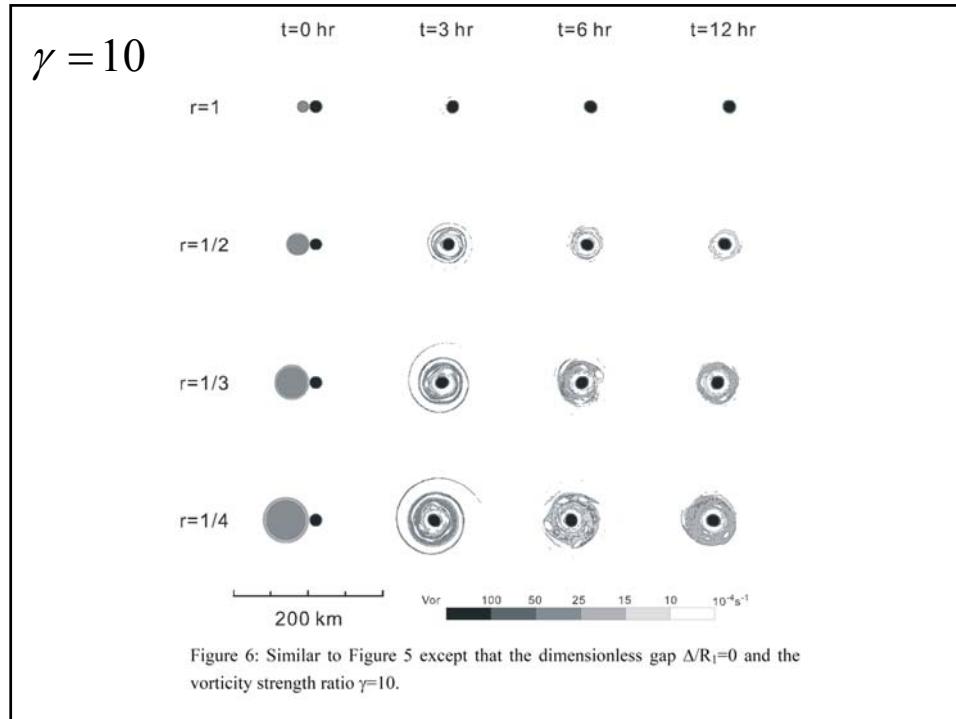
$$\frac{d\zeta}{dt} \sim \zeta \nabla \cdot \mathbf{V}$$

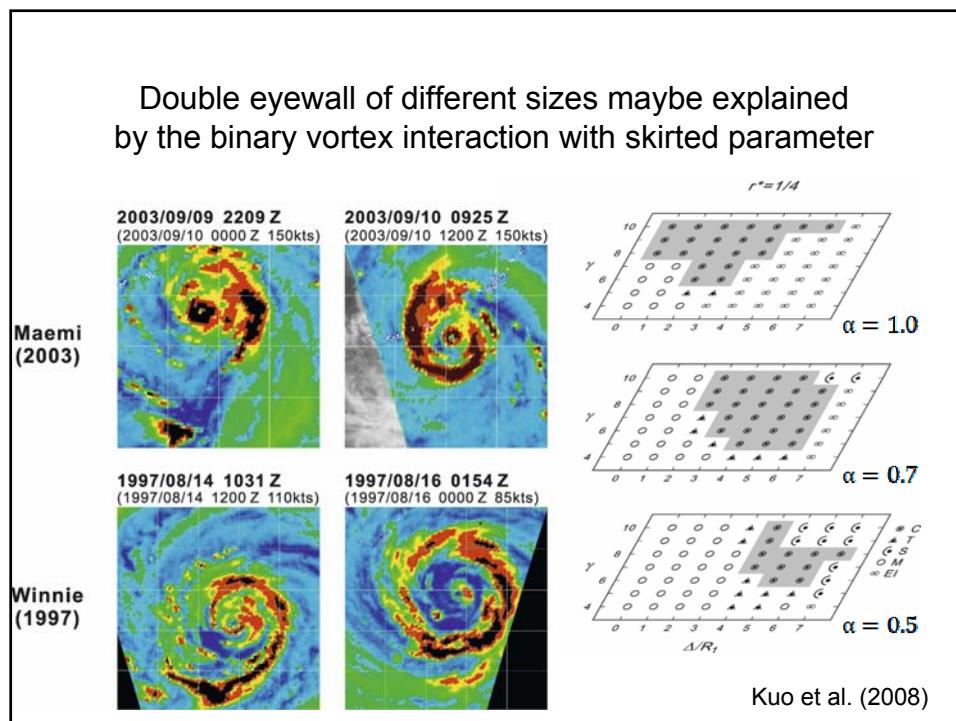
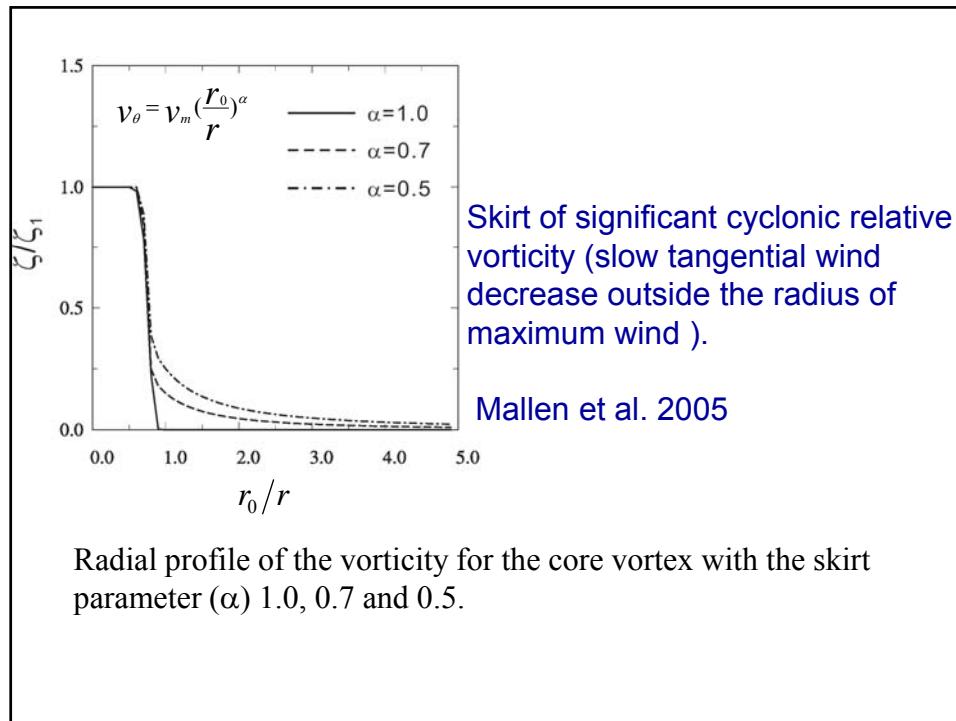
Stronger ζ near the TC core favors the inward response

Symmetrical Model

Moat formation and eyewall replacement are related to the subsidence and the moisture cut-off.

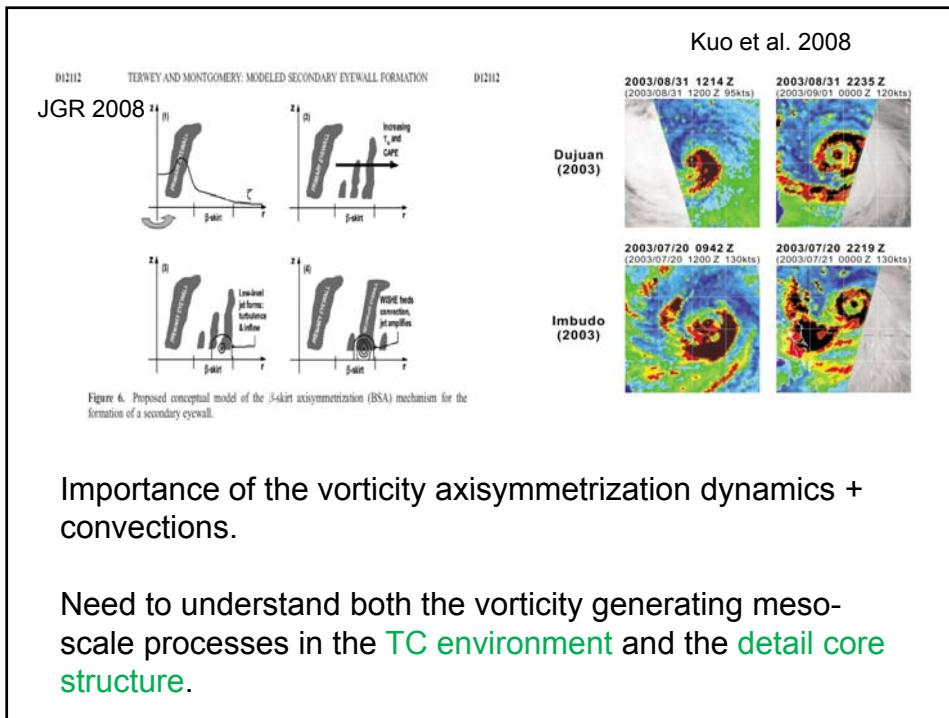






D12112		TERWEY AND MONTGOMERY: MODELED SECONDARY EYEWALL FORMATION	D12112	JGR 2008
Authors	Hypothesis Summary	Relevance to Current Model Results		Type
<i>Willoughby et al. [1982]</i> borrowing from the squall line research of <i>Zipser [1977]</i>	Downdrafts from the primary eyewall force a ring of convective updrafts.	Few downdraft-forced updrafts during this time in the simulations.		O
<i>Willoughby [1979]</i>	Internal resonance between local inertia period and asymmetric friction due to storm motion.	No systematic storm motion in the simulated storms.		A
<i>Hawkins [1983]</i>	Topographic effects	No topographic forcing in the simulations.		O
<i>Willoughby et al. [1984]</i>	Ice microphysics	“Warm-rain” (no-ice) sensitivity case also produces secondary eyewall.		A
<i>Molinari and Skubis [1985] and Molinari and Vallaro [1989]</i>	Synoptic-scale forcings (e.g., inflow surges, upper-level momentum fluxes)	No synoptic-scale forcings in the simulations		O
<i>Montgomery and Kallenbach [1997], Camp and Montgomery [2001] and Terwey and Montgomery [2003]</i>	Internal dynamics-axisymmetrization via sheared vortex Rossby wave processes; collection of wave energy near stagnation or critical radii	Possible explanation		N
<i>Nong and Emanuel [2003]</i>	Sustained eddy momentum fluxes and WISHE feedback	Possible explanation		A
<i>Kuo et al. [2004, 2008]</i>	Axisymmetrization of positive vorticity perturbations around a strong and tight core of vorticity.	Possible explanation		N

^aThe type column refers to the type of model or observations that were used to formulate the hypothesis. O stands for observationally-based; A stands for axisymmetric model; N stands for nonaxisymmetric model.



Summary

Tropical cyclones of sufficient strength (≥ 120 kts) often form double eyewalls. Inner eyewall weakens and/or die.

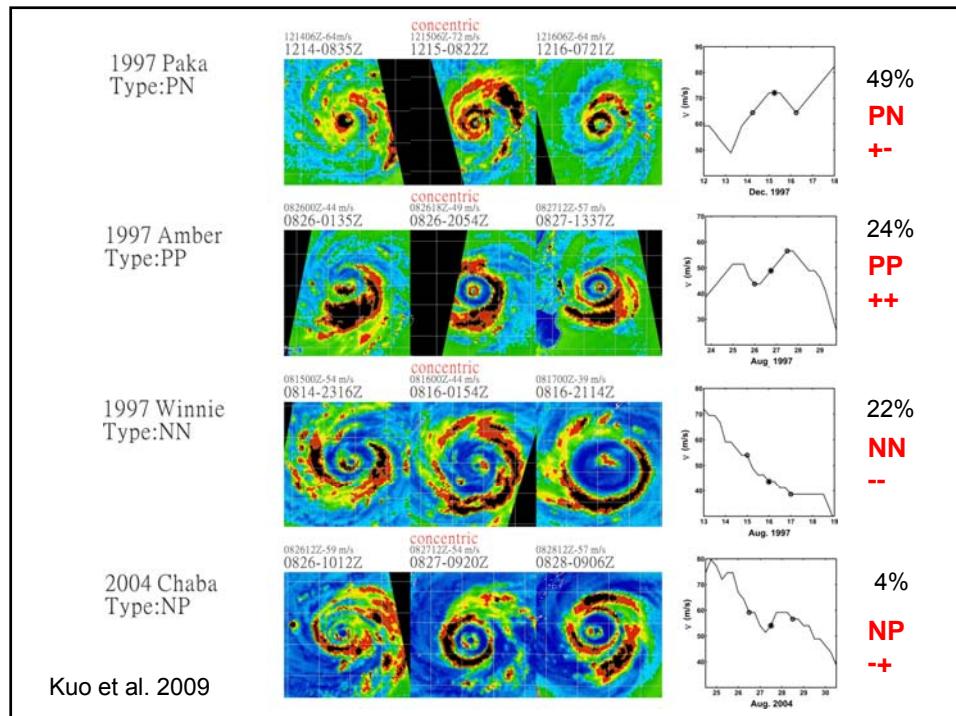
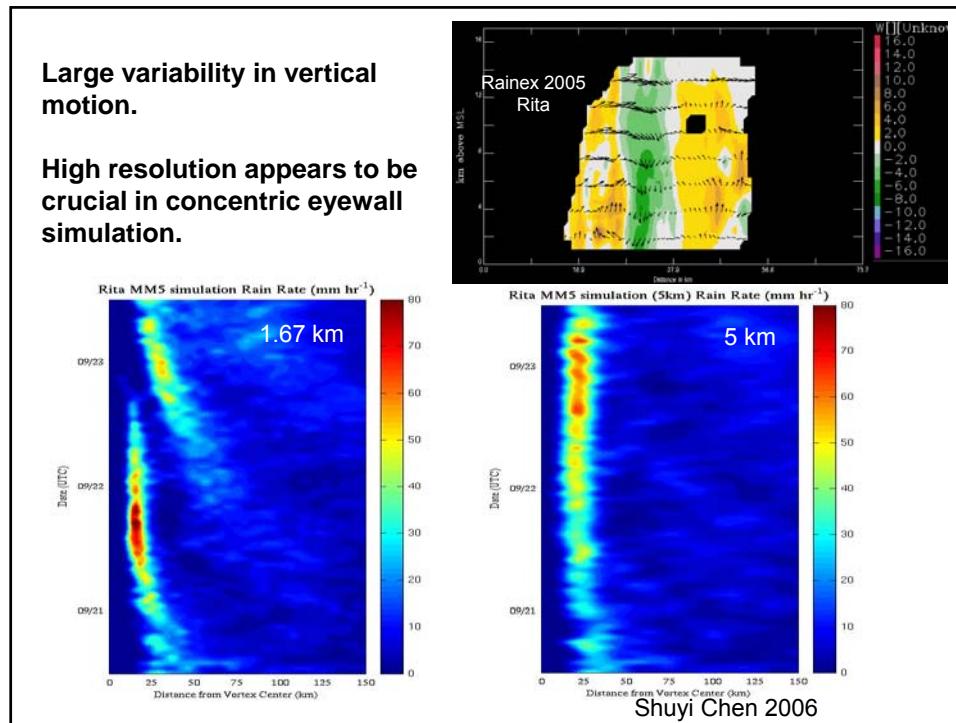
Area of asymmetric convection outside the core vortex that wraps around the inner eyewall to form the concentric eyewalls in about 12 hours.

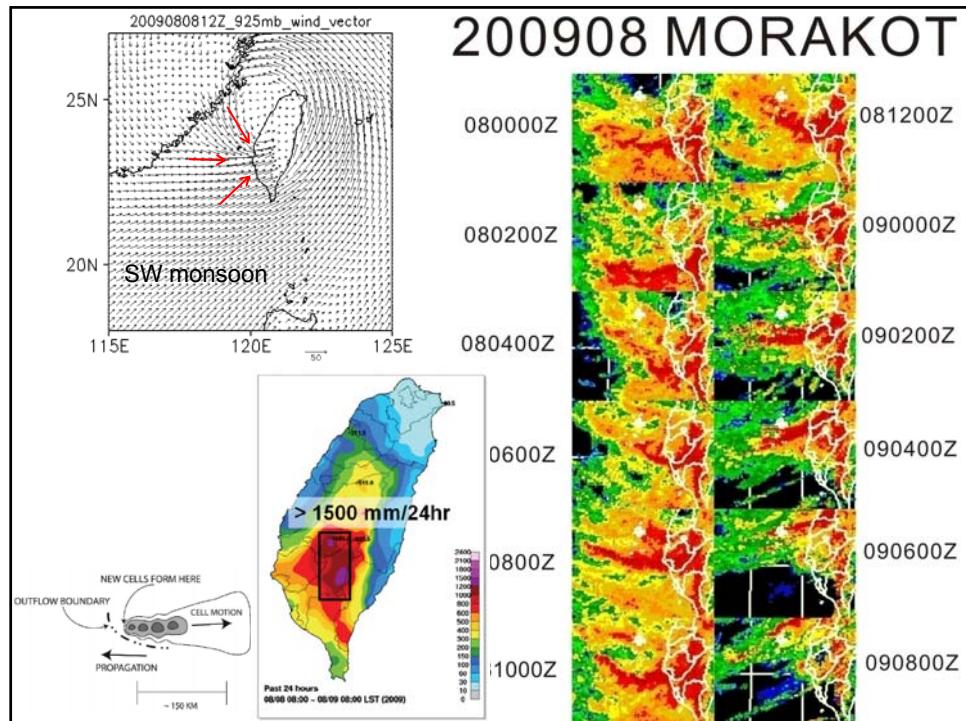
The contraction of the secondary wind maximum and the formation of the moat are features of the vorticity dynamics. The moat formation by subsidence, rapid filamentation, and advective dynamics.

Double eyewall of different sizes maybe explained by the binary vortex interaction with skirted parameter.

The pivotal role of the vorticity strength of the core vortex in maintaining itself, and in stretching, organizing and stabilizing the outer vorticity field, and the shielding effect of the moat to prevent further merger and enstrophy cascade processes in concentric eyewall dynamics.

Vorticity generation in the core and in the environment (via mesoscale convective systems) are of great importance!





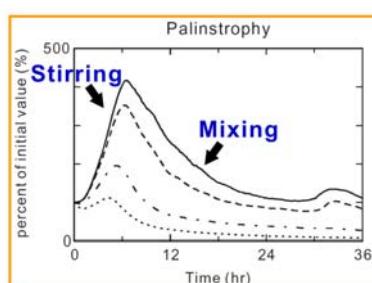
一杯咖啡，古今往事盡付笑談中。
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + \bar{V} \cdot \nabla \theta = v \nabla^2 \theta$$

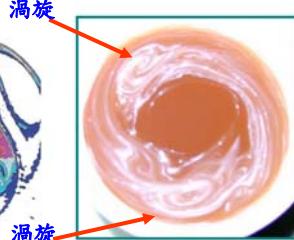
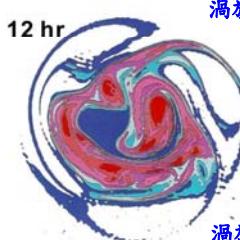
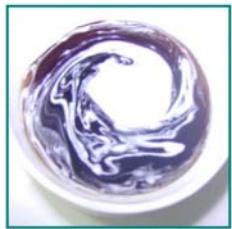
$$C = \frac{1}{2} \int \nabla \theta \cdot \nabla \theta \, dV$$

$$\frac{dC}{dt} = \int (\bar{V} \cdot \nabla \theta) \nabla^2 \theta \, dV - v \int (\nabla^2 \theta) \, dV$$

Stirring Mixing



Coffee with white

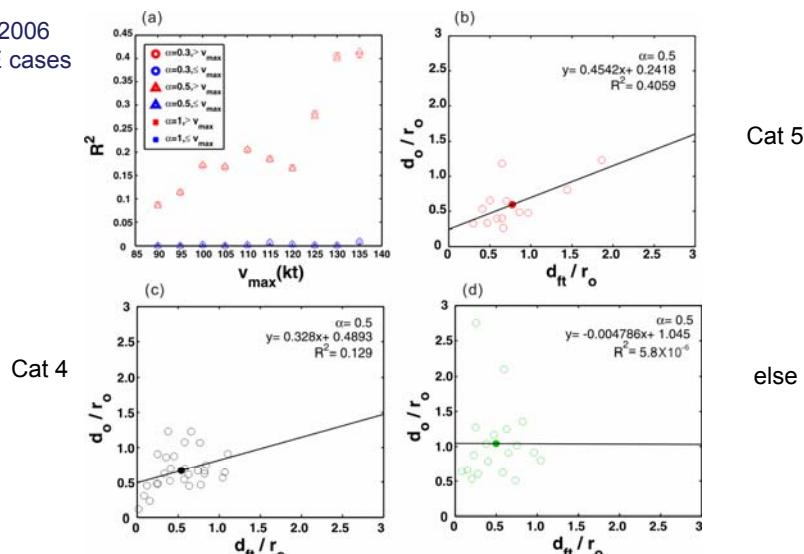


Thank you!

A painting with vortices and filaments!



1997-2006
62 CE cases



The rapid filamentation process tends to make an important contribution to the organization of the moat in strong typhoons, especially in cases where the maximum winds are greater than 130 kts.

Kuo et al. (Nov. 2009 MWR)

Filamentation time (Rozoff et al., 2006)

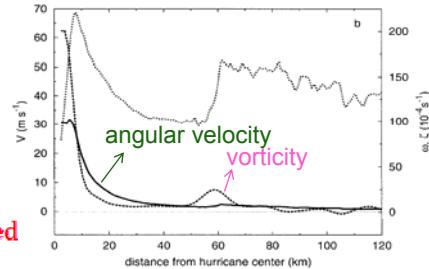
$$\frac{D}{Dt}(\nabla\zeta) = -J(\nabla\psi, \zeta)$$

$\frac{1}{2}S_1 = u_x - v_y$, stretching deformation

$\frac{1}{2}S_2 = v_x + u_y$, shearing deformation

(i) when $S_1^2 + S_2^2 - \zeta^2 < 0$, rotation dominated

$$\nabla\zeta(t) \propto \exp\left[\frac{1}{2}i(\zeta^2 - S_1^2 - S_2^2)^{\frac{1}{2}}t\right]$$



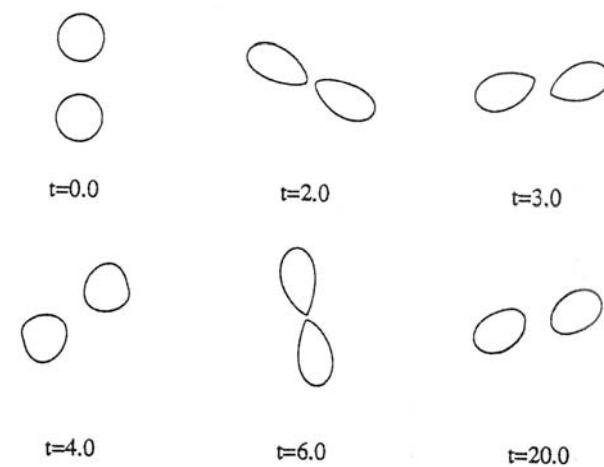
➤ The strong differential rotation associated with the core vortex

(ii) when $S_1^2 + S_2^2 - \zeta^2 > 0$, strain dominated

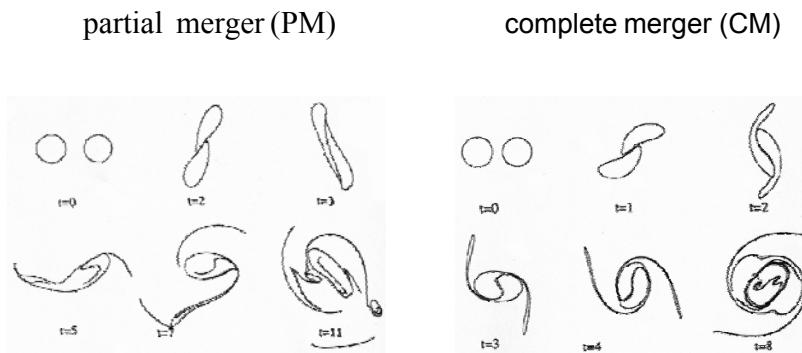
$$\nabla\zeta(t) \propto \exp\left[\frac{1}{2}(S_1^2 + S_2^2 - \zeta^2)^{\frac{1}{2}}t\right]$$

➤ $\tau_{fil} = 2(S_1^2 + S_2^2 - \zeta^2)^{-1/2} < 30 \text{ min}$: rapid filamentation time

Elastic interaction regime



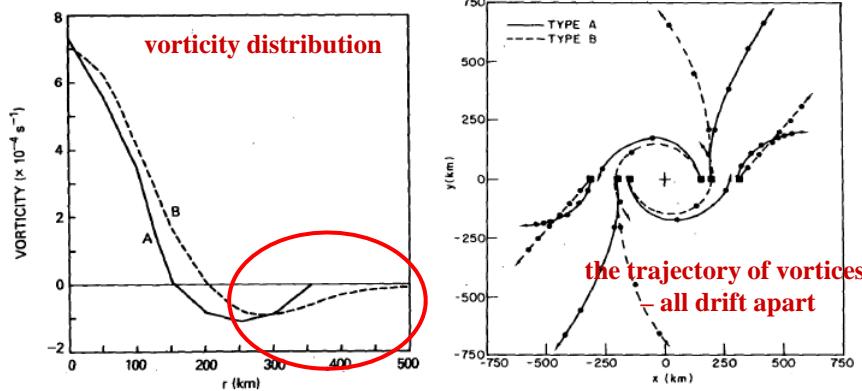
Merger regime



Why ‘merger’ ?

- Chang (1983) – diabatic heating
- DeMaria & Chan (1984) – vortex vorticity gradient
- Dritschel and Waugh (1992)
– advection + selective decay of 2D turbulence

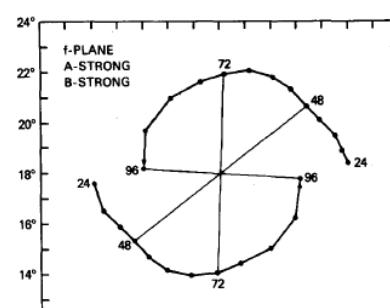
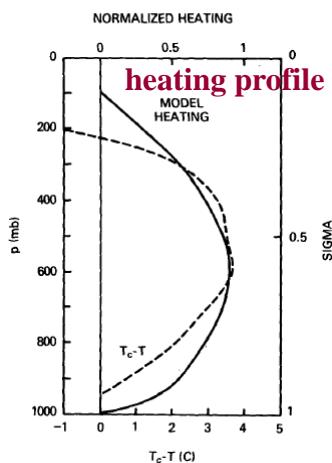
Chang (1983)



non-diabatic heating

Chang (1983)

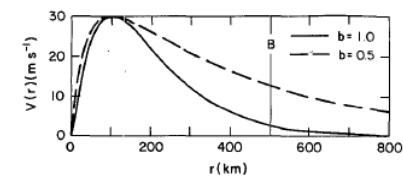
diabatic heating



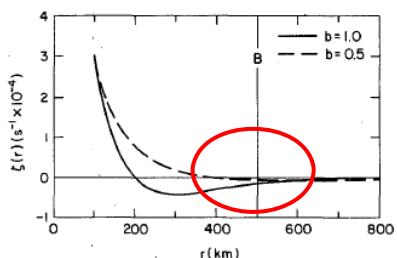
$$\frac{dQ}{dt} = 200 (\text{K day}^{-1})$$

The distance between two vortices decreases with time

DeMaria & Chan (1984)



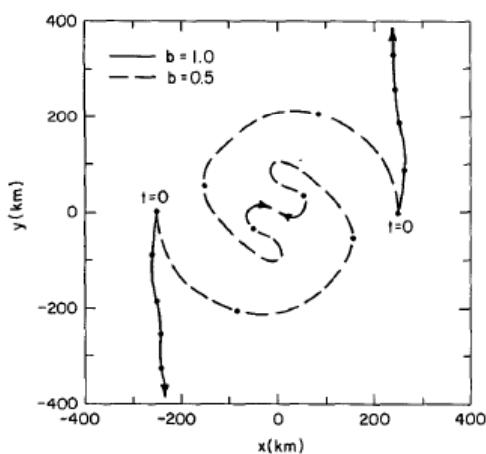
$$V(r) = V_m \left(\frac{r}{r_m} \right) \exp \left\{ \frac{1}{b} \left[1 - \left(\frac{r}{r_m} \right)^b \right] \right\},$$



$$\xi(r) = \frac{2V_m}{r_m} \left[1 - \frac{1}{2} \left(\frac{r}{r_m} \right)^b \right] \exp \left\{ \frac{1}{b} \left[1 - \left(\frac{r}{r_m} \right)^b \right] \right\}$$

b : the factor determines the rate of tangential wind decays

DeMaria & Chan (1984)



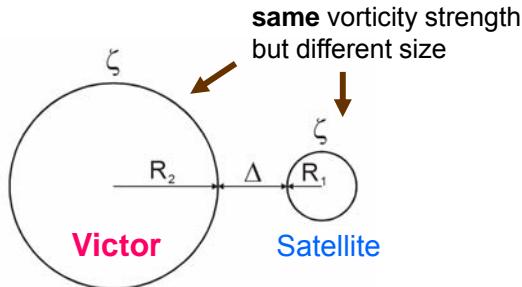
b = 1.0 drift apart

b = 0.5 merge

Binary vortex interaction Dritschel and Waugh (1992)

【Variables】

$$\begin{aligned} R_1, R_2 \\ \Delta \\ \zeta \end{aligned}$$



【Parameters】

- Vortex radius ratio (r) = $\frac{R_1}{R_2}$
- Dimensionless gap ($\frac{\Delta}{R_1}$)

【Conclusion】

- Elastic Interaction (EI)
- Partial straining-out (PSO)
- Complete straining-out (CSO)
- Partial merger (PM)
- Complete merger (CM)

Binary vortex interaction

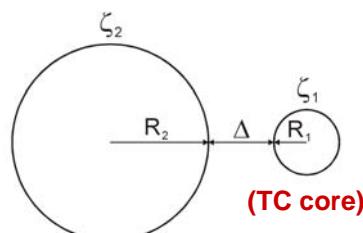
Kuo et al. (2004)

【Variables】

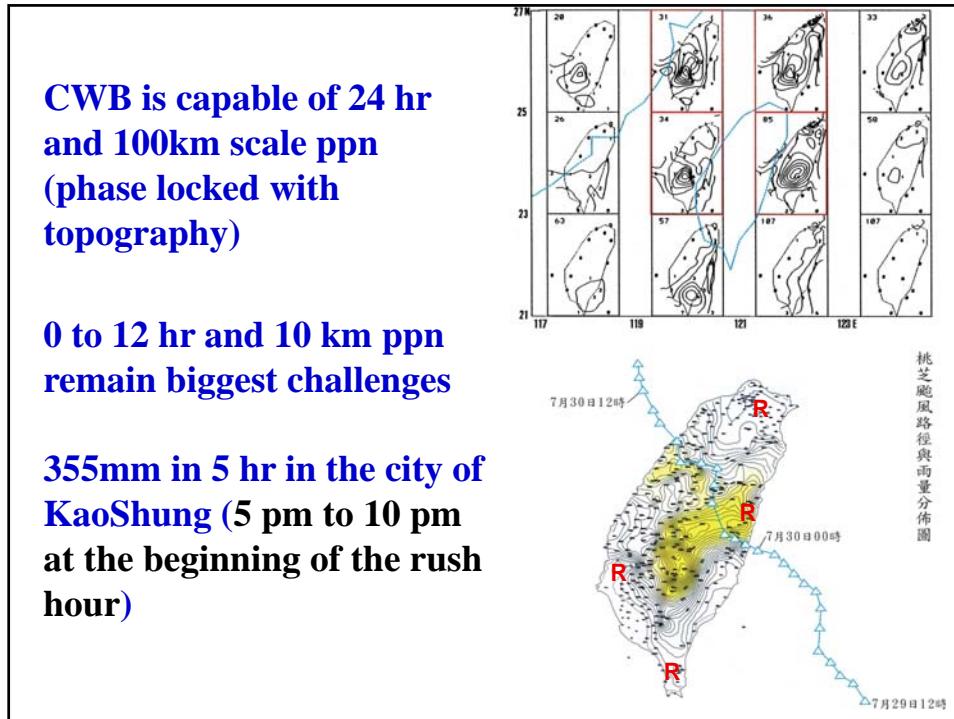
$$R_1, R_2 ; \Delta ; \zeta_1, \zeta_2$$

【Parameters】

- Vortex radius ratio (r) = $\frac{R_1}{R_2}$
- Dimensionless gap ($\frac{\Delta}{R_1}$)
- Vortex strength ratio (γ) = $\frac{\zeta_1}{\zeta_2}$



- An extension of Dritschel and Waugh's (1992) work.
- In addition to the radii ratio and the normalized distance between the two vortices, the vorticity ratio is added as a third external parameters.



颱風潛熱與其它能量的比較

颱風的全台灣平均總雨量為 400mm
 $400\text{ mm} = 0.4\text{ m}$
 $0.4\text{ m} * 1000\text{ kg m}^{-3} * 2.5 \times 10^6\text{ J kg}^{-1}$
 $= 10^9\text{ J m}^2$
 $10^9\text{ J m}^2 * 3.5 \times 10^{10}\text{ m}^2$
 $= 3.5 \times 10^{19}\text{ J} \sim 10^{20}\text{ J}$

${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{56}^{142}\text{Ba} + {}_{36}^{91}\text{Kr} + 3 {}_0^1\text{n}$

$1.68 * m * 10^{13}\text{J/mol}$
 $\Rightarrow 1.46 \times 10^6\text{ kg U}^{235} (6 \times 10^6\text{ mol})$

能量估計值		備註
颱風降雨總潛熱能量	10^{20} J	可使台灣整層大氣增溫100度
台灣一年用電量	$5 \times 10^{17}\text{ J}$	需數百年用電量才相當
全世界核子彈爆炸釋放能量	$2 \times 10^{19} \sim 2 \times 10^{20}\text{ J}$	與颱風同等級
核戰後燃燒釋放能量	$2 \times 10^{20}\text{ J}$	與颱風同等級
地球一天接受的太陽能量	$1.5 \times 10^{22}\text{ J}$	數百個颱風
Tunguska隕石撞地球 (西元1908年，西伯利亞)	10^{16} J	颱風的萬分之一
火流星撞地球 (恐龍滅絕?)	$4 \times 10^{23}\text{ J}$	數千個颱風

Rozoff et al. (2006)

The strong differential rotation outside the radius of maximum wind of the core vortex may also contribute to the formation and maintenance of the moat.

The Rapid Filamentation Zone: A zone with the filamentation time smaller than the 30 min convective turnover time.

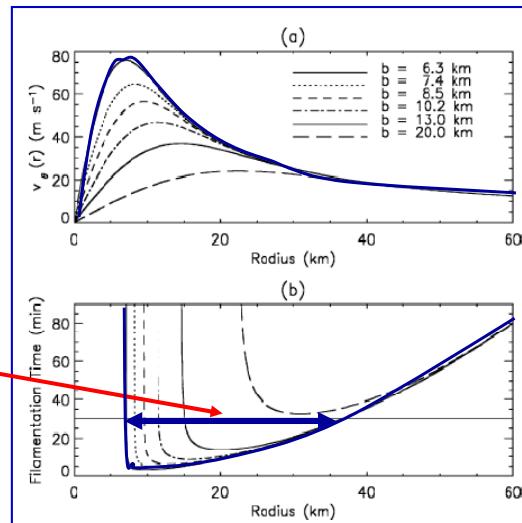


FIG. 2. (a) Radial profiles of $v_\theta(r)$ as given by (15) for six different values of the parameter b . (b) Corresponding radial profiles of $\tau_{fi}(r)$ as given by (16). The filamentation times $\tau_{fi}(r)$ are plotted only in the strain-dominated regions, where $S_1^2 + S_2^2 - \zeta^2 > 0$.

Huang and Robinson
1998

Turbulence
↑
Rhines curve
↓
Waves

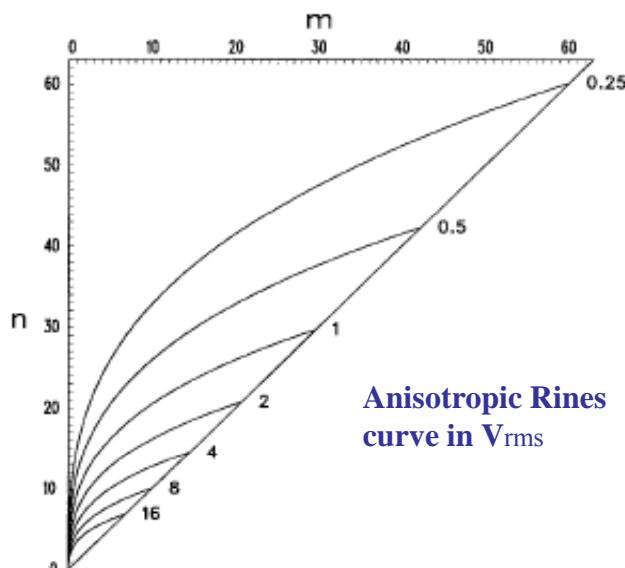


FIG. 1. Anisotropic Rhines curve on the wavenumber plane based on Eq. (3). Dimensional values of V_{rms} (in m s^{-1}) are labeled at right.