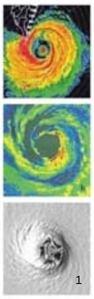
數學建模 科學研究 颱風動力

郭鴻基
教育部國家講座教授
臺大終身職特聘教授

Politics are for the moment
An equation is for eternity



9/22/2009
臺大大氣科學系

1

Now we only see models,
like reflections in a mirror;
but then we shall see face to face.
Now I only know partially;
but then I shall know as fully as
I am myself known.
St. Paul, 1st letter to the Corinthians, 13:12

Models、經典、聖哲就如鏡子，讓我們看到自己，讓我們瞭解自己的侷限，更進而體會完整的人性。

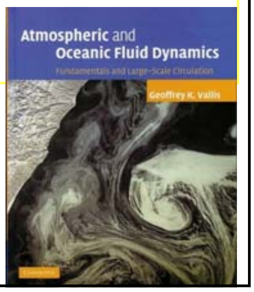
「數學科學模式」幫助我們由片面觀察的自然界，統會瞭解共通完整的科學定律。

2

20th Century

Geophysical Fluid Dynamics (GFD)
Atmospheric Oceanic Fluid Dynamics (AOFD)
is for those interested in doing research in the physics, chemistry, and/or biology of Earth fluid environment.

Fig. 9.2 Karman vortex streets in (a) the laboratory, for water flowing past a cylinder [From M. Van Dyke, An Album of Fluid Motion, Parabolic Press, Stanford, Calif. (1982) p. 56.], and (b) in the atmosphere, for a cumulus-topped boundary layer flowing past an island [NASA MODIS imagery].



熱力學 ＋ 流體力學

Euler 1755

$$\frac{d}{dt}\int_{v_m} \rho\vec{v}\, dv = -\int_{\partial v_m} p\, d\vec{s}$$

$$\int_{v_m} \rho\frac{d\vec{v}}{dt}\, dv = -\int_{v_m} \nabla p\, dv$$

$$\rho\frac{d\vec{v}}{dt} = -\nabla p$$

Lagrange 1781

$$\frac{\partial\vec{u}}{\partial t} + \vec{\zeta}\times\vec{u} = -\frac{1}{\rho}\nabla p - \nabla K - \nabla\Phi$$

Rotation，Vortex

Lorentz Force Law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

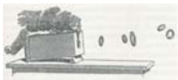
$$\mathbf{F} = q(-\nabla V + \mathbf{v}\times\mathbf{B})$$

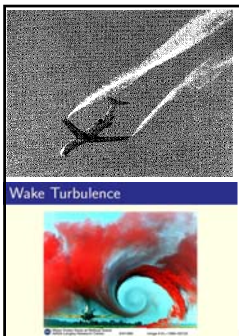
Helmholtz 1858

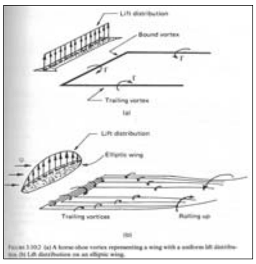
$$\frac{\partial\vec{\zeta}}{\partial t} + \vec{v}\cdot\nabla\vec{\zeta} + \vec{\zeta}\,\nabla\cdot\vec{v} = \vec{\zeta}\cdot\nabla\vec{v} + \vec{B}$$

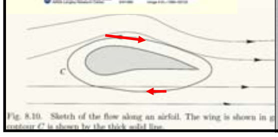
$$\vec{B} = \nabla\times(-\frac{1}{\rho}\nabla p)$$

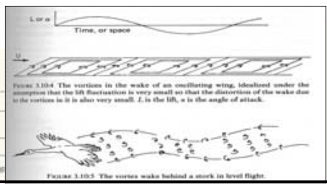
19世紀煙圈表演

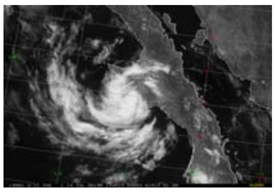


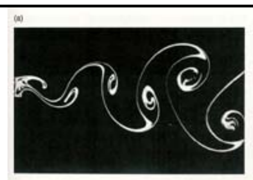


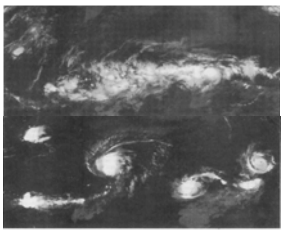


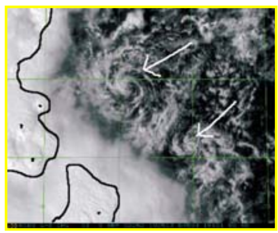








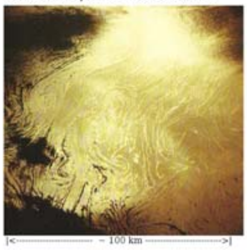




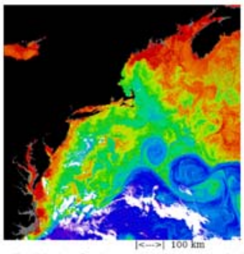
The eddies are small, numerous, and with long filaments.

Tortured Ocean?!

A serendipitous observation



Photograph taken on board Space Shuttle *Challenger* over Mediterranean Sea (Source: NASA)



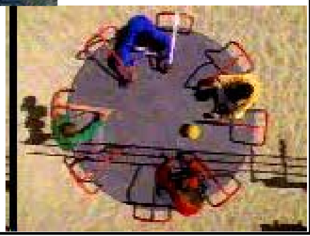
Satellite (Nimbus-7) observation of "**ocean color**" (concentration of phytoplankton), NW North Atlantic Source: NASA GSFC

Courtesy of H.P. Huang

旋轉
Rotation

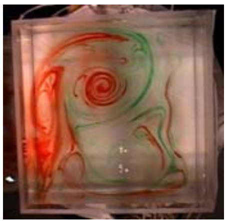
Coriolis Force

Non-inertial Frame



2D Turbulence

Stratification and/or Rotation
Vortex Waves Turbulence



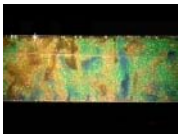
$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = \nu \nabla^2 \zeta$$

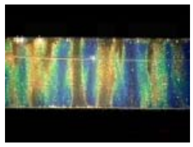
$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}.$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial(\psi, \zeta)}{\partial(x, y)} = \nu \nabla^2 \zeta$$

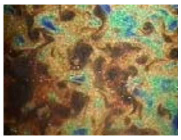
3D

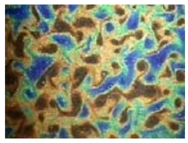
2D (strong rotation)





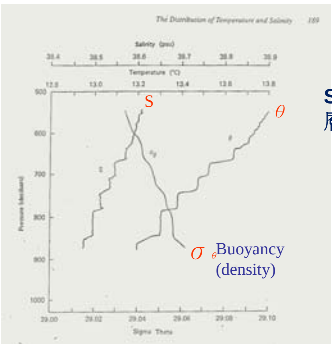
Taylor columns Vortex Tubes





Vortices with sharp edge

Ocean Spice



Stratification
層化

Multiple Scale Interactions in Vortex

Wave mean flow interaction in stable stratified fluid
Turbulent feed back to the vortex mean flow

2D turbulence

Non-divergent barotropic model (Nearly Inviscid Fluid)

$$\frac{\partial}{\partial t}\zeta + J(\psi,\zeta) = \upsilon \nabla^2 \zeta \qquad \boxed{\nabla^2\psi = \zeta}$$

The energy and enstrophy relations

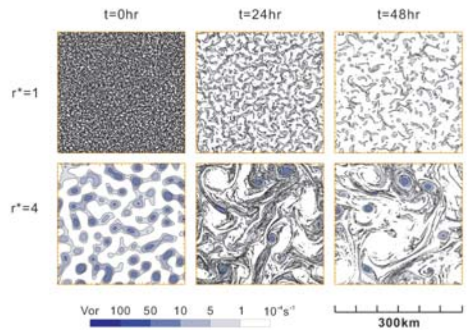
$$\frac{d\mathcal{E}}{dt} = -2\,\upsilon\,\mathcal{Z} \qquad \mathcal{E} = \iint \frac{1}{2}\left(u^2+v^2\right)dx\,dy \quad \text{kinetic energy}$$

$$\mathcal{Z} = \iint \frac{1}{2}\zeta^2\,dx\,dy \quad \text{enstrophy}$$

$$\frac{d\mathcal{Z}}{dt} = -2\,\upsilon\,\mathcal{P} \qquad \mathcal{P} = \iint \frac{1}{2}\nabla\zeta\cdot\nabla\zeta\,dx\,dy \quad \text{palinstrophy}$$

Batchelor 1969

13



小尺度變大尺度

Fewer and stronger vortices !!!

Coherent structure with filamentations in 2-D turbulence

14

Weiss(1981,1991), Rozoff et al. (2004)

$$\frac{D}{Dt}(\nabla\zeta) = -J(\nabla\psi,\zeta)$$

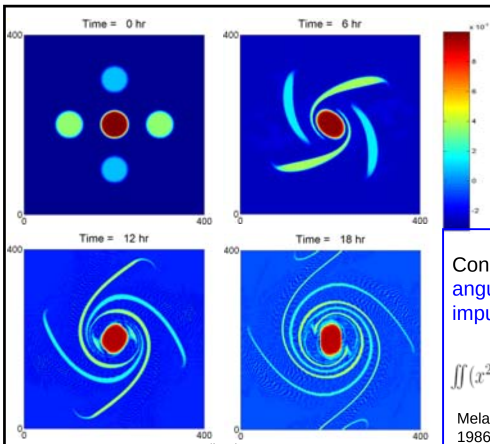
$$\rightarrow \nabla\zeta(t) \propto \exp(\lambda t) \quad \lambda = \pm\frac{1}{2}\sqrt{Q} = \pm\frac{1}{2}\sqrt{S_1^{\,2}+S_2^{\,2}-\zeta^2}$$

$$S_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \quad \text{(stretch deformation)}$$

$$S_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \text{(shear deformation)}$$

Q > 0 (strain dominates)
→vorticity gradient will be stretched
Q < 0 (vorticity dominates)
→vortex is stable (survival of eyewall meso-vortices)

15



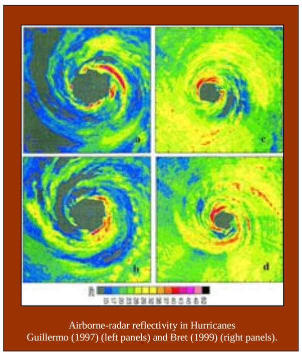
Conserves the angular impulse

$$\iint (x^2+y^2)\zeta\,dxdy$$

Melander et al. 1986, Kuo et al. 2008

16

Spiral Band in Hurricane and Galaxy



Airborne-radar reflectivity in Hurricanes
Guillermo (1997) (left panels) and Bret (1999) (right panels).

Kossin and Schubert 2001

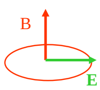
17

Electron density redistribution in experimental plasma physics

single sign charge
+
axial magnetic field confinement

$$\mathrm{E} = -\nabla\psi$$

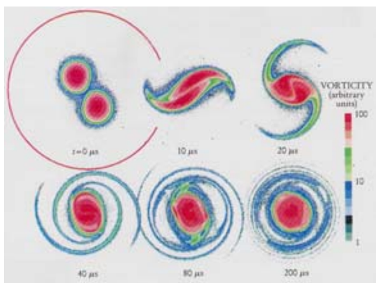
$$\nabla\cdot\mathrm{E} = -\nabla^2\psi = \frac{\rho}{\epsilon}$$



$\vec{E}\times\vec{B}$ drift

Coriolis force

Axisymmetrization 軸對稱化



Core is protected, thin filaments from edges

18

Bowmen and Mangus (1993)

臭氧洞衛星觀測

Observations of deformation and mixing of the total ozone field in the Antarctic polar vortex

核心空氣被渦旋鎖住

細絲帶

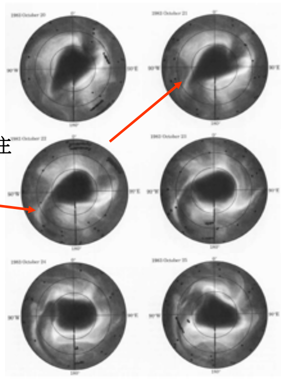


Fig.1: Daily TOMS images of total ozone in the Southern Hemisphere for six consecutive days in October 1983. Latitude circles are drawn at 40°,60°, and 80 °S. The outermost latitude is 20 °S.

19

Huang and Robinson 1998

Forcing added

Inverse energy cascade to zonal Harmonics

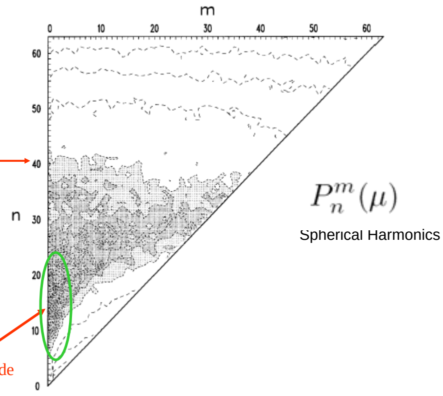


FIG. 2. Energy spectrum $E(m, n)$ of an ensemble mean at day 80 of 10 decaying turbulence experiments. The magnitude of the spectrum is normalized by the maximum value on the map. Contour levels are 0.0001, 0.001, 0.01, 0.1–0.9 with increment 0.1. Area with $E(m, n) > 0.1$ is lightly shaded, $E(m, n) > 0.2$ heavily shaded.

Jupiter Rotational period 9.84hr

Huang and Robinson 1998

The Great Red Spot

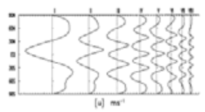
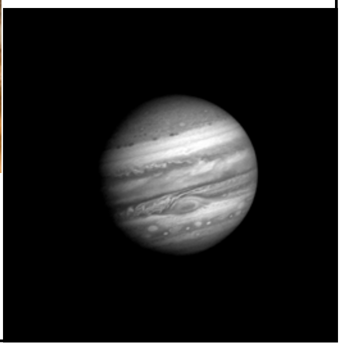
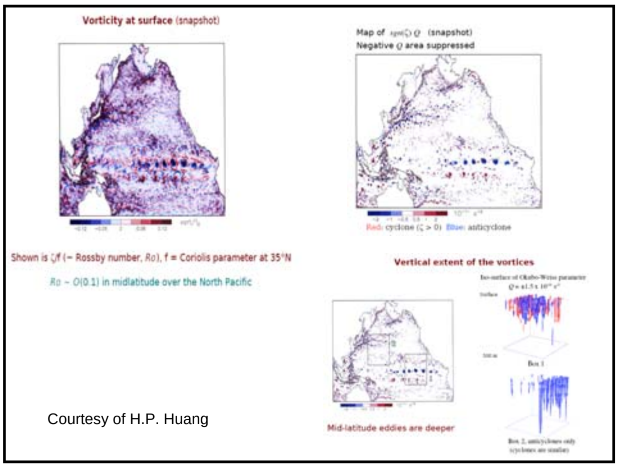


FIG. 4. Time-mean zonal-mean zonal wind profiles for cases I–VIII in Table 1 (the eight open circles in Fig. 3). Each grid on the abscissa represents 1 m s⁻¹.

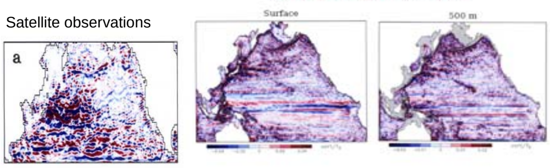




Courtesy of H.P. Huang

Satellite observations

One-year average of vorticity field



Multiple zonal-jet like structures emerge with long-term average

- The presence of multiple zonal jets can critically affect the **transport of heat, biota, and chemical constituents** that are important for climate and life on Earth.

  Eddy diffusivities for tracer transport along and across zonal jets are dramatically different (e.g., Smith 2005)

- Clarification of the interaction between eddies/zonal jets and ocean gyre may lead to **revision of the conventional view of ocean gyre**.

Courtesy of H.P. Huang 22

一杯咖啡，古今往事盡付笑談中。

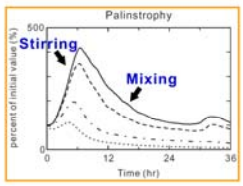
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta\, dV$$

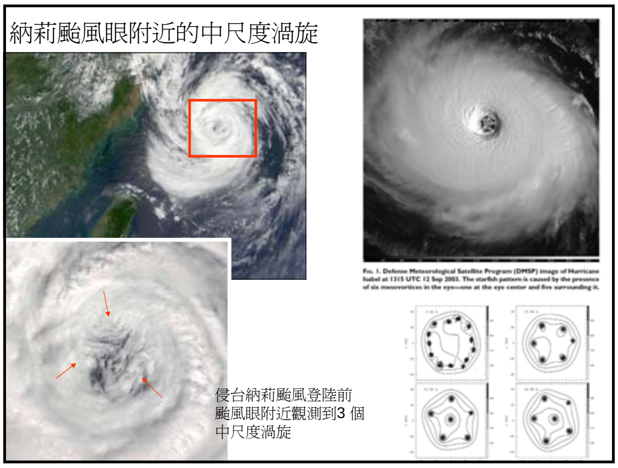
$$\frac{dC}{dt} = \int(\vec{V}\cdot\nabla\theta)\nabla^2\theta\, dV - \nu\int(\nabla^2\theta)\, dV$$

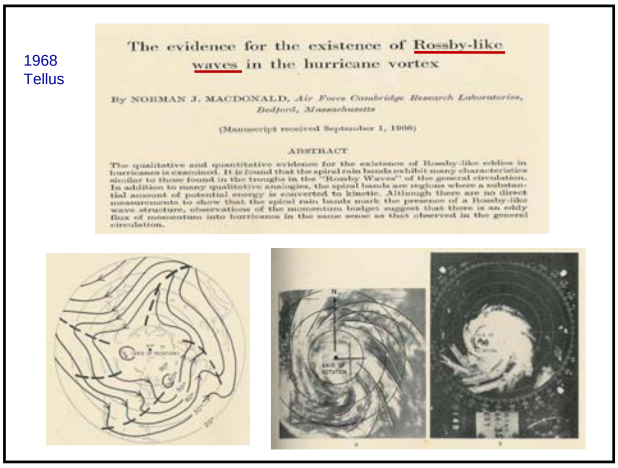
Stirring **Mixing**

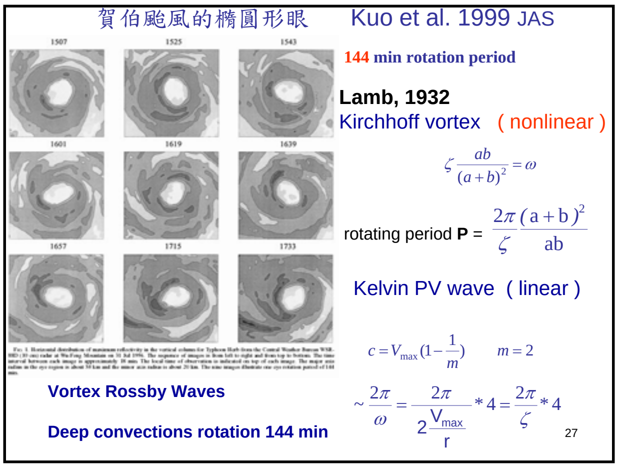


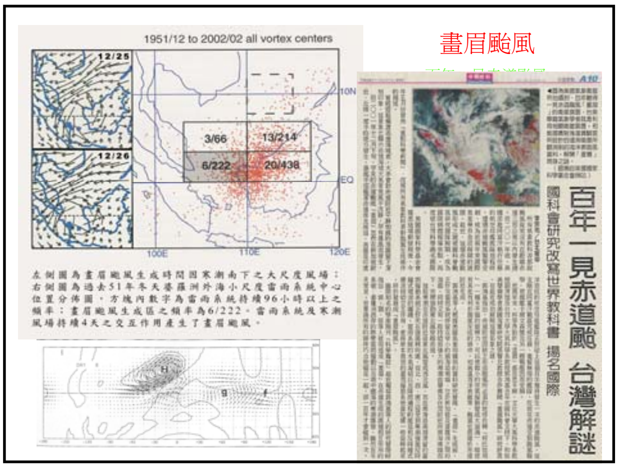
Coffee with white

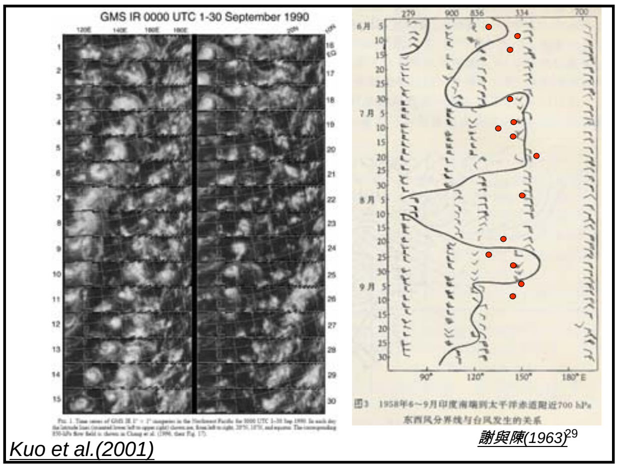
4

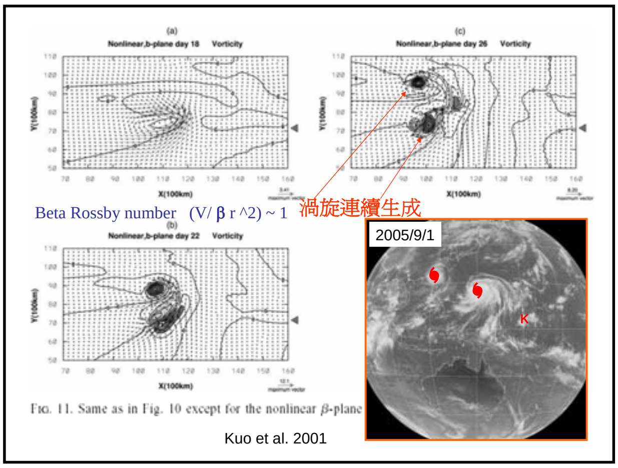

納莉颱風眼附近的中尺度渦旋
侵台納莉颱風登陸前
颱風眼附近觀測到3 個
中尺度渦旋

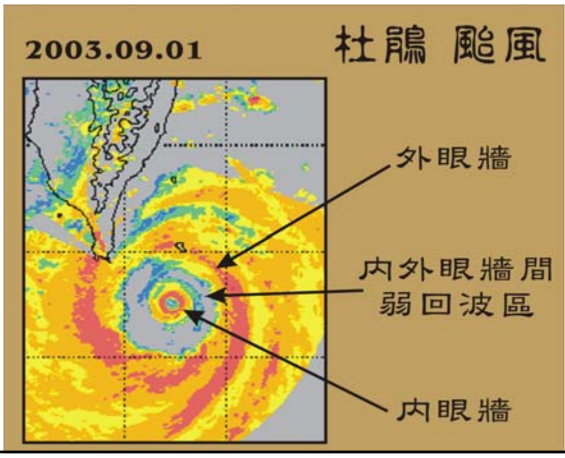

1968
Tellus
The evidence for the existence of Rossby-like waves in the hurricane vortex
By NORMAN J. MACDONALD, Air Force Cambridge Research Laboratories, Bedford, Massachusetts
(Manuscript received September 1, 1966)
ABSTRACT


賀伯颱風的橢圓形眼
Kuo et al. 1999 JAS
144 min rotation period
Lamb, 1932
Kirchhoff vortex （nonlinear）
ζ ab/(a+b)^2 = ω
rotating period P = 2π(a+b)^2/(ζ ab)
Kelvin PV wave （linear）
c = Vmax(1 − 1/m)   m = 2
~ 2π/ω = 2π/(2 Vmax/r) * 4 = 2π/ζ * 4
Vortex Rossby Waves
Deep convections rotation 144 min
27


畫眉颱風
1951/12 to 2002/02 all vortex centers
12/25
12/26
3/66
13/214
6/222
20/438
左側圖為畫眉颱風生成時間因寒潮南下之大尺度風場；右側圖為過去51年冬天婆羅洲外海小尺度雷雨系統中心位置分佈圖。方塊內數字為雷雨系統持續96小時以上之頻率：畫眉颱風生成區之頻率為6/222。雷雨系統及寒潮風場持續4天之交互作用產生了畫眉颱風。
百年一見赤道颱 台灣解謎
國科會研究改寫世界教科書 揚名國際


GMS IR 0000 UTC 1-30 September 1990
图3 1958年6~9月印度南端到太平洋赤道附近700 hPa东西风分界线与台风发生的关系
Kuo et al.(2001)
謝與陳(1963)
29

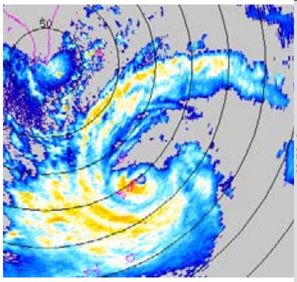

Nonlinear,b-plane day 18 Vorticity
Nonlinear,b-plane day 26 Vorticity
Beta Rossby number (V/ β r ^2) ~ 1
渦旋連續生成
Nonlinear,b-plane day 22 Vorticity
2005/9/1
FIG. 11. Same as in Fig. 10 except for the nonlinear β-plane
Kuo et al. 2001



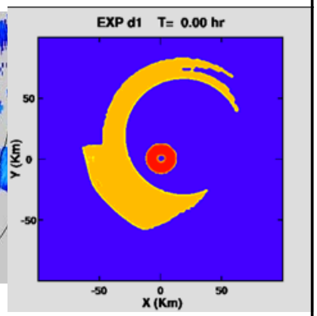
雙颱風眼生成動力研究

Kuo, H.-C., L.-Y. Lin, C.-P. Chang, and R. T. Williams, 2004: The formation of concentric vorticity structure in typhoons. *J. Atmos. Sci.*, **61**, 2722-2734.

0935-1935 LST



Typhoon Lekima (2001)



Binary vortex interaction

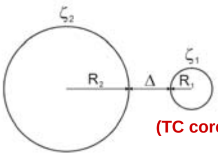
Kuo et al. (2004)

【Variables】

$R_1, R_2;\ \Delta;\ \zeta_1, \zeta_2$

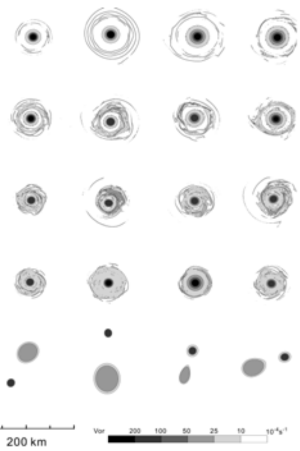
【Parameters】

- Vortex radius ratio $(r) = \frac{R_1}{R_2}$
- Dimensionless gap $(\frac{\Delta}{R_1})$
- Vortex strength ratio $(\gamma) = \frac{\zeta_1}{\zeta_2}$



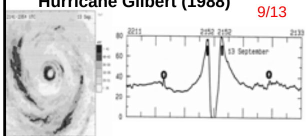
- An extension of Dritschel and Waugh's (1992) work.
- In addition to the radii ratio and the normalized distance between the two vortices, the vorticity ratio is added as a third external parameters. 33

Examples of the vorticity field at hour 12, showing different classifications of binary vortex interactions involving a skirted core vortex.

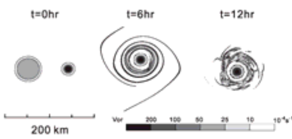


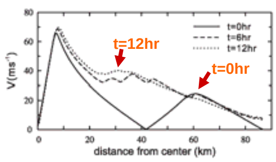
Black and Willoughby (1992)
Hurricane Gilbert (1988)

9/13

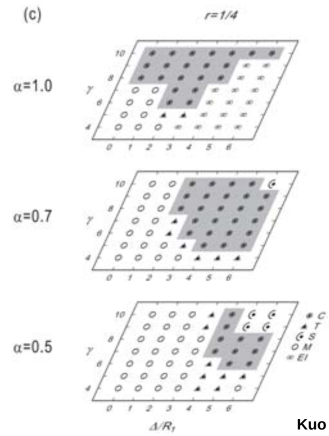


Kuo et al. 2008



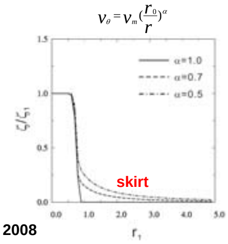


The contraction and the increase of the secondary wind maximum by nonlinear advection dynamics. 35



Rankine vortex (α = 1.0) favors the formation of a concentric structure closer to the core vortex, while the α = 0.7 and α = 0.5 vortices favor the formation of concentric structures farther from the core vortex.

$v_\theta = v_m (\frac{r_0}{r})^\alpha$



Kuo et al. 2008

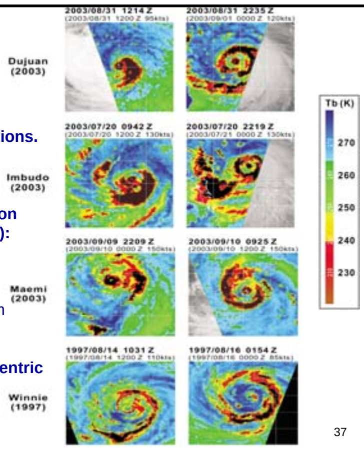
Examples of asymmetric → symmetric concentric formations.

~ 12 hours.

Initial Δ (outer deep convection region - vortex core distance):

Typhoon Dujuan: nearly 0 km
Typhoon Imbudo nearly 50 km
Typhoon Maemi: nearly 100 km
Typhoon Winnie nearly 260km

A wide range of radii of concentric eyewalls



Kuo et al. 2008

37

Terwey and Montgomery, June JGR 2008

D12112 TERWEY AND MONTGOMERY: MODELED SECONDARY EYEWALL FORMATION D12112

Table 1. List of Secondary Eyewall Formation Hypotheses With Summary of Relevance to our Modeled Hurricanes[a]

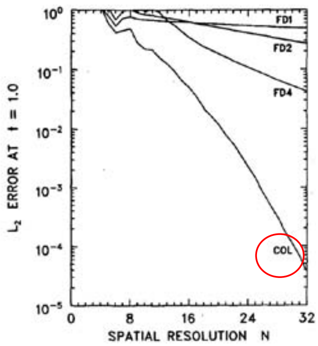
Authors	Hypothesis Summary	Relevance to Current Model Results	Type
Willoughby et al. [1982] borrowing from the squall line research of *Zipser* [1977]	Downdrafts from the primary eyewall force a ring of convective updrafts.	Few downdraft-forced updrafts during this time in the simulations.	O
Willoughby [1979]	Internal resonance between local inertia period and asymmetric friction due to storm motion.	No systematic storm motion in the simulated storms.	A
Hawkins [1983]	Topographic effects	No topographic forcing in the simulations.	O
Willoughby et al. [1984]	Ice microphysics	"Warm-rain" (no-ice) sensitivity case also produces secondary eyewall.	A
Molinari and Skubis [1985] and *Molinari and Vallaro* [1989]	Synoptic-scale forcings (e.g., inflow surges, upper-level momentum fluxes)	No synoptic-scale forcings in the simulations	O
Montgomery and Kallenbach [1997], *Camp and Montgomery* [2001] and *Terwey and Montgomery* [2003]	Internal dynamics-axisymmetrization via sheared vortex Rossby wave processes; collection of wave energy near stagnation or critical radii	Possible explanation	N
Nong and Emanuel [2003]	Sustained eddy momentum fluxes and WISHE feedback	Possible explanation	A
Kuo et al. [2004, 2008]	Axisymmetrization of positive vorticity perturbations around a strong and tight core of vorticity	Possible explanation	N

[a]The type column refers to the type of model or observations that were used to formulate the hypothesis. O stands for observationally-based; A stands for axisymmetric model; N stands for nonaxisymmetric model.

38

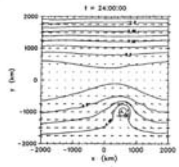
Pafnuty Lvovich Chebyshev

Russian mathematician
(1821～1894)



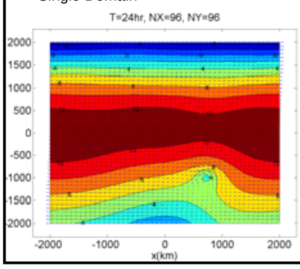
Exponential Convergence
Algebraic Convergence

39

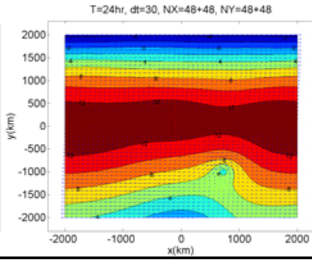


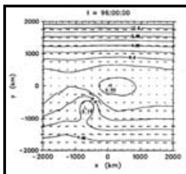
Domain Decomposition Numerical Results

Single Domain



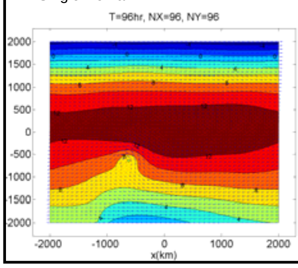
Domain Decomposition(2*2)



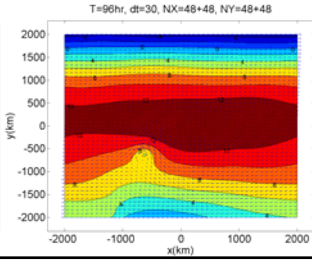


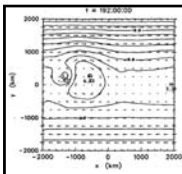
Domain Decomposition Numerical Results

Single Domain



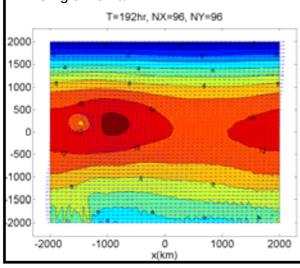
Domain Decomposition(2*2)



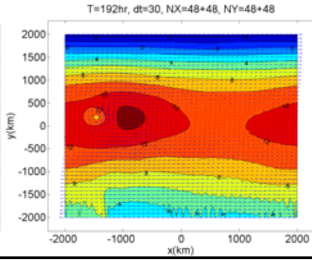


Domain Decomposition Numerical Results

Single Domain



Domain Decomposition(2*2)



颱風潛熱與其它能量的比較

賀伯颱風的全台灣平均總雨量爲400*mm*

$400\ mm = 0.4\ m$

$0.4\ m * 1000\ kg\ m^{-3} * 2.5\times10^{6}\ J\ kg^{-1}$

$= 10^{9}\ J\ m^{2}$

$10^{9}\ J\ m^{2} * 3.5\times10^{10}\ m^{2}$

$= 3.5\times10^{19}\ J \sim 10^{20} J$

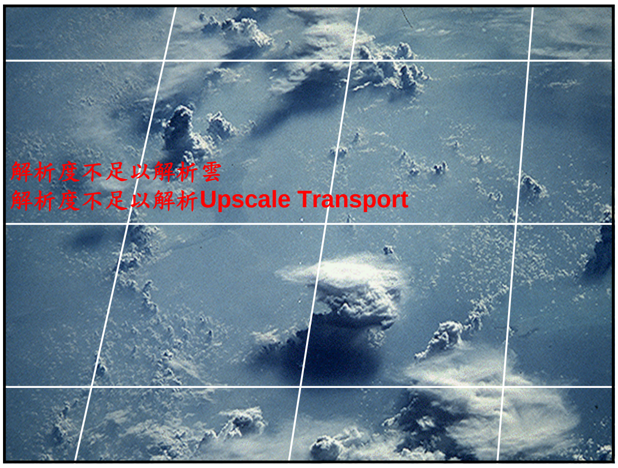
$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{142}_{56}Ba + ^{91}_{36}Kr + 3^{1}_{0}n$

$1.68 * m * 10^{13} J/mol$

$\Rightarrow 1.46\times10^{6}\ kg\ U^{235}\ (6*10^{6}\ mol)$

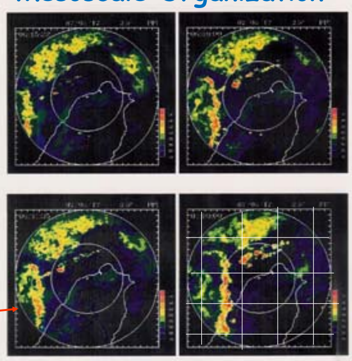
能量估計值		備註
賀伯颱風降雨總潛熱能量	10^{20} J	可使台灣整層大氣增溫100度
台灣一年用電量	$5*10^{17}$ J	需數百年用電量才相當
全世界核子彈爆炸釋放能量	$2*10^{19} \sim 2*10^{20}$ J	與賀伯颱風同等級
核戰後燃燒釋放能量	$2*10^{20}$ J	與賀伯颱風同等級
地球一天接受的太陽能量	$1.5*10^{22}$ J	數百個賀伯颱風
Tunguska隕石撞地球 （西元1908年，西伯利亞）	10^{16} J	賀伯颱風的萬分之一
火流星撞地球（恐龍滅絕？）	$4*10^{23}$ J	數千個賀伯颱風

43



Mesoscale Organization

狂風不終朝
暴雨不終日
孰為此者
天地
天地尚不能久
而況人乎



颮線

Squall line

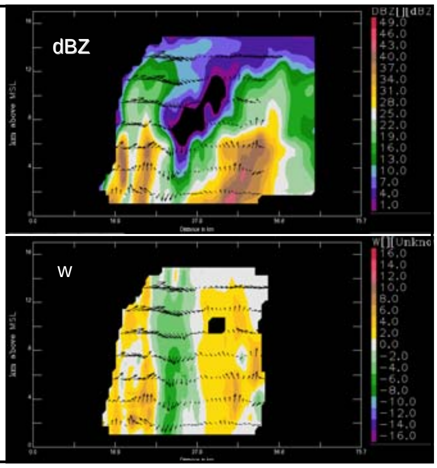
1987 年 5 月 17 日鋒前颮線之雷達回波圖

模式解析？

Hurricane Rita RAINEX (2005)

ELDORA data show downward motion between the two eyewalls

Large variability in the vertical motion



Courtesy of WC Lee

君子務本 本立道生

關鍵基礎能力 語文能力 能專精方能跨領域

誠不以富 亦祇以異

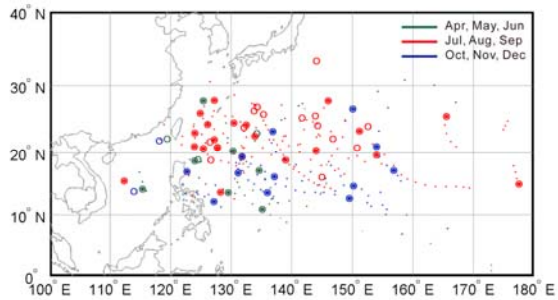
不同立場有不同地位　　特色 專業水準 眼光

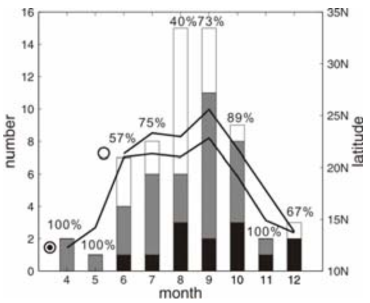
做自己有興趣且有長處的事業

It is easy to say!! 需要許多過程，自我追尋、自我瞭解、自我訓練，才能找到自己的路。

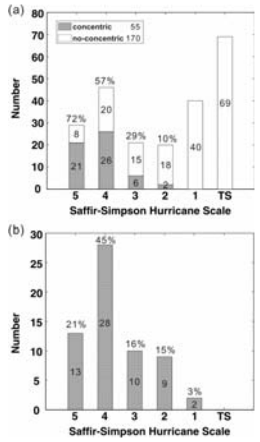
好的工作習慣
好的基礎功夫------讀、算、寫
眼界、Vision、選重要的問題

47

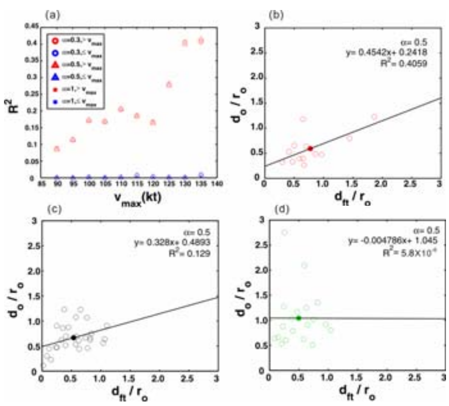




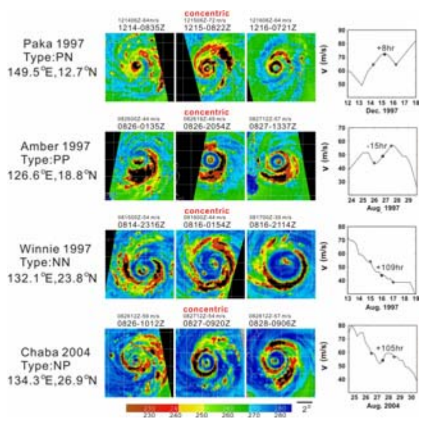
49



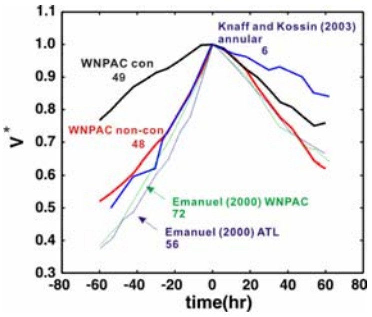
50



51



52



53

$$\frac{d}{dt}\int E(k)dk = 0. \qquad \frac{d}{dt}\left(\int k^2 E(k)dk\right) = \frac{d}{dt}\int Z(k)dk = 0.$$

$$\frac{d}{dt}\left(\int (k-k_1)^2 E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\int k^2 E(k)dk + k_1^2 \int E(k)dk - 2k_1 \int kE(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\frac{\int kE(k)dk}{\int E(k)dk}\right) < 0,$$

Kinetic energy moves toward large scales

$$\frac{d}{dt}\left(\int (k^2-k_1^2)^2 E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\int k^2 Z(k)dk + k_1^4 \int E(k)dk - 2k_1^2 \int k^2 E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\frac{\int k^2 Z(k)dk}{\int Z(k)dk}\right) > 0,$$

Enstrophy moves toward small scales

54