數學模式與科學研究

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3/20/2010
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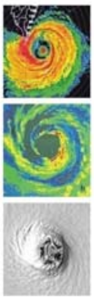


數學是科學的語言，數學模式與科學計算是探索未知的利器；隨著電腦的進步，科學計算更成為打開非線性科學研究的敲門磚。數學模式建立（建模）、科學計算、數位資料分析詮釋與驗證等過程，更是現今數理科學的典範。

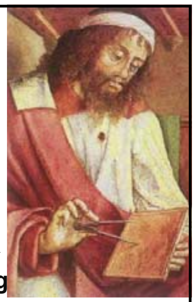
Politics are for the moment
An equation is for eternity

Fovell, Taipei, 2008

The profound study of nature is the most fertile source of mathematical discoveries.
Fourier 1768-1830

讀 算 寫

幾何
代數
微積分
電腦計算繪圖
數學建模/科學計算
Mathematical Modeling
Scientific Computing



+ -	x /
加、減	乘、除
線性	非線性
大題大作	小題大作

2005—2055 科技探索 Institute For The Future

生物模擬與計算

數學模型與生物結合

健康醫療規劃

生物資訊分析(Bioinformatics)

透過人腦、電腦介面，強化人體功能

生物檢測

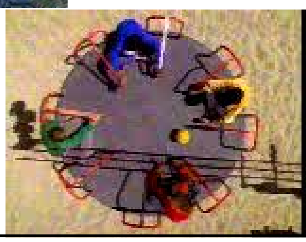
複雜系統：生態、經濟、氣象等數學模式應用議題
…………………………………………

數量化、數位化
數學模式--動力系統

旋轉
Rotation

Coriolis Force

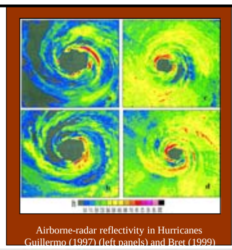
Non-inertial Frame

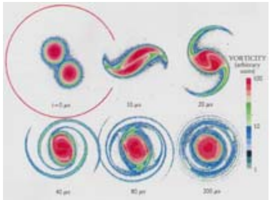


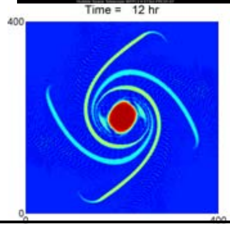
Jean Leon Foucault 1851

$$\frac{d^2\mathbf{x}}{dt^2} = \mathbf{g} + \frac{\mathbf{T}}{m} - 2\boldsymbol{\Omega}\times\frac{d\mathbf{x}}{dt}$$

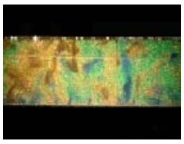
$$\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho}\nabla p - 2\boldsymbol{\Omega}\times\mathbf{V} + \mathbf{g} + \nu\nabla^2\mathbf{V}$$



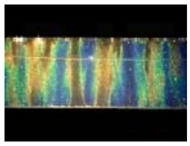




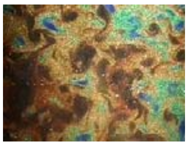
3D

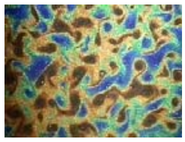


2D (strong rotation)



Taylor columns Vortex Tubes





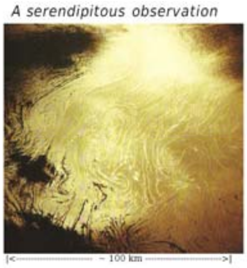
Vortices with sharp edge

Kyoto Univ. GFD group

The eddies are small, numerous, and with long filaments.

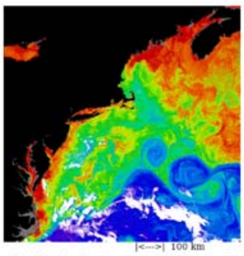
Tortured Ocean?!

A serendipitous observation



|<——— ~ 100 km ———>|

Photograph taken on board Space Shuttle *Challenger* over Mediterranean Sea (Source: NASA)



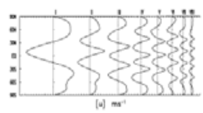
|<—>| 100 km

Satellite (Nimbus-7) observation of "**ocean color**" (concentration of phytoplankton), NW North Atlantic Source: NASA GSFC

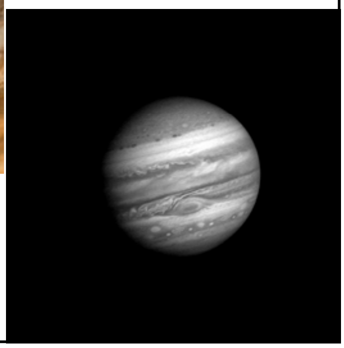
Courtesy of H.P. Huang

Jupiter Rotational period 9.84hr

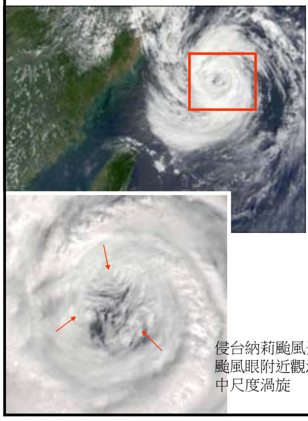
The Great Red Spot

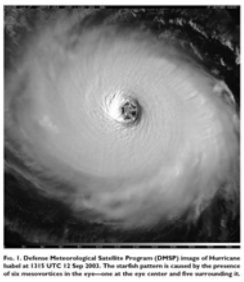


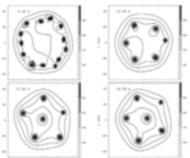
Huang and Robinson 1998



納莉颱風眼附近的中尺度渦旋



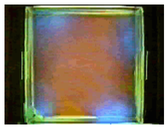




侵台納莉颱風登陸前
颱風眼附近觀測到3 個
中尺度渦旋

Waves with zero potential vorticity

Non-rotation — rotation — rotation



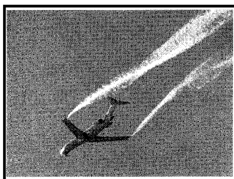
Gravity waves

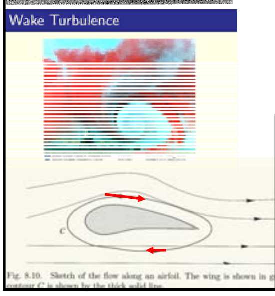
Kelvin Waves
Edge waves

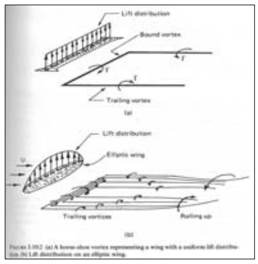
Kelvin-Helmholtz Instability

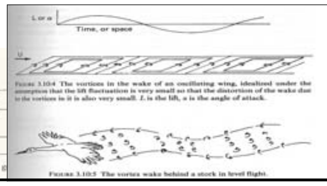
Turbulent Mixing 晴空亂流

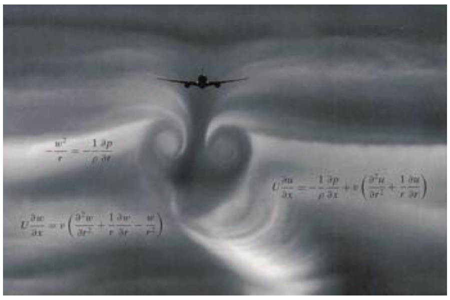
Intermediate wavelength





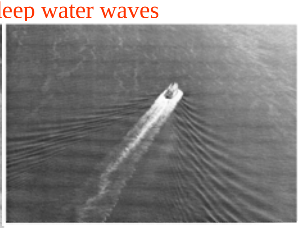




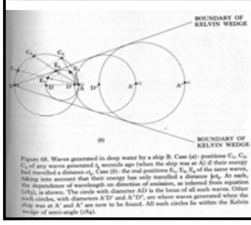


A British Airways Boeing 777-200 aircraft is approaching to land at Gatwick Airport traveling at 170 kts at approximately 1800 ft. The cloud base Is 2200 ft, RH = 83%, T = 16.8, Td = 14.5, p = 1022.2 hPa, wind = 6.4 km/h.

Kelvin wedge deep water waves



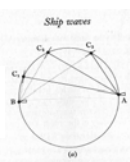
[Courtesy of Aerofilms Ltd.]



$$2*\sin^{-1}\left(\tfrac{1}{3}\right)=2*19.5^{o}$$

$$u\cos\phi = c$$

Stationary waves

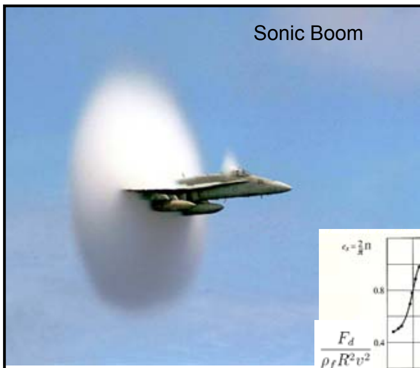


Multiple Scale Interactions in Vortex

Wave mean flow interaction in stable stratified fluid
Turbulent feed back to the vortex mean flow

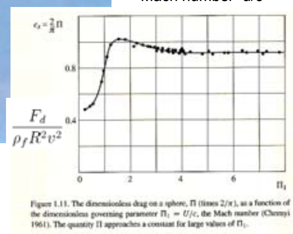
2D turbulence

Kyoto Univ. GFD group



u/c ~ 1

Mach number u/c



Adiabatic Sound

Fovell, 2008 高雄

This model will be a simplification and an idealization, and consequently a falsification. It is to be hoped that the features retained for discussion are those of greatest importance in the present stage of knowledge.

Turing The Chemical Basis of Morphogenesis

以特殊事實為憑藉，逐漸推廣引伸，成立概念式定律的系統，以便籠罩更複雜耿廣泛的對象，科學家依據事實為前提來證明普遍的結論。 方東美

吾生也有涯，吾知也無涯，已有涯逐無涯，殆矣。
有限時空之觀察或有限資料去推導無限時空的科學定律。

有限事實 → 理想（數學）模式 → 解釋 / 驗證 / 預測

False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness.
Darwin, The Origin of Man, chapter 6

數學模式　　驗證資料

理論、解釋資料 interprets experiences on a higher than purely descriptive level. von Neumann
預測　準確性　預測能力
資料同化　利用科學數學模式整合有限的觀測，建構出較完整的資料

數學模式

Formulation　微分、差分方程式

Solution / Analysis　分析、解

Interpretation　科學詮釋

中階課程：微分方程(ODE,PDE)
統計、線性代數
程式、計算與繪圖

數量化、數位化
數學化--模式--動力系統

"Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum." Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。通典

共49個字，假設中文常用字為1000字，共有10^147個選擇

地球歷史 10^18 sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 10^4，

10^10*10^18*10^4 = 10^32

10^32/10^147=10^(-115) ~ 0 機率為零 ，不可能的巧合！

研究學問是苦心孤詣的事業！ 不要人云亦云！

Function y = f(x)

Commonly Occurring Functions

Polynomials: approximate with a high degree of accuracy, almost any existing function

Trigonometric functions cos sin

Exponential functions e

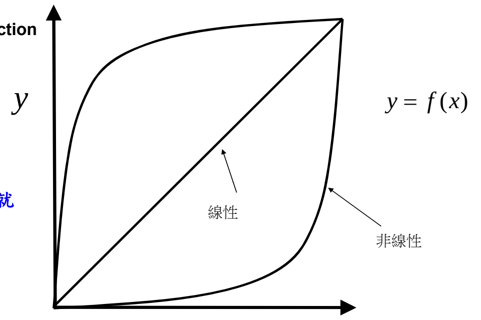
Logarithmic function log ln
s=k log w

一分耕耘 ， 一分收穫？？

函數 Function

(因果）

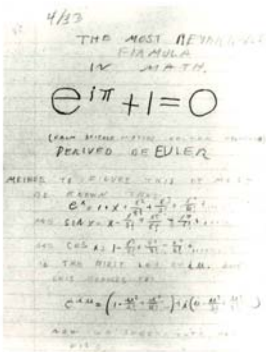
y : 成就

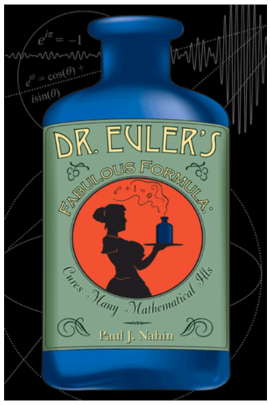


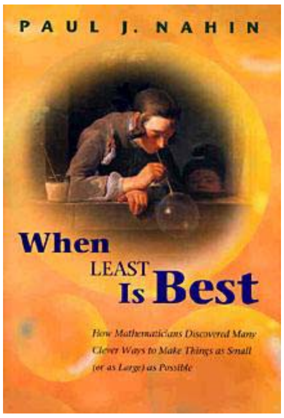
$$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ?$$

x : 努力

Note of Feynman at age of 15







A universe made by God must be a perfect universe, and consequently should always operate with economy.

Shortest path reflection (Hero of Alexandria, 2nd century B.C.)

"Every action done by nature is done in the shortest way."
(Leonardo da Vinci 1452-1519)

Principle of least time (Fermat 1658)

Principle of least action
(Maupertuis 1747)

Hamilton Principle (Sir William Rowan Hamilton 1805-1865)

微積分數學

the Mathematics of Change

莊子：一尺之錘、日取其半、萬世不絕。

$\triangle x \to 0$

Rate of Change

$$\lim_{\triangle \to 0} \frac{f(\triangle)}{g(\triangle)} = \frac{df}{dg}$$

$$\lim_{\epsilon \to 0} \frac{\sin \epsilon}{\epsilon} = ?$$

$u = u(x, y)$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

偏微分

$$\frac{\partial x^2 y}{\partial y} = x^2$$

只對y變數微分，不改變x變數

Rene Descartes (1596-1650)
Pierre de Fermat (1601-1665)
Issac Newton (1642-1727)
Gottfried Leibniz (1646-1716)

"The derivative was first *used*,
it was then *discovered*;
it was then *explored and developed*;
and it was finally *defined*." Grabiner 1983

How to maximize the product from a divided constant? (Fermat 1637)

$M = x(C-x)$ $\quad \hat{x}^2 - C\hat{x} + M \approx (\hat{x}+E)^2 - C(\hat{x}+E) + M$

$x^2 - Cx + M = 0$ $\quad 0 \approx 2\hat{x}E + E^2 - CE$ $\quad f(\hat{x}) \approx f(\hat{x}+E)$

$x = \frac{C \pm \sqrt{C^2-4M}}{2}$ $\quad 0 \approx 2\hat{x} + E - C$ $\quad f(\hat{x}+E) - f(\hat{x}) \approx 0$

$M = \frac{1}{4}C^2$ $\quad 0 = 2\hat{X} - c$ $\quad \frac{f(\hat{x}+E) - f(\hat{x})}{E} \approx 0$

$x = \frac{1}{2}C$

Czech mathematician Bernard Bolzano (1781-1848) 1817 $\quad \lim_{E\to 0} \frac{f(\hat{x}+E) - f(\hat{x})}{E} = \frac{df}{dx} = f'(x)$

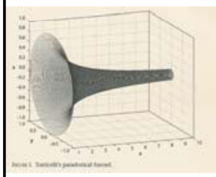
Evangelista Torricelli (1608-1647)

Torricelli designed first accurate barometer.

A finite volume bounded an infinite surface.
[The hyperbola xy =1rotate about x axis;
Gabriel's horn in Biblical Story]

"Torricelli's paradox funnel"

At 1672, English philosopher Thomas Hobbes declared that one would have to be crazy to believe Torricelli.



$$\triangle V \approx \pi y^2 \triangle x$$

$$V = \int dV = \pi \int_a^\infty y^2 dx$$

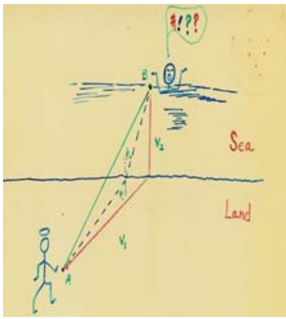
$$V = \pi \int_a^\infty \frac{dx}{x^2} = \frac{\pi}{a}$$

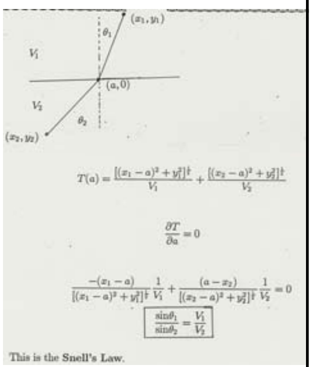
$$A = \int_a^\infty y\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$A = \int_a^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}dx = \int_a^\infty \frac{\sqrt{x^4+1}}{x^3}dx > \int_a^\infty \frac{\sqrt{x^4}}{x^3}dx = \int_a^\infty \frac{1}{x}dx \sim \infty$$

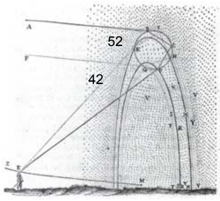
Principle of Least Time (Fermat's principle)





"it was all as ephemeral as a rainbow"

To the lighthouse, Virginia Woolf



René Descartes

Noah's Thanksoffering (c.1803) by Joseph Anton Koch. Noah builds an altar to the Lord after being delivered from the Flood; God sends the rainbow as a sign of his covenant (Genesis 8-9).

一樣觀魚多樣情！

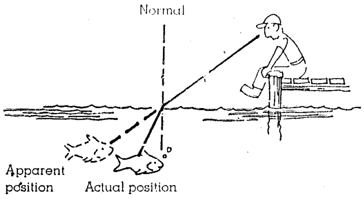


FIGURE 5.13 The refraction of light as it passes from the water into the less-dense air causes a fish to appear closer to the surface than it actually is.

（1）魚快樂嗎？
（2）熱血沸騰，立志革命！
（3）折射定律，最小原理。

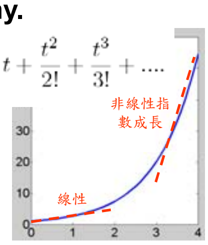
Exponential functions are both man's best friend and worst enemy.

$$\frac{dx}{dt} = -x$$

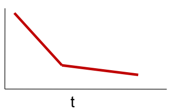
$$x(t) = x(0)e^{-t}$$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \ldots.$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots.$$



指數遞減
e-folding time
（類似半衰期）



$$\lim_{t\to\infty}\frac{t^n}{e^t} = 0$$

Exponential Growth
指數成長

$$\frac{dN}{dt} = -kN$$

$$N(t) = N(t_0)e^{-k(t-t_0)}$$

$N(t)$ $N(t_0)$ k t_0

Exponential Decay

Dating 定年

Half life 半衰期

Finding k from two observations

$$N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$N(t_2) = N(t_0)e^{-k(t_2-t_0)}$$

$$p = \frac{N(t_1)}{N(t_2)} = e^{-k(t_1-t_2)}$$

$$k = \frac{\ln p}{t_2 - t_1}$$

Radiocarbon Dating

Libby Nobel Prize for Chemistry in 1960

C14 half life ~5700 years:
Ratio of C14 to C12 is constant for living matter,
Begin to decay when dead.

$$k = \frac{\ln 2}{5700} \approx 1.216 \times 10^{-4}$$

$$N_0 e^{-kt_{1/2}} = \frac{1}{2}N_0$$

$$N(t_1) = pN(t_0)$$ Assumed to be the same as present

$$pN(t_0) = N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$t_0 = t_1 + \frac{\ln p}{k}$$

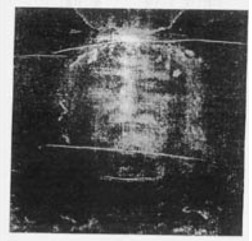
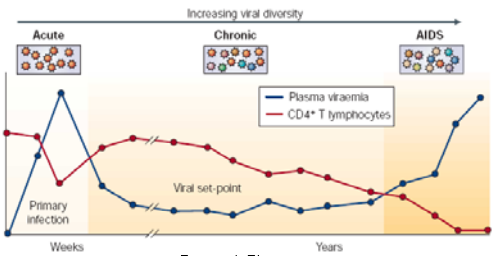


Figure 3.9 Shroud of Turin

Lascaux cave painting



Dormant Phase
潛伏期

HIV Modeling

Perelson and Nelson (1999)

$$\frac{dV}{dt} = P - cV,$$ 藥物治療

$$\frac{dT}{dt} = kT_0V - \alpha T,$$

$$P = N\alpha T.$$

$$P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \quad (1/(day \cdot ml))$$

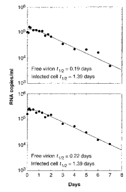


Figure 4.5. Log of plasma concentrations (copies per mL) of HIV-1 RNA (circles) for two representative patients (upper panel, patient 104; lower panel, patient 107) after *ritonavir* treatment was begun on day 0. The solid line is a nonlinear least square fit to the data. HIV-1 RNA level is an easier measure of HIV virions since each HIV virion contains two RNA molecules. (See exercise 5 for more details.) (From Perelson et al. [1996], used by permission of Alan S. Perelson.)

觀察病人服藥後反應決定C

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
: non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

何大一雞尾酒療法

Periodic phenomena are actually everywhere in the biological world.

What else can you think of?

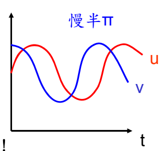
Romantic Romeo and Fickle Juliet (Strogatz 1988)

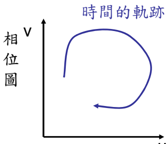
$$\frac{dR}{dt} = J \qquad \frac{dJ}{dt} = -R$$

$$\int_0^{2\pi} \cos t \sin t dt = 0$$

u v

$$\overline{uv} = 0$$



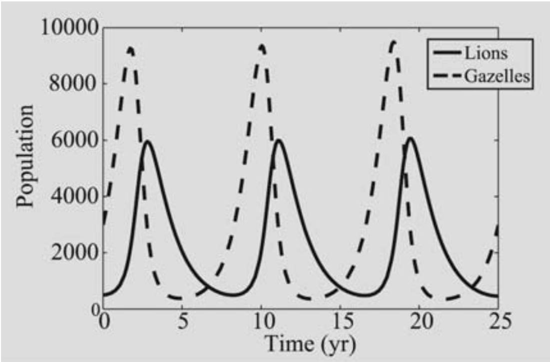


相位圖

Cos 和 Sin 零相關、不來電！

過程可以很熱鬧

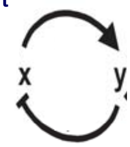
瓊瑤小說？



Example 8-11

Negative Feedback Oscillators

X Cost　Y Sin t



$$\frac{dy}{dt} = x$$

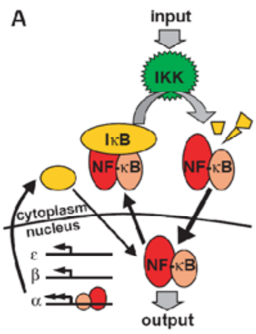
$$\frac{dx}{dt} = -y$$

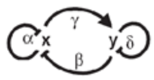
物廉價美　顧客增加消費　價格上揚

價格上揚　顧客減少消費　價格下滑

負回饋

NF-κB and IκB Model





X: nucleus NF-κB

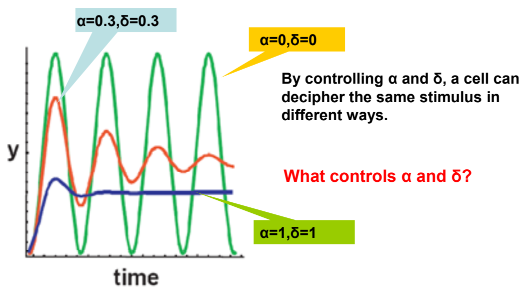
Y: IκB

$$\frac{d}{dt}x = S - \alpha x - \beta y$$

$$\frac{d}{dt}y = \gamma x - \delta y$$

Science **298:** 1241-1245.

Summary of NF-κB and IκB Model



By controlling α and δ, a cell can decipher the same stimulus in different ways.

What controls α and δ?

A beautiful woman !

$$\Pi = \frac{r_f}{E^{1/5} t^{2/5} \rho_0^{-1/5}}$$

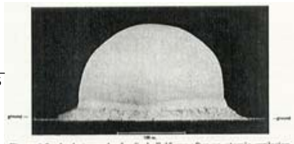


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963).

G.I. Taylor 1950

Symbol	*Definition*	*Representative value or first guess*
R	radius of wavefront	10^2 m
t	time	10^{-2} s
p_0	ambient pressure	10^5 Pa
ρ_0	ambient density	1 kg m^{-3}
E	energy released	10^{14} J

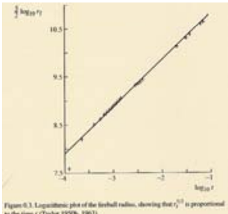


Figure 0.3. Logarithmic plot of the fireball radius, showing that $r_f^{5/2}$ is proportional to the time t (Taylor 1950b, 1963).

原子彈能量～ 10^{14} J

Metabolic rate vs size

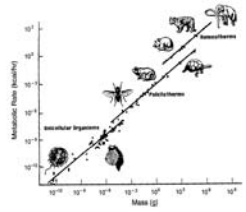


FIGURE 2 Metabolic rate (in kcal/hr) for a series of organisms ranging from the smallest microbes to the largest mammals as a function of mass (in g), exemplifying the persistence of the 3/4-power scaling law (the solid lines) over 20 orders of magnitude (Hemmingsen [32]).

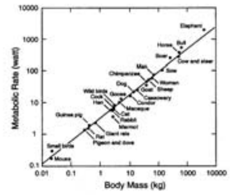


FIGURE 1 Metabolic rate (in watts) for a series of mammals and birds as a function of mass (in kg); the scale is logarithmic and exemplifies the 3/4-power scaling discovered by Kleiber [2, 22, 27, 29].

$$I = I_0 M^{3/4}$$

Hemmingsen (1960) *Reports of the Steno Memorial Hospital and Nordisk Insulin Laboratorium* **9**, 6-110

Kleiber (1932) Body size and metabolism. *Hilgardia* **6**, 315-353.

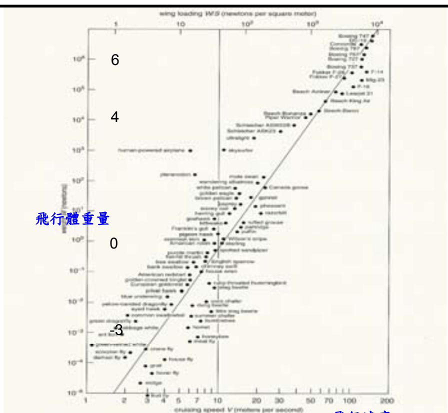
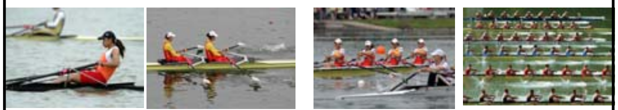


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.



A, G constant

$$F \sim \rho v^2 l^2$$

$$P \sim \rho v^3 l^2$$

$$G \sim \frac{l^3}{n}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$v \sim n^{1/9}$$

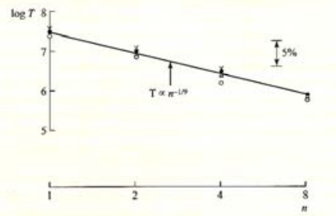


Figure 1.13. The −1/9 power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; ×, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).

Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)

The "***Black Death***" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

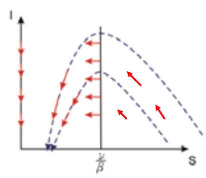
$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu) I$$

$$\frac{dR}{dt} = +\nu I$$

ν Recovery Rate

β Infection Rate

No Death in the model



$S = \frac{\nu}{\beta}$ 有 null cline

Forecast and control of epidemics in a globalized world **PNAS vol.101 no.42**

Hufnagel†, Brockmann, and Geisel

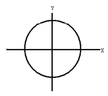
演講者: 陳怡文 日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

Negative Feedback

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

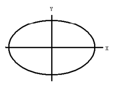


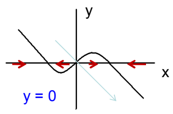
x = sint, y = cos t

NF + Fast and Slow

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -\epsilon x$$

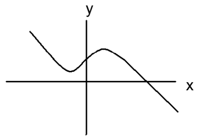


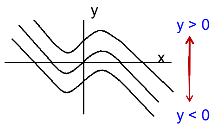


NF + FS + Multiple equilibriums

$$\frac{dx}{dt} = x - x^3 + y$$

$$\frac{dy}{dt} = -\epsilon x$$





London Millennium Bridge

London's Millennium Bridge is the first pedestrian river crossing over the Thames in central London for more than a century.

It is a 325m steel bridge linking the City of London at St. Paul's Cathedral with the Tate Modern Gallery at Bankside.

"Nice" lateral vibrations (20 cm S shape wobble, 1cycle per second) like on Tacoma Bridge developed on the day (June 12, 2000) of the opening……

The Ultimate Problem in Meteorology　Bjerknes 1911
氣象的終極問題

I The Present state of the atmosphere must be characterized as accurately as possible. 正確的觀測大氣現狀 [多重時空尺度]

II The intrinsic laws, according to which the subsequent states develop out of the preceding ones, must be known.
正確的大氣運作規律

Vilhelm Bjerknes (1862–1951)

Numerical Weather Prediction 數值天氣預報
[第一部電腦ENIAC, EBV model, 1950]
The Observation component 觀測
The diagnostic or analysis component 診斷分析
The prognostic component 預報

科氏力(18 19)

Momentum Conservation (18)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \nu\nabla^2 w$$

Mass conservation (18)

$$\frac{\partial \rho}{\partial t} + \frac{\partial u\rho}{\partial x} + \frac{\partial v\rho}{\partial y} + \frac{\partial w\rho}{\partial z} = 0$$

Energy conservation (19)

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = Q$$

Radiation 大氣輻射 (19,20)
Moisture Latent heat

Equation of State(17,18,19)

$$p = \rho R_a T, \quad \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_a}{c_p}}$$

(19)

雲物理 (19,20)

問蒼茫大氣，誰主浮沈？
質量、動量、能量與大氣狀態方程式

Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

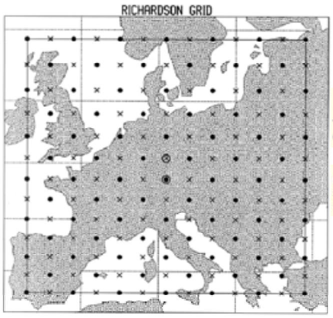
His 'forecast' was a catastrophic failure:

$\Delta p = 145$ hPa in 6 hours

His method was unimpeachable.

So, *what went wrong?*

Peter Lynch

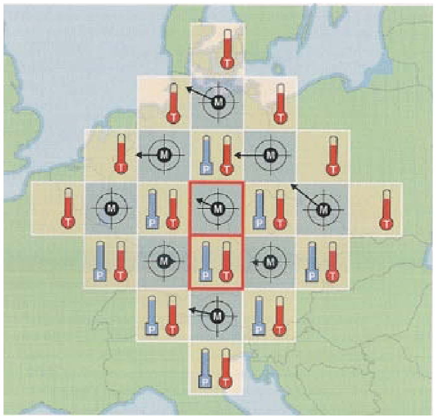


$$\frac{df}{dx} \rightarrow \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

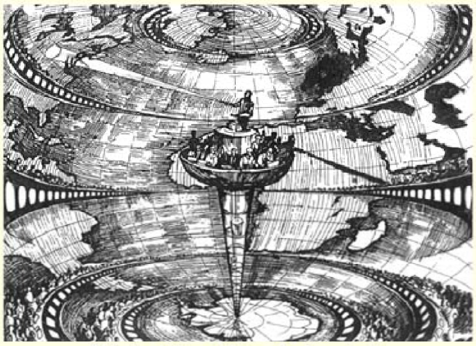
$$\frac{dQ}{dt} \rightarrow \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

13x13=169個ODE

169 自由度



Richardson's Dream

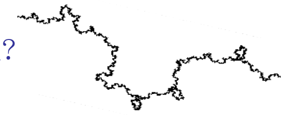


Richardson's Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, ECMWF, 1984

64,000 Computers: The first Massively Parallel Processor

A relation between the probability of two countries going to war and the length of their common border.

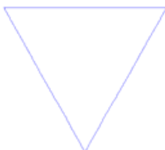
海岸線有多長?

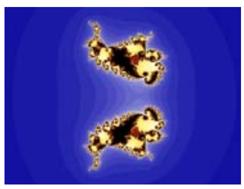


碎形源起

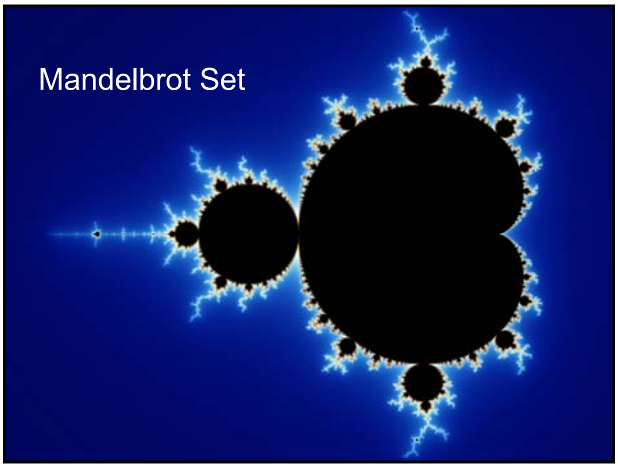
Fractal 碎形

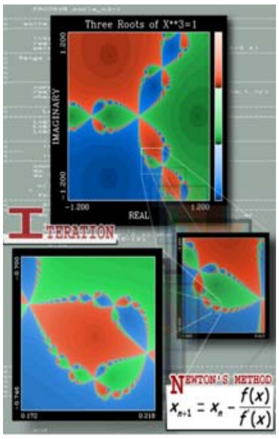
自我相似性
奇異吸子
碎形維度





Mandelbrot Set





自我相似的對稱圖案

Cautious Romeo and Juliet

$$\frac{dR}{dt} = -aR + bJ$$

$$\frac{dJ}{dt} = -aJ + bR$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -a-b \\ -a+b \end{pmatrix}$$

$$\begin{pmatrix} R \\ J \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{(-a-b)t} + \beta \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{(b-a)t}$$

Falling in Love b > a 1/b < 1/a
情網必須墜入（猶豫是感情殺手）

Arm Races

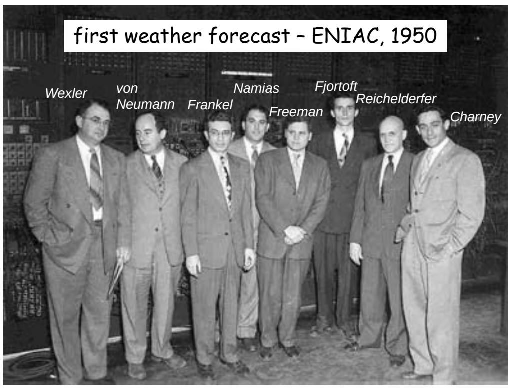
$$\frac{dx}{dt} = -a(x - x_0) + by$$

$$\frac{dy}{dt} = -a(y - y_0) + bx$$

x, y war potential

1/a < 1/b Truce

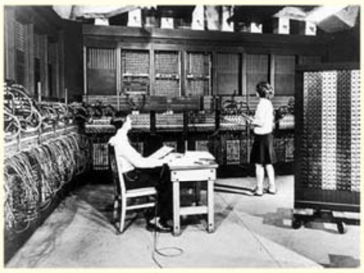
(Peace?)



In front of the Eniac, Aberdeen Proving Ground, April 4, 1950, on the occasion of the first numerical weather computations carried out with the aid of a high-speed computer.

The ENIAC

Electronic Numerical Integrator and Computer



18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

No high-level language
Assembly language

500 Flops
Function Table 0.001 s

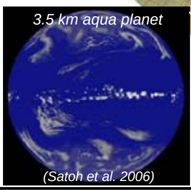
3,700,000,000 times slower than current day large computer

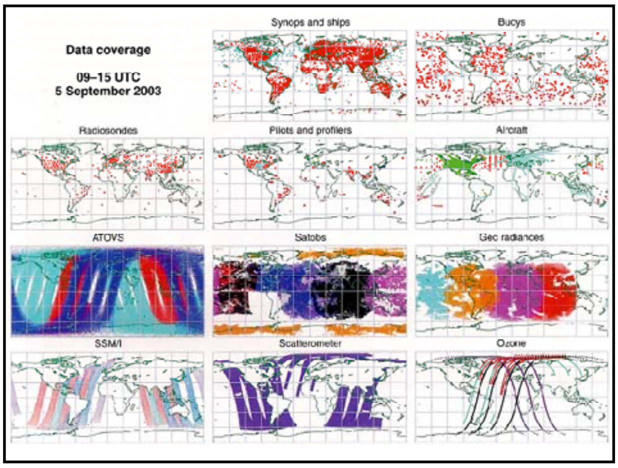
第一部電腦　氣象預報

ENIAC – late 40s

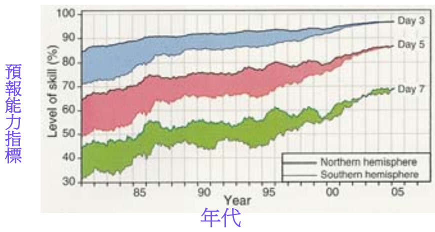
Earth Simulator -- 2002

NASDA, JAERI, JAMSTEC





南北半球 對於3，5，7天之預報能力隨時間的進展



南北差異日漸減少主要是由於近年來衛星觀測以及資料同化技術日漸成熟

7天預報一年進步約1.5%，3天預報一年進步0.3%

一杯咖啡，古今往事盡付笑談中。

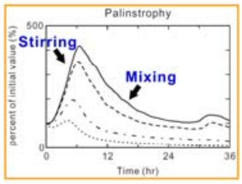
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

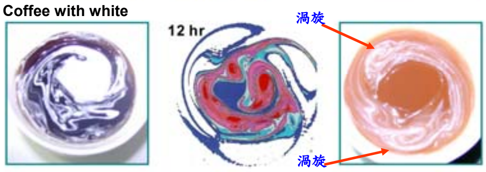
$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta\, dV$$

$$\frac{dC}{dt} = \int(\vec{V}\cdot\nabla\theta)\nabla^2\theta\, dV - \nu\int(\nabla^2\theta)\, dV$$

Stirring **Mixing**



Coffee with white



Thank you!

A painting with filamentations!

Now we only see models,
like reflections in a mirror;
but then we shall see face to face.
Now I only know partially;
but then I shall know as fully as
I am myself known.

St. Paul, 1st letter to the Corinthians, 13:12

Models、經典、聖哲就如鏡子，讓我們看到自己，讓我們瞭解自己的侷限，更進而體會完整的人性。

「數學科學模式」幫助我們由片面觀察的自然界，統會瞭解共通完整的科學定律。