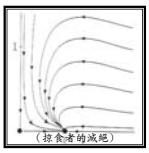
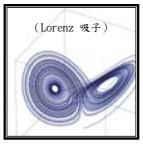
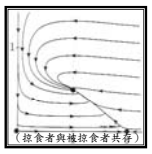
數學建模與科學研究

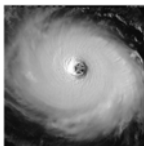
郭鴻基 教授

教育部國家講座教授
台灣大學終身特聘教授
中興大學物理講座教授









內容

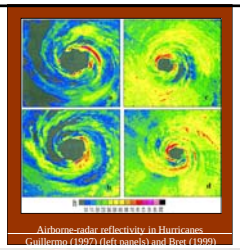
科學研究是探索未知知識的建構過程，而數學是科學的語言。隨著電腦的進步，資料大量的數位化，科學計算成為非線性科學研究的敲門磚。數學建模、科學計算、分析詮釋與驗證等過程，更是現今數學科學的典範。

2009/12/22 建國中學

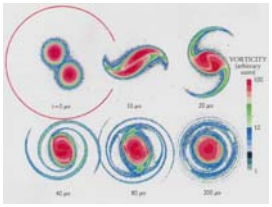
Politics are for the moment
An equation is for eternity

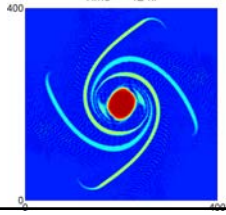
Fovell, Taipei, 2008

The profound study of nature is the most fertile source of mathematical discoveries.
Fourier 1768-1830


Airborne radar reflectivity in Hurricanes Guillermo (1997) (left panels) and Bret (1999)

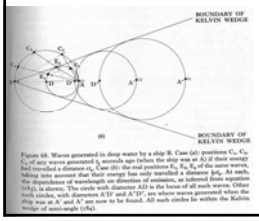

Time = 12 hr





Kelvin wedge deep water waves

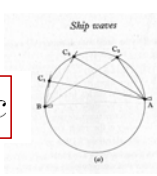
[*Courtesy of Aerofilms Ltd.*]



$$2 * \sin^{-1}\left(\tfrac{1}{3}\right) = 2 * 19.5^{\circ}$$

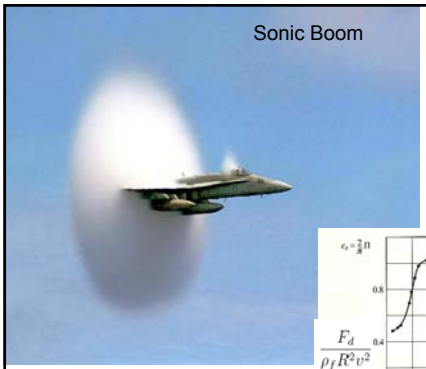
$$u \cos\phi = c$$

Stationary waves

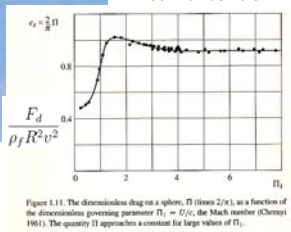


Sonic Boom

u/c ~ 1



Mach number u/c



Adiabatic Sound

數學模式

Formulation　　微分、差分方程式

Solution / Analysis　　分析、解

Interpretation　　科學詮釋

中階課程：微分方程(ODE,PDE)　**數量化、數位化**
統計、線性代數
程式、計算與繪圖　**數學化--模式--動力系統**

Fovell, 2008 高雄

This model will be a simplification and an idealization, and consequently a falsification. It is to be hoped that the features retained for discussion are those of greatest importance in the present stage of knowledge.

Turing　The Chemical Basis of Morphogenesis

"Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum."　Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。通典

共49個字，假設中文常用字為1000字，共有10^147個選擇

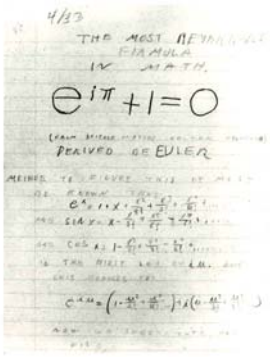
地球歷史 10^18 sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 10^4，

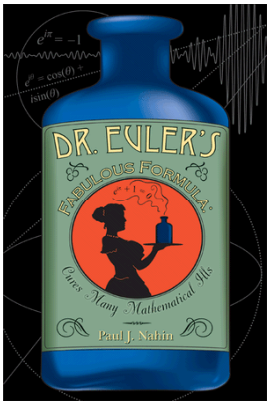
10^10*10^18*10^4 = 10^32

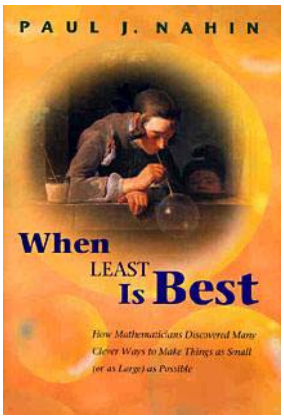
10^32/10^147=10^(-115) ~ 0　機率為零，不可能的巧合！

研究學問是苦心孤詣的事業！　不要人云亦云！

Note of Feynman at age of 15







A universe made by God must be a perfect universe, and consequently should always operate with economy.

Shortest path reflection (Hero of Alexandria, 2nd century B.C.)

"Every action done by nature is done in the shortest way." (Leonardo da Vinci 1452-1519)

Principle of least time (Fermat 1658)

Principle of least action (Maupertuis 1747)

Hamilton Principle (Sir William Rowan Hamilton 1805-1865)

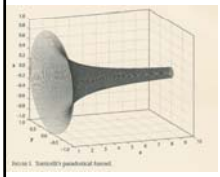
Evangelista Torricelli (1608-1647)

Torricelli designed first accurate barometer.

A finite volume bounded an infinite surface.
[The hyperbola xy =1rotate about x axis; Gabriel's horn in Biblical Story]

"Torricelli's paradox funnel"

At 1672, English philosopher Thomas Hobbes declared that one would have to be crazy to believe Torricelli.



$$\triangle V \approx \pi y^2 \triangle x$$

$$V = \int dV = \pi \int_a^\infty y^2 dx$$

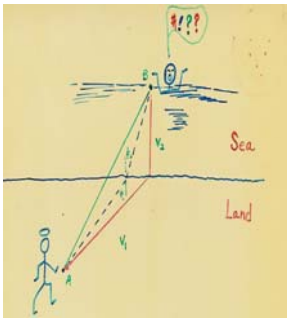
$$V = \pi \int_a^\infty \frac{dx}{x^2} = \frac{\pi}{a}$$

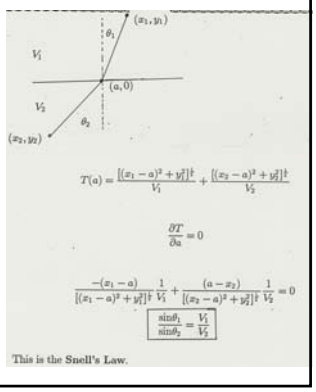
$$A = \int_a^\infty y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

$$A = \int_a^\infty \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx = \int_a^\infty \frac{\sqrt{x^4+1}}{x^3}dx > \int_a^\infty \frac{\sqrt{x^4}}{x^3}dx = \int_a^\infty \frac{1}{x}dx \sim \infty$$

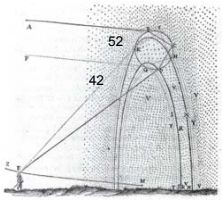
Principle of Least Time (Fermat's principle)



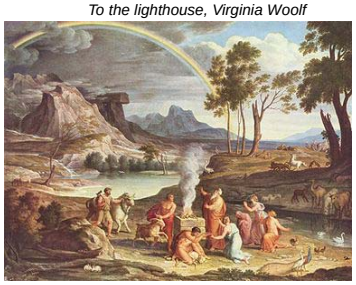


"it was all as ephemeral as a rainbow"

To the lighthouse, Virginia Woolf



René Descartes



Noah's Thanksoffering (c.1803) by Joseph Anton Koch. Noah builds an altar to the Lord after being delivered from the Flood; God sends the rainbow as a sign of his covenant (Genesis 8-9).

一樣觀魚多樣情！

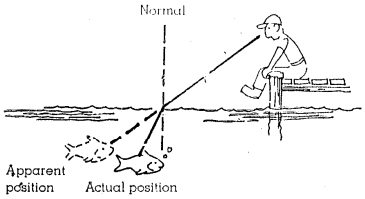


FIGURE 5.13 The refraction of light as it passes from the water into the less-dense air causes a fish to appear closer to the surface than it actually is.

（1）魚快樂嗎？
（2）熱血沸騰，立志革命！
（3）折射定律，最小原理。

Function y = f(x)

Commonly Occurring Functions

Polynomials: approximate with a high degree of accuracy, almost any existing function

Trigonometric functions cos sin

Exponential functions e

Logarithmic function log ln

s=k log w

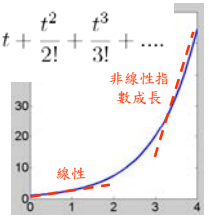
Exponential functions are both man's best friend and worst enemy.

$$\frac{dx}{dt} = -x$$

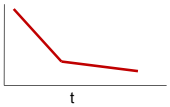
$$x(t) = x(0)e^{-t}$$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + ...$$

$$e^{t} = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...$$



指數遞減
e-folding time
（類似半衰期）



$$\lim_{t\to\infty} \frac{t^n}{e^t} = 0$$

Exponential Growth
指數成長

$$\frac{dN}{dt} = -kN$$

$$N(t) = N(t_0)e^{-k(t-t_0)}$$

$N(t)$ $N(t_0)$ k t_0

Exponential Decay

Dating 定年

Half life 半衰期

Finding *k* from two observations

$$N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$N(t_2) = N(t_0)e^{-k(t_2-t_0)}$$

$$p = \frac{N(t_1)}{N(t_2)} = e^{-k(t_1-t_2)}$$

$$k = \frac{\ln p}{t_2 - t_1}$$

Radiocarbon Dating

Libby Nobel Prize for Chemistry in 1960

C14 half life ~5700 years:
Ratio of C14 to C12 is constant for living matter,
Begin to decay when dead.

$$k = \frac{\ln 2}{5700} \approx 1.216 \times 10^{-4}$$

$$N_0 e^{-kt_{1/2}} = \frac{1}{2} N_0$$

$$N(t_1) = pN(t_0)$$

Assumed to be the same as present

$$pN(t_0) = N(t_1) = N(t_0)e^{-k(t_1 - t_0)}$$

$$t_0 = t_1 + \frac{\ln p}{k}$$

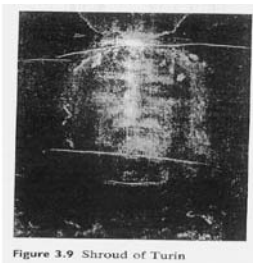
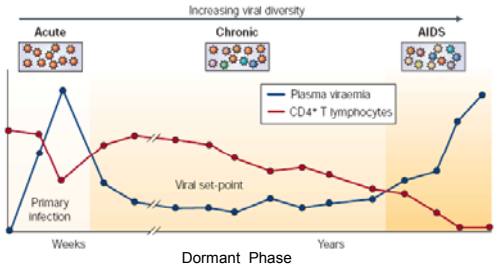


Figure 3.9 Shroud of Turin

Lascaux cave painting



Dormant Phase
潛伏期

HIV Modeling

Perelson and Nelson (1999)

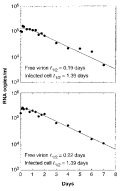
$$\frac{dV}{dt} = P - cV,$$

藥物治療

$$\frac{dT}{dt} = kT_0V - \alpha T,$$

$$P = N\alpha T.$$

$$P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \ \ (1/(day \cdot ml))$$



觀察病人服藥後反應決定C

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
: non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

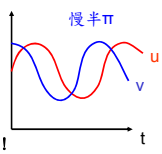
何大一雞尾酒療法

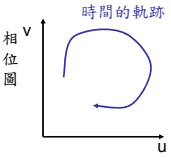
Romantic Romeo and Fickle Juliet (Strogatz 1988)

$$\frac{dR}{dt} = J \quad \frac{dJ}{dt} = -R$$

$$\int_0^{2\pi} \underset{u}{\cos t} \underset{v}{\sin t} dt = 0$$

$$\overline{uv} = 0$$





Cos 和 Sin 零相關、不來電！

過程可以很熱鬧

瓊瑤小說？

相位圖

The profound study of nature is the most fertile source of mathematical discoveries.

$$f(x) = \sum \hat{f}_k e^{ikx}$$

$$\hat{f}_k = \frac{1}{2\pi} \int_0^{2\pi} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}_k e^{ikx} dx$$

$$\hat{f}_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

Fourier, Jean Baptiste Joseph

1768-1830

1807 at age 39; argued with Lagrange and Laplace on the representation of a triangle wave with cosine and sine functions.

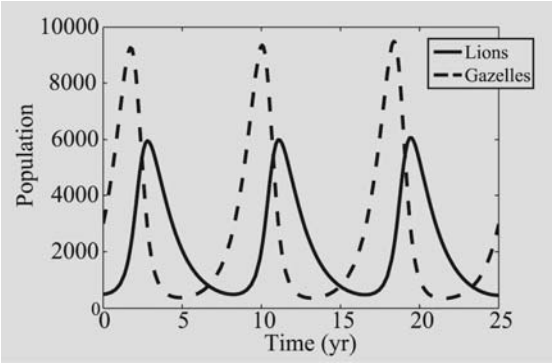
Heat emission or diffusion (by IR)

His calculations showed a very cold surface (No green house effect)

f(x) does not have to be analytical;
f(x) does not have to be periodic.

Periodic phenomena are actually everywhere in the biological world.

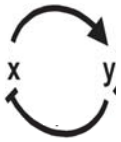
What else can you think of?



Example 8-11

Negative Feedback Oscillators

X Cost Y Sin t



$$\frac{dy}{dt} = x$$

$$\frac{dx}{dt} = -y$$

物廉價美 顧客增加消費 價格上揚

價格上揚 顧客減少消費 價格下滑

負回饋

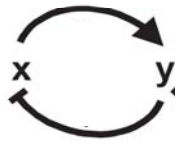
Negative Feedback Oscillators

負回饋 恩將仇報 以德報怨

反者道之動 常

天之道其猶張弓 損有餘 補不足

$$\frac{dy}{dt} = x$$



$$\frac{dx}{dt} = -y$$

What can X and Y be? X Cost Y Sin t

仁者數學

Feedback 回饋

Positive Feedback

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = x$$

其勢不可久

回也，其心三月不違仁，其餘則日月至焉而已矣。

不常

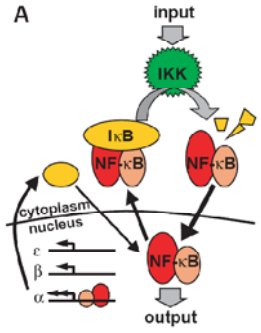
Negative Feedback

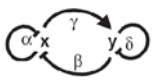
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

反者道之動
一陰一陽之謂道
天之道其猶張弓 損有餘 補不足
坤 直方大
天道好還週而復始
復 常

NF-κB and IκB Model





X: nucleus NF-κB

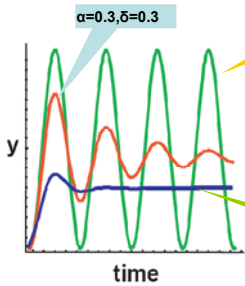
Y: IκB

$$\frac{d}{dt}x = S - \alpha x - \beta y$$

$$\frac{d}{dt}y = \gamma x - \delta y$$

Science **298:** 1241-1245.

Summary of NF-κB and IκB Model



By controlling α and δ, a cell can decipher the same stimulus in different ways.

What controls α and δ?

A beautiful woman !

Anything different ?

Lincoln !!

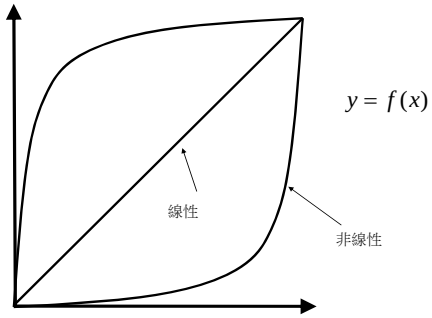
一分耕耘 ， 一分收穫？？

函數 Function

(因果)

y

y：成就

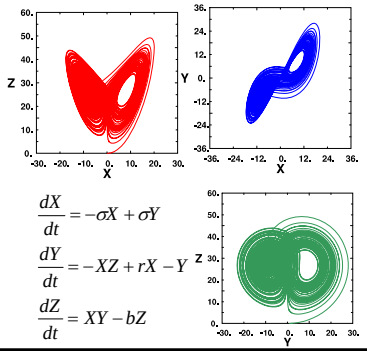


$y = f(x)$

$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ?$ x x：努力

Edward Norton Lorenz (1917～2008)

American Mathematician & Meterologist



$\sigma = 10,\ r = 28,\ b = \frac{8}{3}$

$$\frac{dX}{dt} = -\sigma X + \sigma Y$$
$$\frac{dY}{dt} = -XZ + rX - Y$$
$$\frac{dZ}{dt} = XY - bZ$$

蝴蝶效應 Butterfly Effect Chaos 混沌

混沌 非線性 $y = x^{30}$ 精確度有限 非線性

$$0.02 \begin{cases} x = 0.99 \\ x = 1.01 \end{cases} \qquad 0.61 \begin{cases} y \approx 0.74 \\ y \approx 1.35 \end{cases}$$

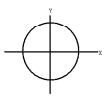
預報能力的喪失！！

"Sensitivity dependence on initial condition."

H Poincare

Negative Feedback

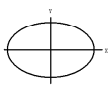
$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -x$$

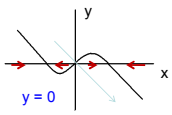


x = sint, y = cos t

NF + Fast and Slow

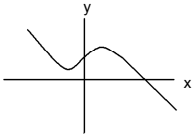
$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -\epsilon x$$

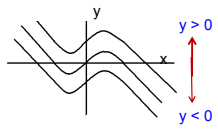




NF + FS + Multiple equilibriums

$$\frac{dx}{dt} = x - x^3 + y$$
$$\frac{dy}{dt} = -\epsilon x$$





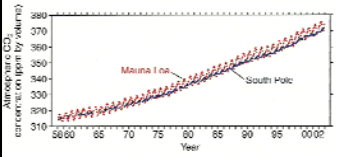
London Millennium Bridge

London's Millennium Bridge is the first **pedestrian** river crossing over the Thames in central London for more than a century.

It is a **325m** steel bridge linking the City of London at St. Paul's Cathedral with the Tate Modern Gallery at Bankside.

"Nice" lateral vibrations **(20 cm S shape wobble, 1cycle per second)** like on Tacoma Bridge developed on the day (June 12, 2000) of the opening……

Keeling Curve (1958-)



Charles Keeling (left) at the dedication of the Keeling Building at the Mauna Loa Observatory, Hawaii (1997)

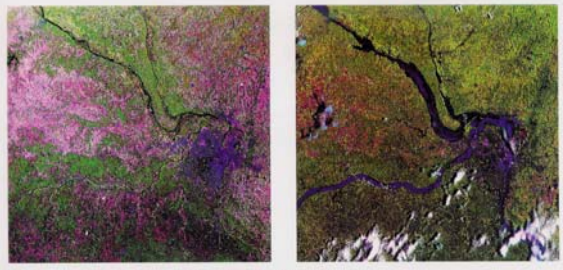
凝結尾　**人為**

Fig. 10.43 Condensation trails. [Photograph courtesy of Art Rangno.]

During the 3 days after the 9/11 /2001, statistical significant increase of 1.1C in the average diurnal temperature range for ground station across US.

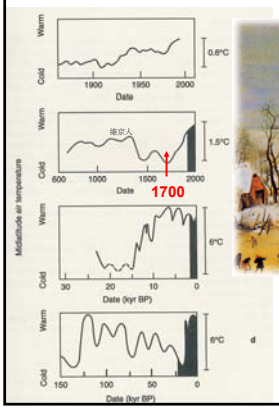
風雨之不時，是無世而不常有之。　荀子天論

1988　**年際變化 季節預報**　1993



Heavy rains in the summer of 1993 produced floods along most of the Mississippi River in the central United States, as shown in these Earth satellite photographs of St. Louis, Missouri on July 4, 1988 (left) and July 18,1993 (right). Extreme climatic events may be increasing in frequency as a consequence of added radiative absorbing gases in the atmosphere.

氣候變遷　**Warming trend begins 1700A.D.**



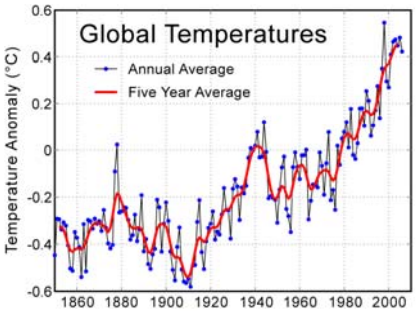
Bruegel, Pieter, the Younger
< Winter Landscape (1601) >

自然？？　　人為？？

夏蟲不可語冰　**WHY??**

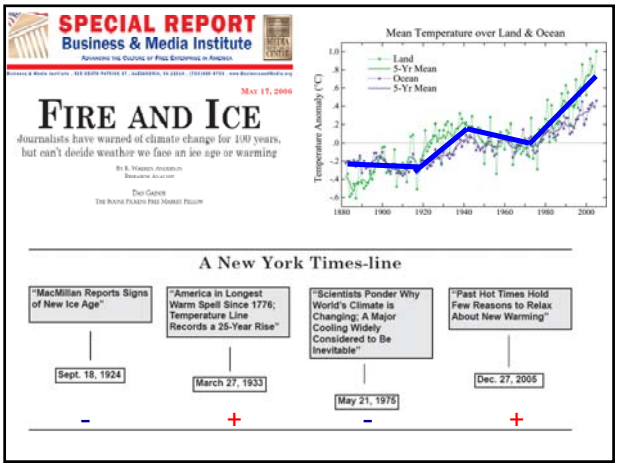
Extrapolation is absolutely unstable!

氣候變遷(全球暖化)現況



(Source: Global Warming Art)

1900以前資料可信度較不足；
1910-1940增溫相當快速，尚未能完全解釋；
1950年代的降溫常是爭議焦點，可能解釋為氣懸膠的冷卻作用；
1965以後太陽幅射強度幾無變化；
1980以後增溫持續而明顯。



January, 1977

April, 2006

Thank you!

A painting with filamentations!

Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)

The "***Black Death***" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

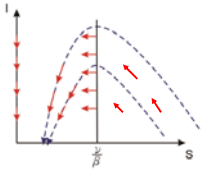
$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu)I$$

$$\frac{dR}{dt} = +\nu I$$

ν Recovery Rate

β Infection Rate

No Death in the model



$S = \frac{\nu}{\beta}$有 null cline

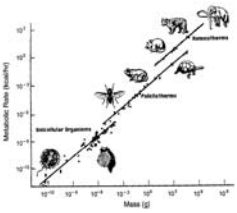
Forecast and control of epidemics in a globalized world PNAS vol.101 no.42

Hufnagel†, Brockmann, and Geisel

演講者: 陳怡文　日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

Metabolic rate vs size



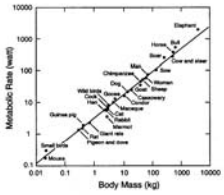


FIGURE 1 Metabolic rate (in watts) for a series of mammals and birds as a fu of mass (in kg); the scale is logarithmic and exemplifies the 3/4-power scalin discovered by Kleiber [2, 22, 27, 29].

$$I = I_0 M^{3/4}$$

Hemmingsen (1960) *Reports of the Steno Memorial Hospital and Nordisk Insulin Laboratorium* **9**, 6-110

Kleiber (1932) Body size and metabolism. *Hilgardia* **6**, 315-353.

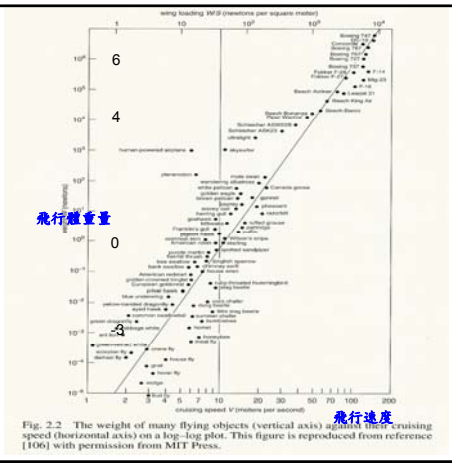


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.

$$\Pi = \frac{r_f}{E^{1/5} t^{2/5} \rho_0^{-1/5}}$$

G.I. Taylor 1950

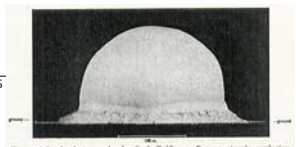
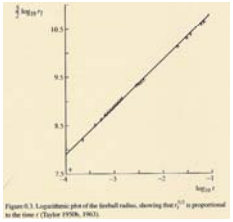


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963).

Symbol	*Definition*	*Representative value or first guess*
R	radius of wavefront	10^2 m
t	time	10^{-2} s
p_0	ambient pressure	10^5 Pa
ρ_0	ambient density	1 kg m^{-3}
E	energy released	10^{14} J



原子彈能量～ 10^{14} J

A, G constant

$$E \sim \rho v^2 l^2$$

$$P \sim \rho v^3 l^2$$

$$G \sim \frac{l^3}{n}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$v \sim n^{1/9}$$

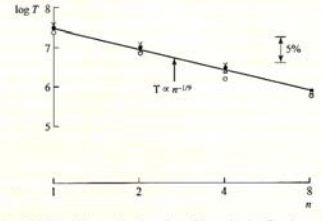


Figure 1.13. The $-1/9$ power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; ×, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).

動力系統

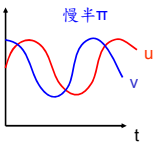
$$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}, \gamma_i)$$

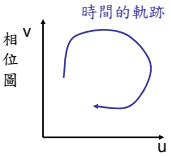
時間變化謂之動力　　變數　　許多外在及內在控制參數

$$\int_0^{2\pi} \cos t \sin t dt = 0$$

u　v

$$\overline{uv} = 0$$





Cos 和 Sin 零相關、不來電！

推背圖：前知三百年，後知三百年

可以解釋〝已知〞，可以預測〝未來〞

相位圖

你快樂嗎？一個簡單的生涯規劃動力系統

u：快樂指數

x:考試作業量

y:玩魔獸的時間

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

天縱英明的資優生　<0　>0　<0　<0　>0

考試越少越不快樂，

玩魔獸的時間越多越不快樂

$\frac{\partial u}{\partial x}$>0 考試越多越快樂

$\frac{\partial u}{\partial y}$<0 電動越玩越不快樂

→ 人的個性

$\frac{dx}{dt}$, $\frac{dy}{dt}$ → 人的境遇

個性+境遇=人生

相形不如論心

論心不如則術

形不勝心

心不勝術　荀子非相