數學建模與科學研究

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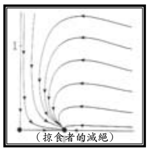
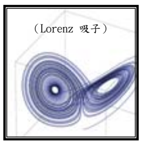
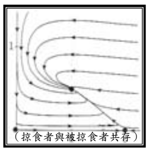

（掠食者的滅絕）

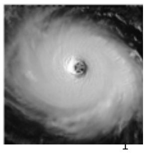

（Lorenz 吸子）

內容：

科學研究是探索未知知識的建構過程，而數學是科學的語言。隨著電腦的進步，資料大量的數位化，科學計算成為非線性科學研究的敲門磚。數學建模、科學計算、分析詮釋與驗證等過程，更是現今數學科學的典範。數學函數本身是無因次，但是科學量化卻有單位，演講將簡介因次分析，並舉例說明數學建模、科學計算、因次分析詮釋等過程。

2009/7/21-24成功數學系


（掠食者與被掠食者共存）



Politics are for the moment
An equation is for eternity

但覺高歌有鬼神
不知餓死填溝壑

讀 算 寫

幾何
代數
微積分
電腦計算繪圖
數學建模/科學計算
Mathematical Modeling
Scientific Computing

+ -	x /
加、減	乘、除
線性	非線性
大題大作	小題大作

2005—2055 科技探索 Institute For The Future

生物模擬與計算

數學模型與生物結合

數量化、數位化
數學模式--動力系統

健康醫療規劃

生物資訊分析(Bioinformatics)

透過人腦、電腦介面，強化人體功能

生物檢測

複雜系統：生態、經濟、氣象等數學模式應用議題
………………………………………

Now we only see models,
like reflections in a mirror;
but then we shall see face to face.
Now I only know partially;
but then I shall know as fully as
I am myself known.
St. Paul, 1st letter to the Corinthians, 13:12

Plato : shadow of reality; Kant : phenomenon and underlying noumenon.

We only see reality through models!!

Mathematical Model is a representation and analysis of reality through mathematical symbols and concepts.

False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for every one takes a salutary pleasure in proving their falseness.
Darwin, The Origin of Man, chapter 6

數學模式

理論、 解釋資料 **interprets experiences on a higher than purely descriptive level. von Neumann**

預測 **準確性 預測能力**

資料同化 利用科學數學模式整合有限的觀測，建構出較完整的資料

數學模式

Formulation　　微分、差分方程式

Solution / Analysis　　分析、解

Interpretation　科學詮釋

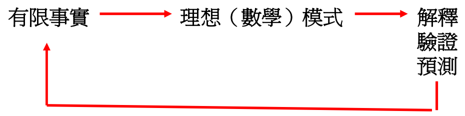
中階課程：微分方程(ODE,PDE)
統計、線性代數
程式、計算與繪圖

數量化、數位化
數學化--模式--動力系統

以特殊事實為憑藉，逐漸推廣引伸，成立概念式定律的系統，
以便籠罩更複雜耿廣泛的對象，
科學家依據事實為前提來證明普遍的結論。　　方東美

吾生也有涯，吾知也無涯，已有涯逐無涯，殆矣。
有限時空之觀察或有限資料去推導無限時空的科學定律。

問蒼茫大地誰主浮沈？
有物有則　因果律　　　　為什麼？　形而上學



有限事實 → 理想（數學）模式 → 解釋
驗證
預測

Bode’s Law of Astronomy　　1778

0　3　6　12　24　48　96　192　384

4　7　10　16　28　52　100　196　388

0.4　0.7　1.0　1.6　2.8　5.2　10　19.6　38.8

Mercury	0.4 (0.39)	Venus	0.7 (0.72)	Earth	1.0 (1.0)
Mars	1.6 (1.52)	Asteroids	2.8 (2.77)	Jupiter	5.2 (5.2)
Saturn	10 (9.54)	Uranus	19.6 (19.19)	Neptune	38.8 (30.07)
Pluto	fails (39.60)				

“Plutoed”

Fovell, 2008 高雄

This model will be a simplification and an idealization, and consequently a falsification. It is to be hoped that the features retained for discussion are those of greatest importance in the present stage of knowledge.

Turning　**The Chemical Basis of Morphogenesis**

微積分數學
the Mathematics of Change

莊子：一尺之錘、日取其半
、萬世不絕。

$$\triangle x \to 0$$

Rate of Change

$$\lim_{\triangle\to 0} \frac{f(\triangle)}{g(\triangle)} = \frac{df}{dg}$$

$$\lim_{\epsilon\to 0} \frac{\sin\epsilon}{\epsilon} =?$$

$$u = u(x, y)$$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

偏微分

$$\frac{\partial x^2 y}{\partial y} = x^2$$

只對y變數微分，不改變x變數

“Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum.”　　Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，
窮終始之要，始可以度其古，中可以行於今。通典

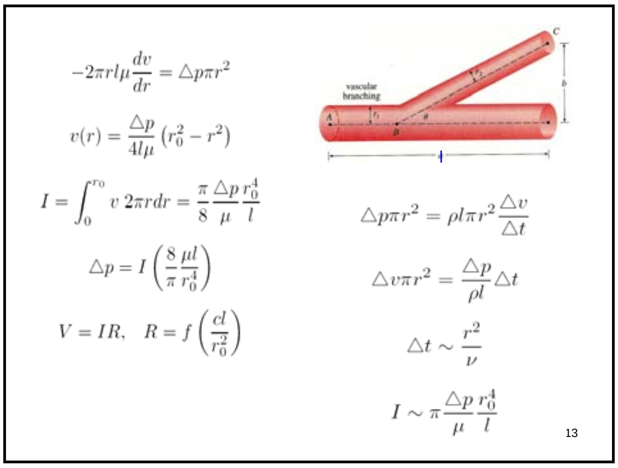
共**49**個字，假設中文常用字爲**1000**字，共有**10^147**個選擇

地球歷史 **10^18** sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 **10^4**，

10^10*10^18*10^4 = 10^32

10^32/10^147=10^(-115)　~　0　機率爲零　，不可能的巧合！

研究學問是苦心孤詣的事業！　　不要人云亦云!



$$-2\pi r l \mu \frac{dv}{dr} = \triangle p \pi r^2$$

$$v(r) = \frac{\triangle p}{4l\mu}\left(r_0^2 - r^2\right)$$

$$I = \int_0^{r_0} v\ 2\pi r dr = \frac{\pi}{8}\frac{\triangle p}{\mu}\frac{r_0^4}{l}$$

$$\triangle p = I\left(\frac{8}{\pi}\frac{\mu l}{r_0^4}\right)$$

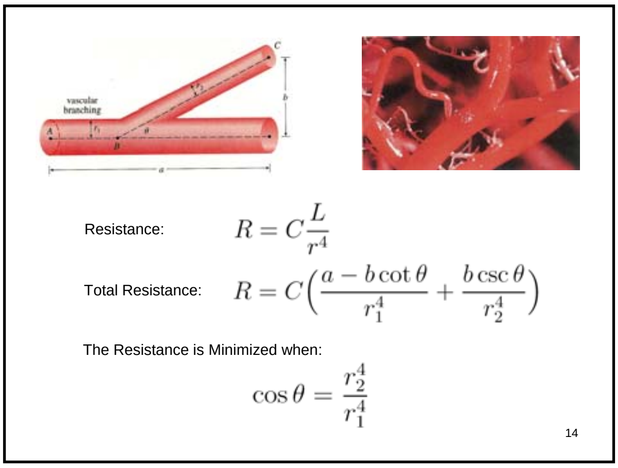
$$V = IR, \quad R = f\left(\frac{cl}{r_0^2}\right)$$

$$\triangle p \pi r^2 = \rho l \pi r^2 \frac{\triangle v}{\triangle t}$$

$$\triangle v \pi r^2 = \frac{\triangle p}{\rho l}\triangle t$$

$$\triangle t \sim \frac{r^2}{\nu}$$

$$I \sim \pi \frac{\triangle p}{\mu}\frac{r_0^4}{l}$$

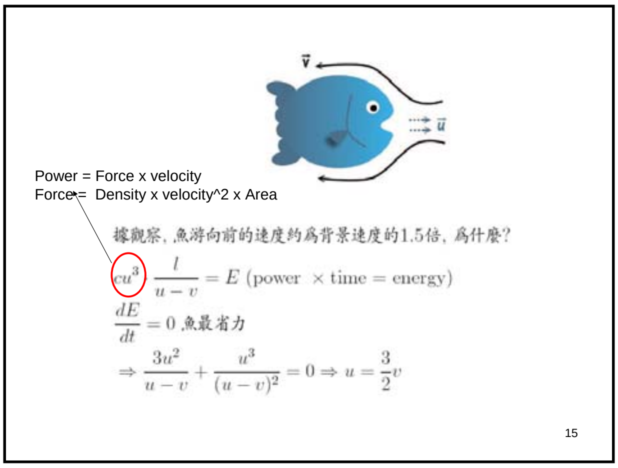


Resistance: $R = C\dfrac{L}{r^4}$

Total Resistance: $R = C\left(\dfrac{a - b\cot\theta}{r_1^4} + \dfrac{b\csc\theta}{r_2^4}\right)$

The Resistance is Minimized when:

$$\cos\theta = \frac{r_2^4}{r_1^4}$$



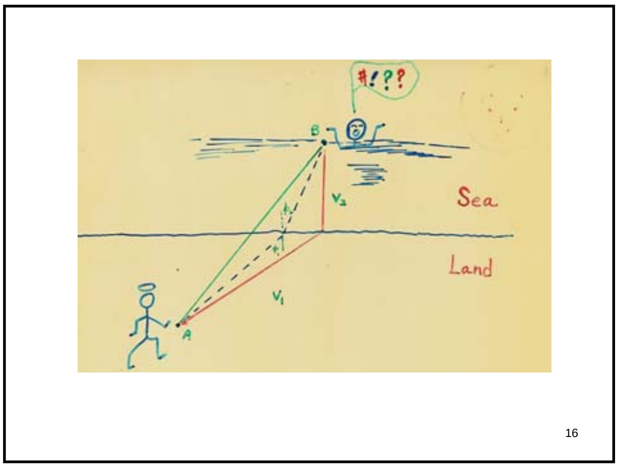
Power = Force x velocity
Force = Density x velocity^2 x Area

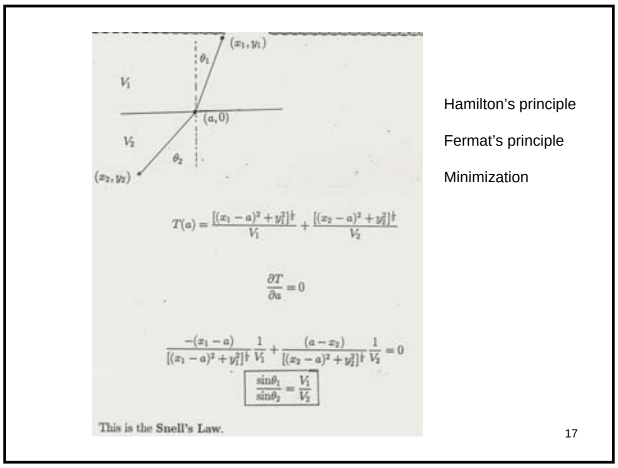
據觀察，魚游向前的速度約爲背景速度的1.5倍，爲什麼?

$$cu^3 \cdot \frac{l}{u-v} = E\ (\text{power } \times \text{time} = \text{energy})$$

$$\frac{dE}{dt} = 0\ \text{魚最省力}$$

$$\Rightarrow \frac{3u^2}{u-v} + \frac{u^3}{(u-v)^2} = 0 \Rightarrow u = \frac{3}{2}v$$





$$T(a) = \frac{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}{V_1} + \frac{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}{V_2}$$

$$\frac{\partial T}{\partial a} = 0$$

$$\frac{-(x_1-a)}{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}\frac{1}{V_1} + \frac{(a-x_2)}{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}\frac{1}{V_2} = 0$$

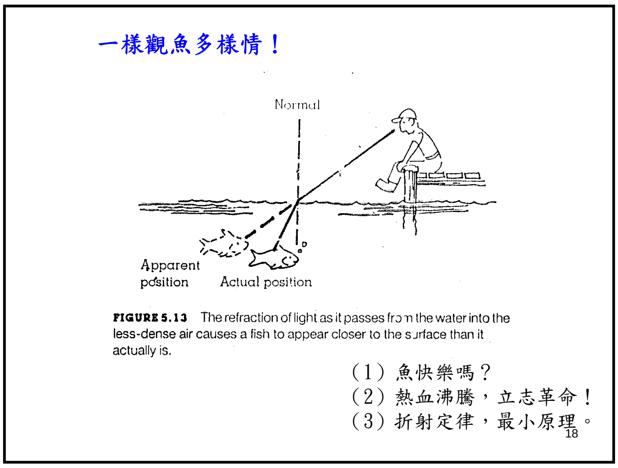
$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2}}$$

This is the Snell's Law.

Hamilton's principle

Fermat's principle

Minimization

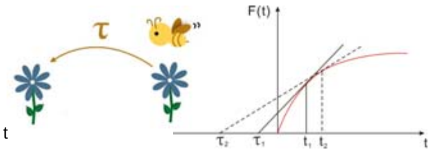


一樣觀魚多樣情！

FIGURE 5.13 The refraction of light as it passes from the water into the less-dense air causes a fish to appear closer to the surface than it actually is.

（1）魚快樂嗎？
（2）熱血沸騰，立志革命！
（3）折射定律，最小原理。

術 方法



一朵花停留的時間 : t

蜜峰從第一朵花到下一朵花的飛行時間： τ

$$g(t) = \frac{f(t)}{t+\tau} \qquad \text{nectar collection rate} = \frac{\text{food per visit}}{\text{time}}$$

$g'(t) = 0$ 有最大採集效率

$$\Rightarrow \frac{f'(t)}{t+\tau} - \frac{f(t)}{(t+\tau)^2} = 0 \quad \Rightarrow f'(t) = \frac{f}{t+\tau}$$

Function y = f(x)

Commonly Occurring Functions

Polynomials: approximate with a high degree of accuracy, almost any existing function

Trigonometric functions **cos** **sin**

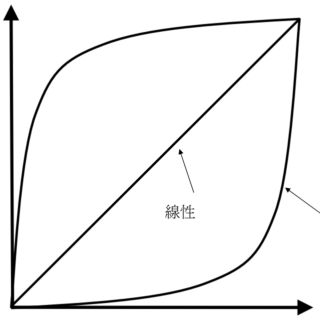
Exponential functions **e**

Logarithmic function **log** **ln**
s=k log w

一分耕耘 ， 一分收穫？？

函數 Function

(因果）



y

$y = f(x)$

y : 成就

$$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ? \qquad x$$

x : 努力

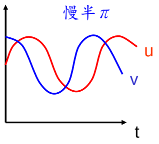
動力系統

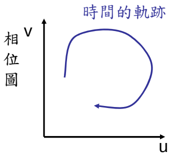
$$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}, \gamma_i)$$

時間變化謂之動力　變數　許多外在及內在控制參數

$$\int_0^{2\pi} \underset{u}{\cos t}\,\underset{v}{\sin t}\,dt = 0$$

$$\overline{uv} = 0$$





Cos 和 Sin 零相關、不來電！

推背圖：前知三百年，後知三百年
可以解釋〝已知〞，可以預測〝未來〞

相位圖

你快樂嗎？一個簡單的生涯規劃動力系統

u：快樂指數
x: 考試作業量
y: 玩魔獸的時間

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

天縱英明的資優生 <0 >0 <0 <0 >0

考試越少越不快樂，
玩魔獸的時間越多越不快樂

$\frac{\partial u}{\partial x}$>0 考試越多越快樂
$\frac{\partial u}{\partial y}$<0 電動越玩越不快樂

人的個性

$\frac{dx}{dt}$
$\frac{dy}{dt}$

人的境遇

個性+境遇=人生
相形不如論心
論心不如則術
形不勝心
心不勝術 **荀子非相**

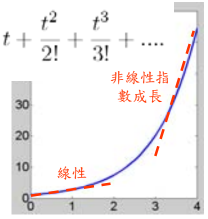
Exponential functions are both man’s best friend and worst enemy.

$$\frac{dx}{dt} = -x$$

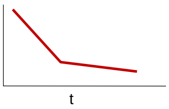
$$x(t) = x(0)e^{-t}$$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + ...$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + ...$$



指數遞減
e-folding time
（類似半衰期）



$$\lim_{t\to\infty} \frac{t^n}{e^t} = 0$$

Exponential Growth
指數成長

$$\frac{dN}{dt} = -kN$$

$$N(t) = N(t_0)e^{-k(t-t_0)}$$

$N(t)$ $N(t_0)$ k t_0

Exponential Decay

Dating 定年

Half life 半衰期

Finding *k* from two observations

$$N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$N(t_2) = N(t_0)e^{-k(t_2-t_0)}$$

$$p = \frac{N(t_1)}{N(t_2)} = e^{-k(t_1-t_2)}$$

$$k = \frac{\ln p}{t_2 - t_1}$$

Age of the Solar System

$$U_1(t) = U^{238}(t)$$

$$U_2(t) = U^{235}(t)$$

$$\frac{dU_1}{dt} = -\lambda_1 U_1$$

$$\frac{dU_2}{dt} = -\lambda_2 U_2$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \ln 2/4.47Gyr \\ \ln 2/0.707 \end{pmatrix}$$

$$\frac{U_1(t)}{U_2(t)} = \frac{U_1(0)}{U_2(0)}e^{(\lambda_1-\lambda_2)t}$$

We do not know the initial U238 and U235 at the beginning of our solar system, but the nuclear Synthesis theory suggest that the ratio is approximately equal to one.

$$\frac{U_1(t)}{U_2(t)} = 137.8$$

$$\frac{U_1(0)}{U_2(0)} \approx 1$$

$$t = \frac{\ln(137.8)}{\lambda_2 - \lambda_1} \cong 5.97Gyr$$

Radiocarbon Dating

Libby Nobel Prize for Chemistry in 1960

C14 half life ~5700 years:
Ratio of C14 to C12 is constant for living matter,
Begin to decay when dead.

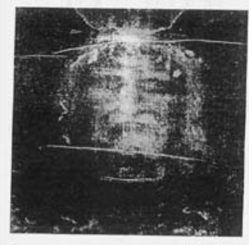


Figure 3.9 Shroud of Turin

$$k = \frac{\ln 2}{5700} \approx 1.216 \times 10^{-4}$$

$$N_0 e^{-kt_{1/2}} = \frac{1}{2}N_0$$

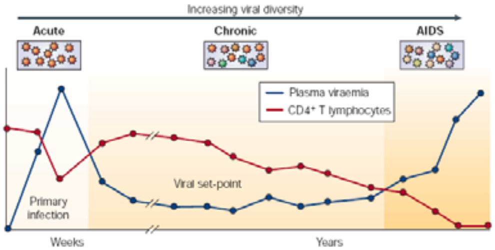
$$N(t_1) = pN(t_0)$$

Assumed to be the same as present

$$pN(t_0) = N(t_1) = N(t_0)e^{-k(t_1-t_0)}$$

$$\boxed{t_0 = t_1 + \frac{\ln p}{k}}$$

Lascaux cave painting



Dormant Phase
潛伏期

HIV Modeling

Perelson and Nelson (1999)

$$\frac{dV}{dt} = P \boxed{-\,cV,}$$

藥物治療

$$\frac{dT}{dt} = kT_0 V - \alpha T,$$

$$P = N\alpha T.$$

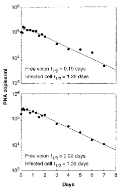


Figure 4.5. Log of plasma concentrations (copies per mL) of HIV-1 RNA (circles) for two representative patients (upper panel, patient 104; lower panel, patient 107) after *ritonavir* treatment was begun on day 0. The solid line is a nonlinear least square fit to the data. HIV-1 RNA level is an easier measure of HIV virions since each HIV virion contains two RNA molecules. (See exercise 5 for more details.) (From Perelson et al. [1996], used by permission of Alan S. Perelson.)

觀察病人服藥後反應決定C

$$\boxed{P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \;\; (1/(day \cdot ml))}$$

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
: non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

何大一雞尾酒療法

Fourier, Jean Baptiste Joseph

1768-1830

Heat emission or diffusion (by IR)

His calculations showed a very cold surface (No green house effect)

The profound study of nature is the most fertile source of mathematical discoveries.

$$f(x) = \sum \hat{f}_k e^{ikx}$$

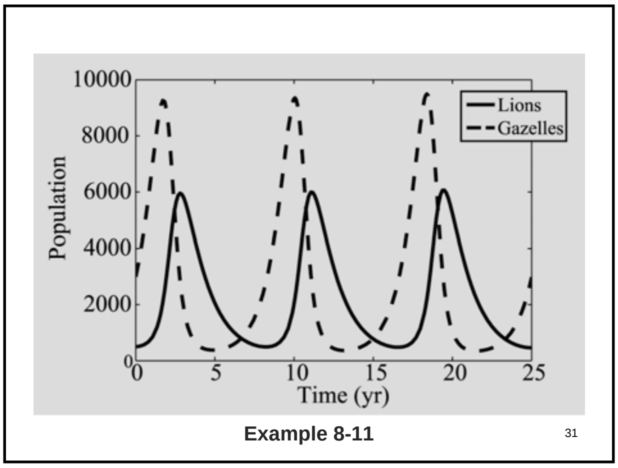
$$\hat{f}_k = \frac{1}{2\pi}\int_0^{2\pi} f(x)e^{-ikx}dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}_k e^{ikx}dx$$

$$\hat{f}_k = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-ikx}dx$$

1807 at age 39; argued with Lagrange and Laplace on the representation of a triangle wave with cosine and sine functions.

f(x) does not have to be analytical;
f(x) does not have to be periodic.



Example 8-11

仁者數學

Feedback　　回饋

Positive Feedback

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = x$$

其勢不可久

回也，其心三月不違仁，
其餘則日月至焉而已矣。

不常

Negative Feedback

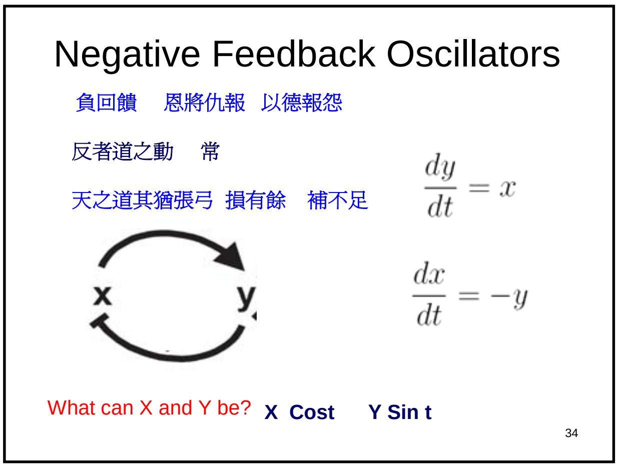
$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

反者道之動
一陰一陽之謂道
天之道其猶張弓　損有餘　補不足
坤 直方大
天道好還週而復始
復　　常

Periodic phenomena are actually everywhere in the biological world.

What else can you think of?



Negative Feedback Oscillators

負回饋　恩將仇報　以德報怨

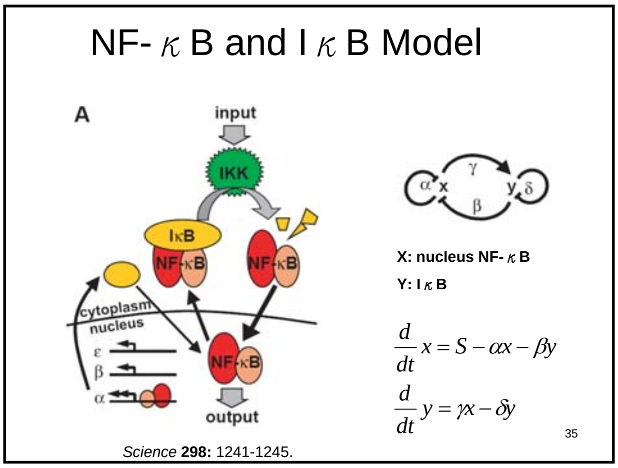
反者道之動　常

天之道其猶張弓　損有餘　補不足

$$\frac{dy}{dt} = x$$

$$\frac{dx}{dt} = -y$$

What can X and Y be?　**X Cost　Y Sin t**



NF-κB and IκB Model

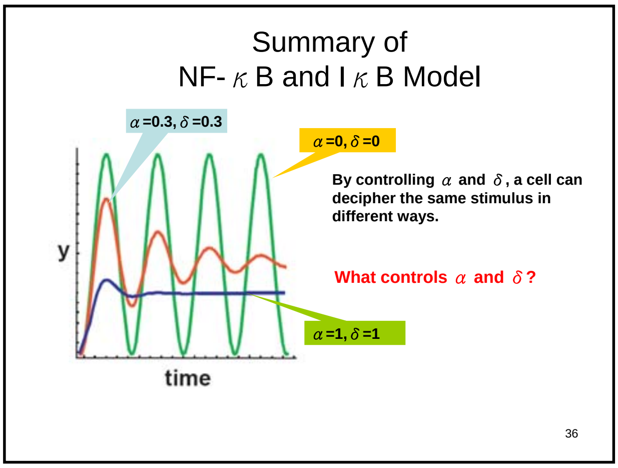
Science **298:** 1241-1245.

X: nucleus NF-κB

Y: IκB

$$\frac{d}{dt}x = S - \alpha x - \beta y$$

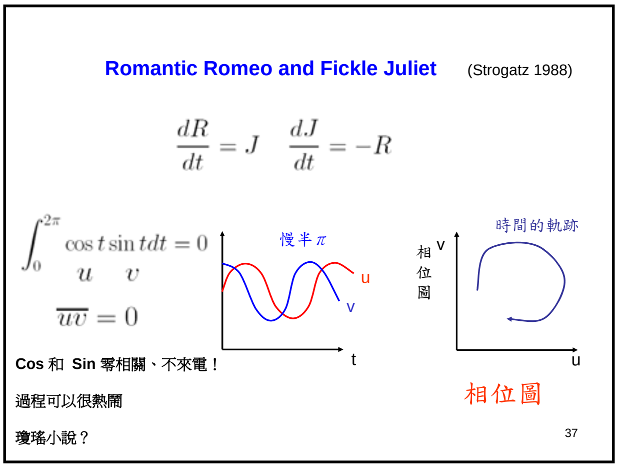
$$\frac{d}{dt}y = \gamma x - \delta y$$



Summary of NF-κB and IκB Model

By controlling α and δ, a cell can decipher the same stimulus in different ways.

What controls α and δ?



Romantic Romeo and Fickle Juliet

(Strogatz 1988)

$$\frac{dR}{dt} = J \quad \frac{dJ}{dt} = -R$$

$$\int_0^{2\pi} \underset{u}{\cos t} \, \underset{v}{\sin t} \, dt = 0$$

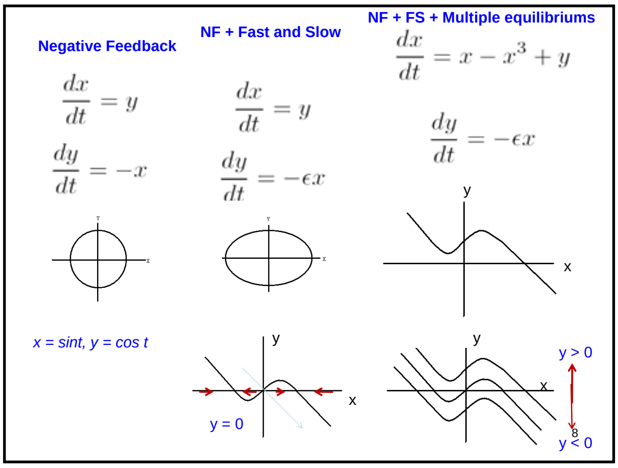
$$\overline{uv} = 0$$

Cos 和 Sin 零相關、不來電！

過程可以很熱鬧

瓊瑤小說？

相位圖



Negative Feedback

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -x$$

x = sint, y = cos t

NF + Fast and Slow

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -\epsilon x$$

NF + FS + Multiple equilibriums

$$\frac{dx}{dt} = x - x^3 + y$$

$$\frac{dy}{dt} = -\epsilon x$$

Cautious Romeo and Juliet

$$\frac{dR}{dt} = -aR + bJ$$

$$\frac{dJ}{dt} = -aJ + bR$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -a-b \\ -a+b \end{pmatrix}$$

$$\begin{pmatrix} R \\ J \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{(-a-b)t} + \beta \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{(b-a)t}$$

Falling in Love b > a 1/b < 1/a
情網必須墜入（猶豫是感情殺手）

Arm Races

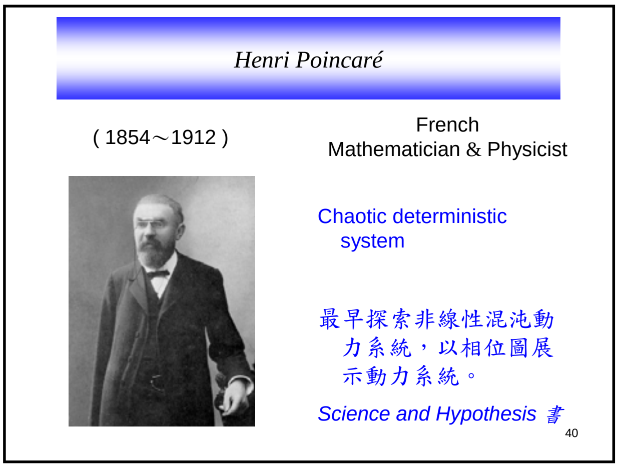
$$\frac{dx}{dt} = -a(x - x_0) + by$$

$$\frac{dy}{dt} = -a(y - y_0) + bx$$

x, y war potential

1/a < 1/b Truce

(Peace?)



Henri Poincaré

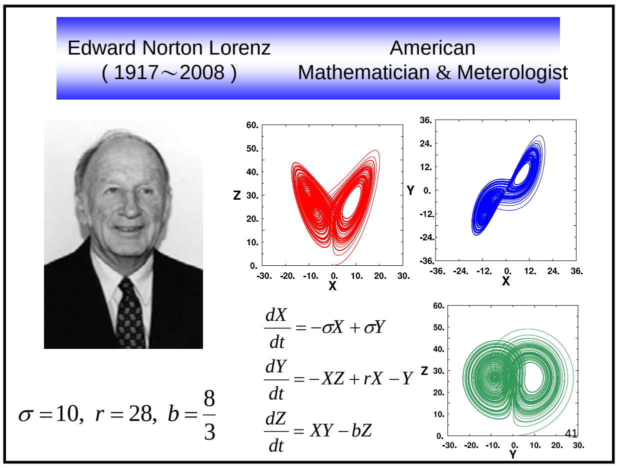
(1854～1912)

French
Mathematician & Physicist

Chaotic deterministic
system

最早探索非線性混沌動力系統，以相位圖展示動力系統。

Science and Hypothesis 書



Edward Norton Lorenz
(1917～2008)

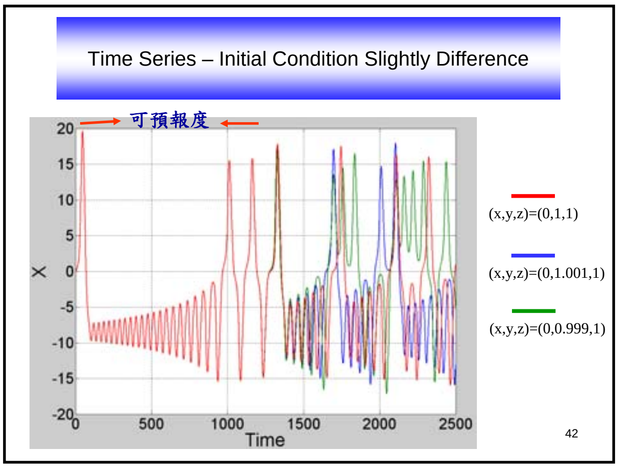
American
Mathematician & Meterologist

$$\sigma = 10, \ r = 28, \ b = \frac{8}{3}$$

$$\frac{dX}{dt} = -\sigma X + \sigma Y$$

$$\frac{dY}{dt} = -XZ + rX - Y$$

$$\frac{dZ}{dt} = XY - bZ$$



Time Series – Initial Condition Slightly Difference

🟥 (x,y,z)=(0,1,1)

🟦 (x,y,z)=(0,1.001,1)

🟩 (x,y,z)=(0,0.999,1)

蝴蝶效應 Butterfly Effect Chaos 混沌

混沌 非線性 $y = x^{30}$ 精確度有限
非線性

$$0.02 \left\{ \begin{array}{l} x = 0.99 \\ x = 1.01 \end{array} \right. \qquad 0.61 \left\{ \begin{array}{l} y \approx 0.74 \\ y \approx 1.35 \end{array} \right.$$

預報能力的喪失！！

"Sensitivity dependence on initial condition."

H Poincare

Thomas Robert Malthus

Wikipedia

(1766～1834)

English demographer and political economist

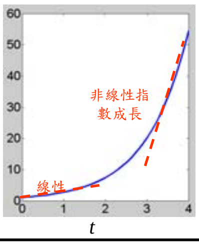
人口學家
政治經濟學家

Malthusian Model

Population Growth

$$\boxed{\lim_{\Delta \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

$$\frac{dp}{dt} = \alpha p \quad \Rightarrow \quad p = p_0 e^{\alpha t}$$



線性 非線性

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

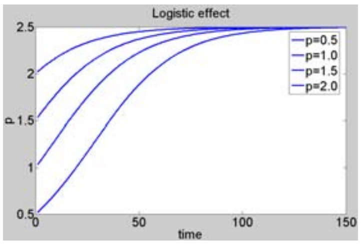
Logistic Effect

Time Series (時間序列)

$$\boxed{\frac{dp}{dt} = \alpha p - \beta p^2 = \alpha p (1 - \frac{p}{\frac{\alpha}{\beta}})}$$

其中 $\frac{\alpha}{\beta}$

→ 環境之承載效應



Lotka-Volterra Model

A Predator-Prey Model

Humberto D'Ancona
1926

World War One 掠食者比例變大

	Port	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
Predator	**Fiume**	12%	21%	22%	21%	36%	27%	16%	16%	15%	11%
Prey	**Trieste**	14%	7%	16%	15%	-	18%	15%	13%	11%	10%

$$\boxed{\frac{dx}{dt} = x - xy} \qquad \boxed{\frac{dy}{dt} = -y + xy}$$

Vito Volterra

Wikipedia

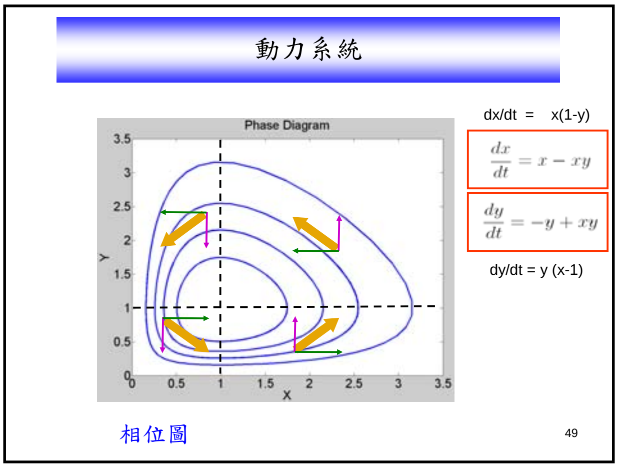
(1860～1940)

Italian
Mathematician & Physicist

Contribution:

1) Mathematical biology

2) Volterra-Lotka equations



動力系統

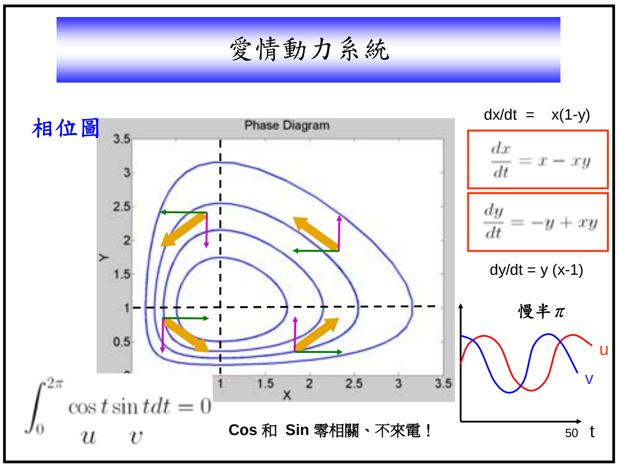
dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

相位圖



愛情動力系統

相位圖

dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

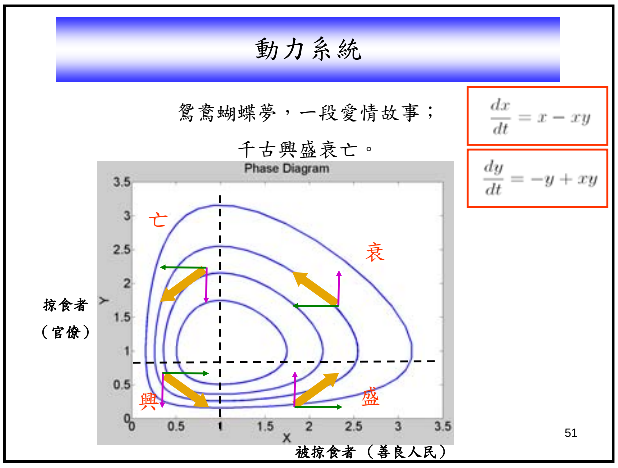
$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

$$\int_0^{2\pi} \cos t \sin t dt = 0$$

$$u \quad v$$

Cos 和 Sin 零相關、不來電！



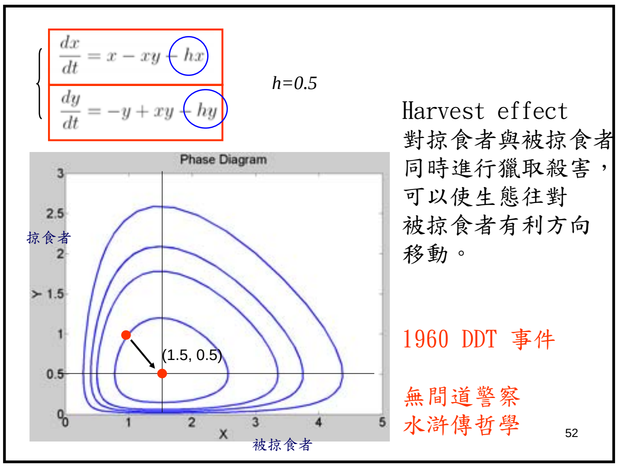
動力系統

鴛鴦蝴蝶夢，一段愛情故事；

千古興盛衰亡。

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$



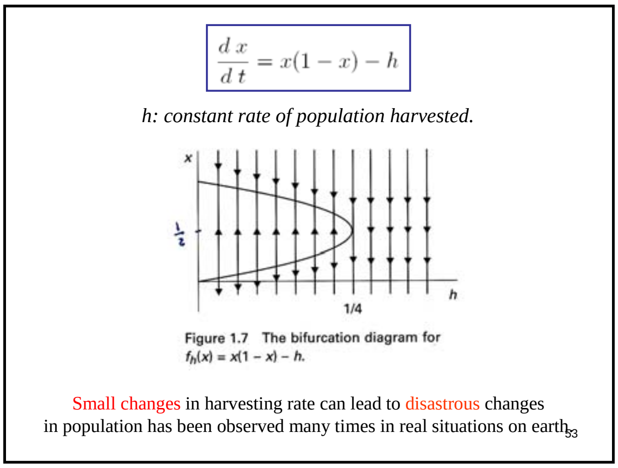
$$\begin{cases} \dfrac{dx}{dt} = x - xy - hx \\ \dfrac{dy}{dt} = -y + xy - hy \end{cases}$$

h=0.5

Harvest effect
對掠食者與被掠食者
同時進行獵取殺害，
可以使生態往對
被掠食者有利方向
移動。

1960 DDT 事件

無間道警察
水滸傳哲學

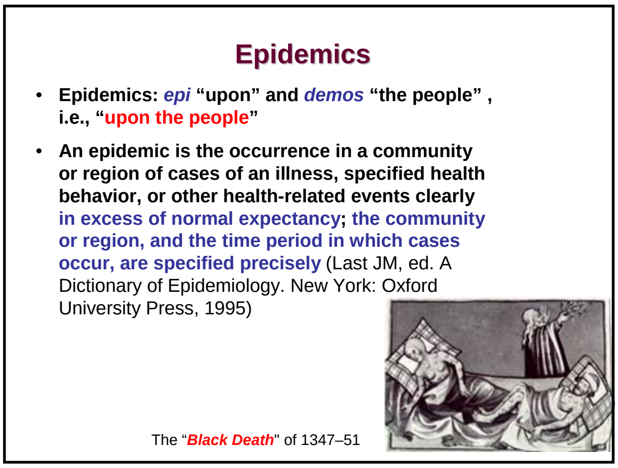


$$\boxed{\frac{dx}{dt} = x(1-x) - h}$$

h: constant rate of population harvested.

Figure 1.7 The bifurcation diagram for $f_h(x) = x(1-x) - h$.

Small changes in harvesting rate can lead to disastrous changes in population has been observed many times in real situations on earth



Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)

The "***Black Death***" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

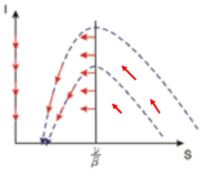
$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu)I$$

$$\frac{dR}{dt} = +\nu I$$

ν Recovery Rate

β Infection Rate

No Death in the model



$S = \frac{\nu}{\beta}$有 null cline

Forecast and control of epidemics in a globalized world PNAS vol.101 no.42

Hufnagel†, Brockmann, and Geisel

演講者: 陳怡文　日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

Synchronization

$$\frac{d\theta_e}{dt} = \omega_e$$

$$\frac{d\theta}{dt} = \omega + \alpha \sin(\theta_e - \theta)$$

$$\phi(t) = \theta_e - \theta \quad \tau = \alpha t$$

$$\delta = \frac{(\omega_e - \omega)}{\alpha}$$

$$\frac{d\phi}{d\tau} = \delta - \sin\phi \quad -\pi \le \phi \le \pi$$

Fast and Slow

$$\frac{dx}{dt} = x - x^3 \boxed{+ y}$$

$$\frac{dy}{dt} = \boxed{-\epsilon x}$$

London Millennium Bridge

London's Millennium Bridge is the first pedestrian river crossing over the Thames in central London for more than a century.

It is a 325m steel bridge linking the City of London at St. Paul's Cathedral with the Tate Modern Gallery at Bankside.

"Nice" lateral vibrations (20 cm S shape wobble, 1cycle per second) like on Tacoma Bridge developed on the day (June 12, 2000) of the opening……

Letter to the Guardian on June 14, 2000 (2 days later)

The Millennium Bridge problem (Millennium bug strikes again, June 13) has little to do with crowds walking in steps. It is connected with what people do as they try to maintain balance if the surface on which they are walking starts to move, and is similar to what can happen if a number of people stand up at the same time in a small boat. It is possible in both cases that the movements that people make as they try to maintain their balance lead to an increase in whatever swaying is already present, so that the swaying goes on getting worse.

Is it true "the bridge never going to fall down", or any rate get damaged, as a result of swaying? That has been said about the bridge before, and those Responsible for this one need to understand, before making such pronouncement, that the problem involves more than engineering principle.

Prof. Brian Joesphson
Department of Physics
University of Cambridge

Nothing like this had been predicted by computer simulations, the safety assessments, the wind tunnel experiments.

"Nice" lateral vibrations 20 cm S shape wobble developed by shaking machine with 1cycle per second frequency. [where the hell does it come from?]

People walk at a pace of about 2 strides per second; these repetitive footfalls is to create a vertical forces.

The culprit is the small sideway force with each step; 1 cycle per second. Shouldn't the no coordination cancel out the force? No bridge standard code on this.

Synchronizations; just like people stand up in the boat. Sync may be accidentally trigger when enough people on the bridge.

Boosting immunity by antiviral drug therapy: A simple relationship among timing, efficacy, and success

Natalia L. Komarova, Eleanor Barnes, Paul Klenerman, and Dominik Wodarz

演講者：李坤珀 生科四
指導老師：郭鴻基教授
2007/03/06

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PNAS

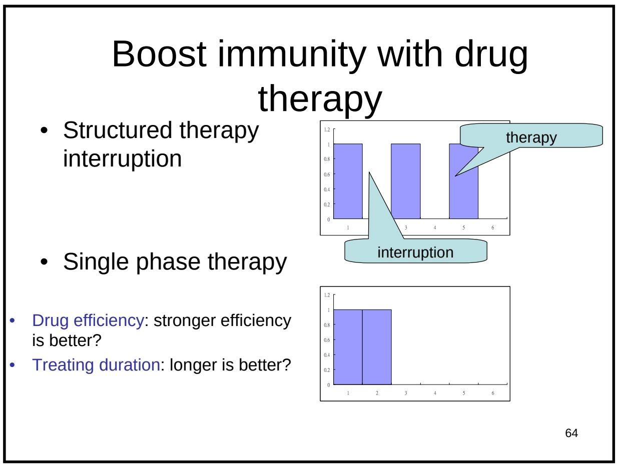
Boost immunity with drug therapy

- Drug efficiency: stronger efficiency is better?
- Treating duration: longer is better?

- Simulate the treatment with mathematical model

Virus suppress immunenity

- Some virus, such as HIV, HBV, and HCV, have ability to suppress immune response
- Human Immunodeficiency Virus (HIV, 人體免疫缺陷病毒)
  - About 40 million people who have been HIV infected
- Hepatitis B Virus (HBV, B型肝炎病毒)
  - 2 billion people who have been HBV infected
- Hepatitis C Virus (HCV, C型肝炎病毒)
  - 1.8 millon people who have been HCV infected

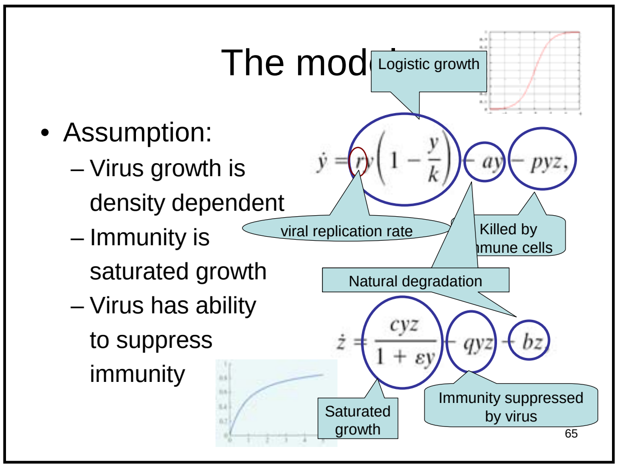


Boost immunity with drug therapy

- Structured therapy interruption

- Single phase therapy

- Drug efficiency: stronger efficiency is better?
- Treating duration: longer is better?



The model

- Assumption:
  - Virus growth is density dependent
  - Immunity is saturated growth
  - Virus has ability to suppress immunity

$$\dot{y} = ry\left(1 - \frac{y}{k}\right) - ay - pyz,$$

$$\dot{z} = \frac{cyz}{1 + \varepsilon y} - qyz - bz,$$

Y 帝國
X 殖民地

$$\begin{cases} \frac{dp}{dt} = K_1 p(1 - \frac{p}{c_1}) - \alpha_1 pq \\ \frac{dq}{dt} = -K_2 q + \alpha_2 pq \end{cases}$$

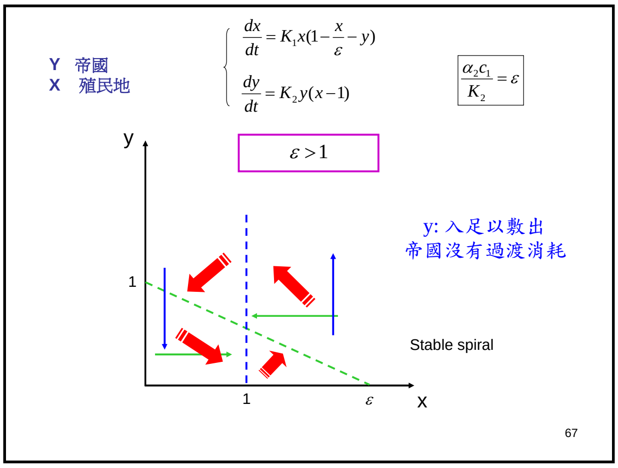
原方程式變數：

K_1	α_1	c_1
K_2	α_2	

令

$$\boxed{x = \frac{p}{\frac{K_2}{\alpha_2}}} \quad \boxed{y = \frac{q}{\frac{K_1}{\alpha_1}}} \quad \boxed{\frac{\alpha_2 c_1}{K_2} = \varepsilon}$$

$$\Rightarrow \begin{cases} \frac{dx}{dt} = K_1 x(1 - \frac{x}{\boxed{\varepsilon}} - y) \\ \frac{dy}{dt} = K_2 y(x - 1) \end{cases}$$


Y 帝國
X 殖民地

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - \frac{x}{\varepsilon} - y) \\ \frac{dy}{dt} = K_2 y(x-1) \end{cases}$$

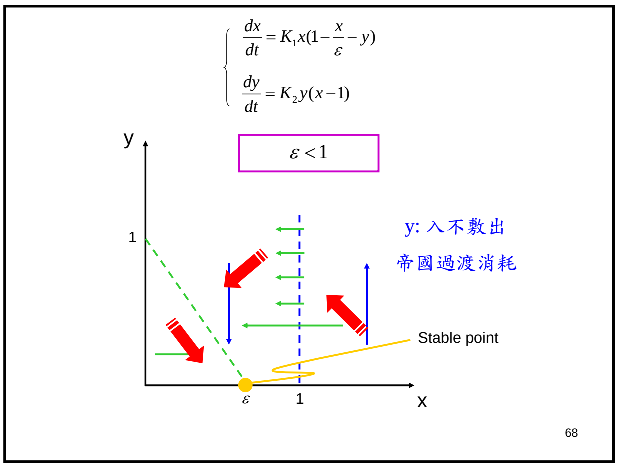
$$\boxed{\frac{\alpha_2 c_1}{K_2} = \varepsilon}$$

$$\boxed{\varepsilon > 1}$$

y: 入足以敷出
帝國沒有過渡消耗

Stable spiral

67

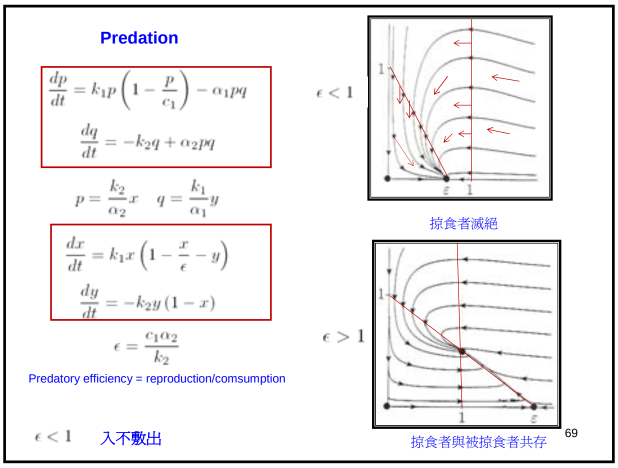

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - \frac{x}{\varepsilon} - y) \\ \frac{dy}{dt} = K_2 y(x-1) \end{cases}$$

$$\boxed{\varepsilon < 1}$$

y: 入不敷出
帝國過渡消耗

Stable point

68


Predation

$$\boxed{\begin{aligned} \frac{dp}{dt} &= k_1 p\left(1 - \frac{p}{c_1}\right) - \alpha_1 pq \\ \frac{dq}{dt} &= -k_2 q + \alpha_2 pq \end{aligned}}$$

$$p = \frac{k_2}{\alpha_2}x \quad q = \frac{k_1}{\alpha_1}y$$

$$\boxed{\begin{aligned} \frac{dx}{dt} &= k_1 x\left(1 - \frac{x}{\epsilon} - y\right) \\ \frac{dy}{dt} &= -k_2 y\,(1-x) \end{aligned}}$$

$$\epsilon = \frac{c_1 \alpha_2}{k_2}$$

Predatory efficiency = reproduction/comsumption

$\epsilon < 1$ 入不敷出

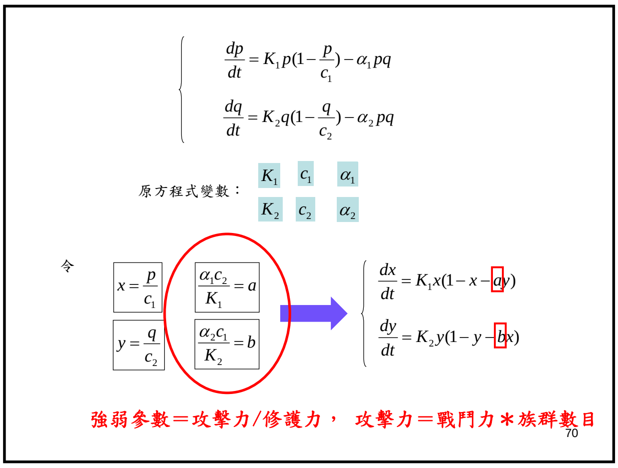
$\epsilon < 1$

掠食者滅絕

$\epsilon > 1$

掠食者與被掠食者共存

69


$$\begin{cases} \frac{dp}{dt} = K_1 p(1 - \frac{p}{c_1}) - \alpha_1 pq \\ \frac{dq}{dt} = K_2 q(1 - \frac{q}{c_2}) - \alpha_2 pq \end{cases}$$

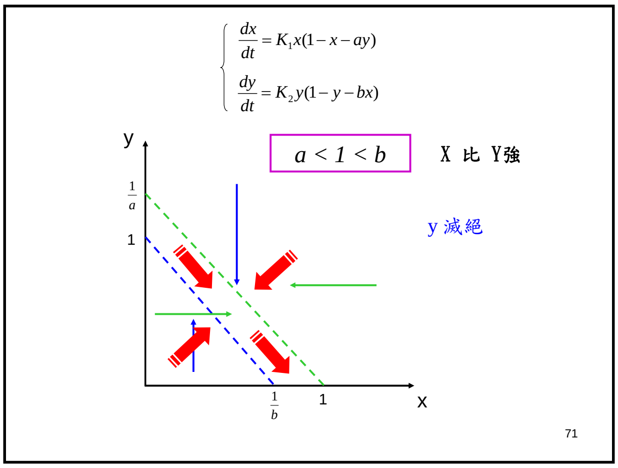
原方程式變數： K_1 c_1 α_1 K_2 c_2 α_2

令 $\boxed{x = \frac{p}{c_1}}$ $\boxed{\frac{\alpha_1 c_2}{K_1} = a}$ $\boxed{y = \frac{q}{c_2}}$ $\boxed{\frac{\alpha_2 c_1}{K_2} = b}$

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - \boxed{a}y) \\ \frac{dy}{dt} = K_2 y(1 - y - \boxed{b}x) \end{cases}$$

強弱參數＝攻擊力/修護力， 攻擊力＝戰鬥力＊族群數目

70

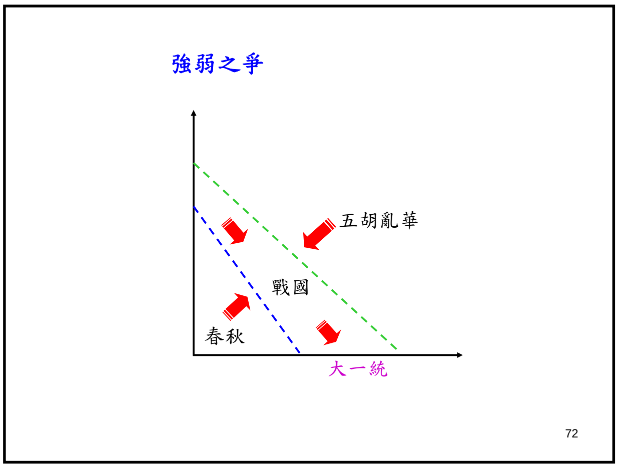

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$

$$\boxed{a < 1 < b}$$

X 比 Y強

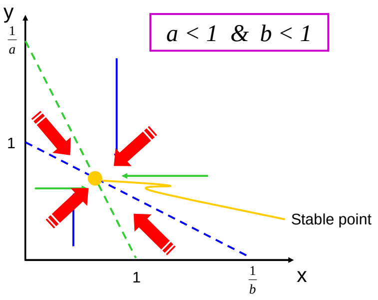
y 減絕

71

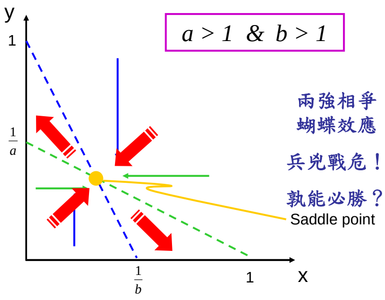

強弱之爭

72

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$



$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$



兩強相爭
蝴蝶效應

兵兇戰危！

孰能必勝？

WHY 無因次化？？

數學是無因次 上帝創造人類 人類創造單位

20分 與 **100**分 ？ 快慢？ 大小？

標準化 **Top 1%**

手術死亡3人 與手術死亡100人 何者爲佳？

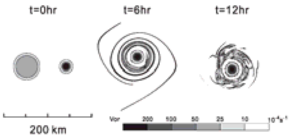
線性與非線性

Scale free

Mathematical elegance and analysis

Important parameter(s)

因次與無因次相輔相成



颱風雙眼牆動力

Buckingham's Pi Theorem

π

n variables can always be combined to form
Exactly (n- r) independent nondimensional variables, where
R is the rank of the dimensional matrix.

T, M, L, g are the 4 variable in the simple pendulum problem

kg, m, s are the rank 3 dimensional unit

4-3 =1 nondimensional va $T \sim \sqrt{\frac{l}{g}}$

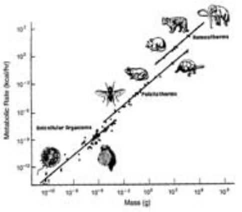
Similar result can be derived from Newton's mechanics or
from the fact that the difference of potential energy and kinetic energy over
a period will be minimized; the Hamilton principle.

To see a world in a grain of sand
And heaven in a wild flower
Hold infinity in the palm of your hand
And eternity in an hour.
William Blake

一沙一世界
一花一天堂
手中掌握無限
剎那即是永恆

Scale free

Metabolic rate vs size



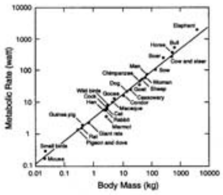


FIGURE 2 Metabolic rate (in kcal/hr) for a series of organisms ranging from the smallest microbes to the largest mammals as a function of mass (in g), exemplifying the persistence of the 3/4-power scaling law (the solid lines) over 20 orders of magnitude (Hemmingsen [12]).

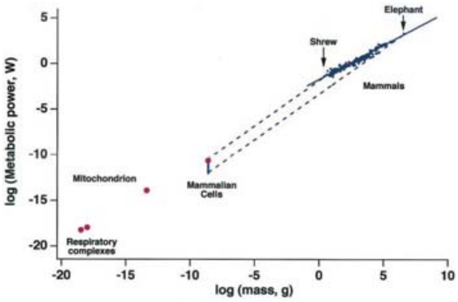
FIGURE 1 Metabolic rate (in watts) for a series of mammals and birds as a fu of mass (in kg); the scale is logarithmic and exemplifies the 3/4-power scalin discovered by Kleiber [2, 22, 27, 29].

$$I = I_0 M^{3/4}$$

Hemmingsen (1960) *Reports of the Steno Memorial Hospital and Nordisk Insulin Laboratorium* **9**, 6-110

Kleiber (1932) Body size and metabolism. *Hilgardia* **6**, 315-353.

“Metabolic rate vs size” down to molecule



West, Woodruff, Brown (2002) PNAS **99**, 2473-2478

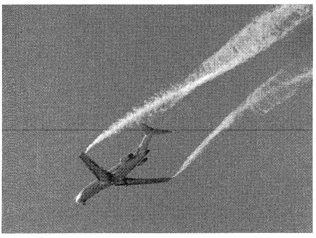


Fig. 8.9. Vortices trailing form the wingtips of a Boeing 727. Figure courtesy of NASA.

Wake Turbulence

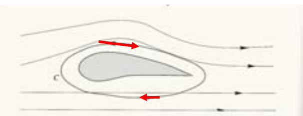


Fig. 8.10. Sketch of the flow along an airfoil. The wing is shown in grey, the contour C is shown by the thick solid line.

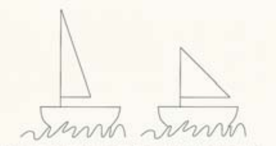
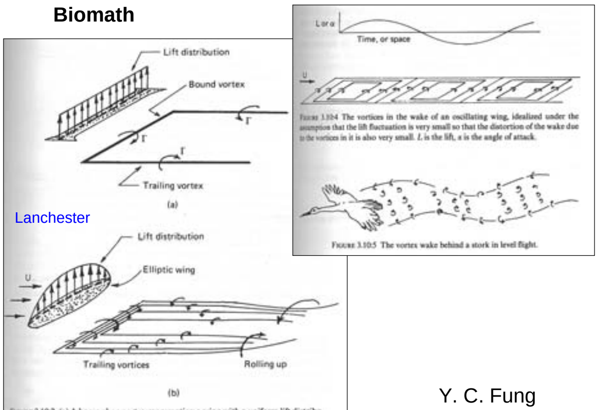


Fig. 8.12. Two boats carrying sails with very different aspect ratios.

Biomath



Lanchester

FIGURE 3.10:2 (a) A horse-shoe vortex representing a wing with a uniform lift distribution. (b) Lift distribution on an elliptic wing.

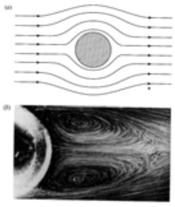
FIGURE 3.10:4 The vortices in the wake of an oscillating wing, idealized under the assumption that the lift fluctuation is very small so that the distortion of the wake due to the vortices in it is also very small. L is the lift, α is the angle of attack.

FIGURE 3.10:5 The vortex wake behind a stork in level flight.

Y. C. Fung

1717-1783

D’Alembert Paradox



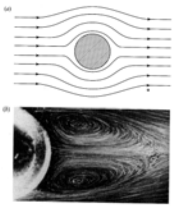
$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathrm{Re} << 1.$$

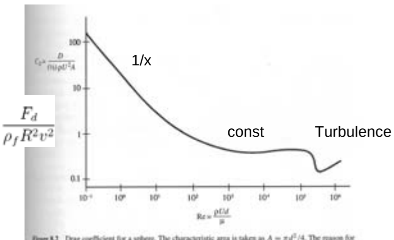
D’Alembert Solution of the Wave Equation
[f(x + ct) and f(x-ct)]

Re small viscosity important
Re large viscosity unimportant

Atmospheric Motion first expressed mathematically
(Won the 1746 Berlin Academy’s Award; aqua-planet
Endorsement of Euler)

Solar and Lunar Force Cause the Atmopheric Motion





$$\frac{F_d}{\rho_f R^2 v^2}$$

Figure 8.2 Drag coefficient for a sphere. The characteristic area is taken as $A = \pi d^2/4$. The reason for the sudden drop of C_D at Re ~ 5×10^5 is the transition of the laminar boun[illegible]lent one, as explained in Chapter 10.

$$\frac{vR}{\nu} = \mathrm{Re}$$

F_d $[M][L][T]^{-2}$ drag force;
ρ_f $[M][L]^{-3}$ density of fluid
R $[L]$ radius of sphere
v $[L][T]^{-1}$ speed
$\boxed{\nu \quad [L]^2[T]^{-1} \quad \text{kinematic viscosity}}$.

$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathrm{Re} << 1.$$

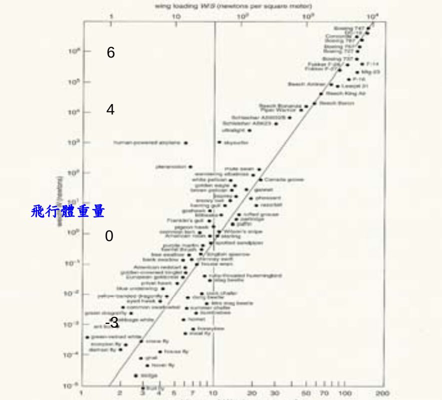


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.

$$F = C_L\, \rho_{air}\, v^2\, S$$

v: speed

V: volumn

S: area

$$W = \rho_f\, V\, g = \rho_f\, S^{1.5}\, g$$

$$F = W$$

$$S^{0.5} = \frac{C_L\, \rho_{air}\, v^2}{\rho_f\, g}$$

$$W = \rho_f\, g\, S^{1.5} = \frac{{C_L}^3\, \rho_{air}^3\, v^6}{\rho_f^2\, g^2}$$

$$\Pi = \frac{r_f}{E^{1/5} t^{2/5} {\rho_0}^{-1/5}}$$

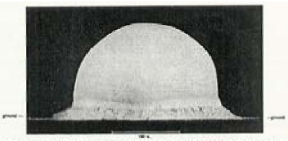


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963).

G.I. Taylor 1950

Symbol	*Definition*	*Representative value or first guess*
R	radius of wavefront	10^2 m
t	time	10^{-2} s
p_0	ambient pressure	10^5 Pa
ρ_0	ambient density	1 kg m^{-3}
E	energy released	10^{14} J

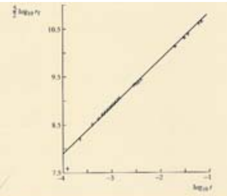


Figure 6.3. Logarithmic plot of the fireball radius, showing that $r_f^{5/2}$ is proportional to the time t (Taylor 1950b, 1963).

原子彈能量～ 10^{14} J

$$r = r(t, E, \rho, p, e)$$

$$[e] = [L^2T^{-2}]; \quad [p] = [ML^{-1}T^{-2}]; \quad [t] = [T]$$

$$[\rho] = [ML^{-3}]; \quad [E] = [ML^2T^{-2}]; \quad [r] = [L].$$

$$\frac{\rho r^5}{Et^2} = f\left(\frac{pr^3}{E}, \frac{p}{\rho e}\right)$$

$pr^3/E \to \infty$ **Minor explosion**

$$f_1\,(\gamma - 1) = \gamma$$

$$\frac{\rho r^5}{Et^2} = f\left(\frac{pr^3}{E}, \gamma - 1\right)$$

$$r^2 = \frac{\gamma p}{\rho} t^2$$

$$\frac{\rho r^5}{Et^2} = \frac{pr^3}{E} f_1\,(\gamma - 1)$$

$$\frac{dr}{dt} = C_s$$

$$\frac{\rho r^5}{Et^2} = f\left(\frac{pr^3}{E}, \frac{p}{\rho e}\right)$$

$pr^3/E \to 0$

Major Explosion

$$\frac{\rho r^5}{Et^2} = f_2\,(\gamma - 1)\,,$$

$$r = \left(\frac{E}{\rho}\right)^{1/5} t^{2/5} f_2^{1/5}.$$

$$\frac{dr}{dt} = \frac{2}{5}\left(\frac{E}{\rho}\right)^{1/5} t^{-3/5} f_2^{1/5}$$

$$\frac{dr}{dt} = \frac{2}{5}\left(\frac{E}{\rho}\right)^{1/2} r^{-3/2} f_2^{1/2}.$$

$$E \sim \rho r^5 t^{-2} = 1kg/m^3 (10^2 m)^5 (10^{-2} s)^{-2} \sim 10^{14} J.$$

A, G constant

$$F \sim \rho v^2 l^2$$

$$P \sim \rho v^3 l^2$$

$$\boxed{G \sim \frac{l^3}{n}}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$\boxed{v \sim n^{1/9}}$$

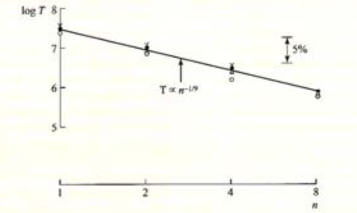
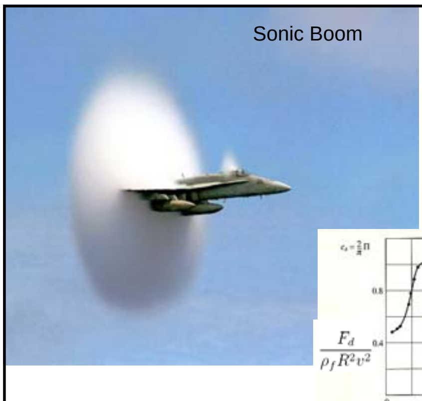


Figure 1.13. The −1/9 power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; ×, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).

Sonic Boom



u/c ~ 1

Mach number u/c

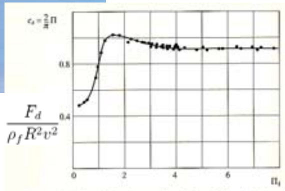
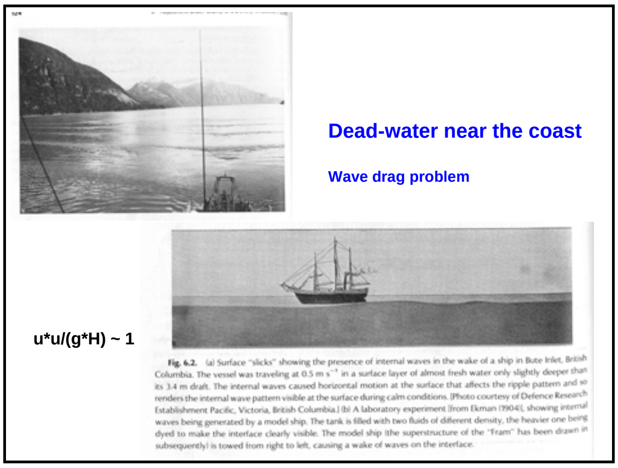


Figure 1.11. The dimensionless drag on a sphere, Π (times 2/π), as a function of the dimensionless governing parameter $\Pi_1 = U/c$, the Mach number (Chernyi 1961). The quantity Π approaches a constant for large values of Π_1.

Adiabatic Sound

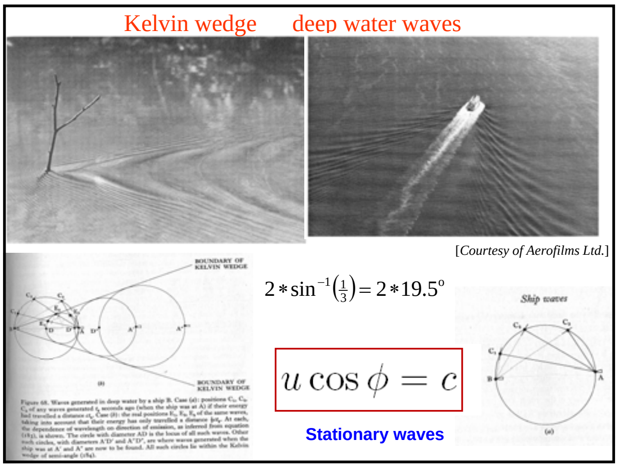


Dead-water near the coast

Wave drag problem

u*u/(g*H) ~ 1

Fig. 6.2. (a) Surface "slicks" showing the presence of internal waves in the wake of a ship in Bute Inlet, British Columbia. The vessel was traveling at 0.5 m s^{-1} in a surface layer of almost fresh water only slightly deeper than its 3.4 m draft. The internal waves caused horizontal motion at the surface that affects the ripple pattern and so renders the internal wave pattern visible at the surface during calm conditions. [Photo courtesy of Defence Research Establishment Pacific, Victoria, British Columbia.] (b) A laboratory experiment [from Ekman (1904)], showing internal waves being generated by a model ship. The tank is filled with two fluids of different density, the heavier one being dyed to make the interface clearly visible. The model ship (the superstructure of the "Fram" has been drawn in subsequently) is towed from right to left, causing a wake of waves on the interface.



Kelvin wedge deep water waves

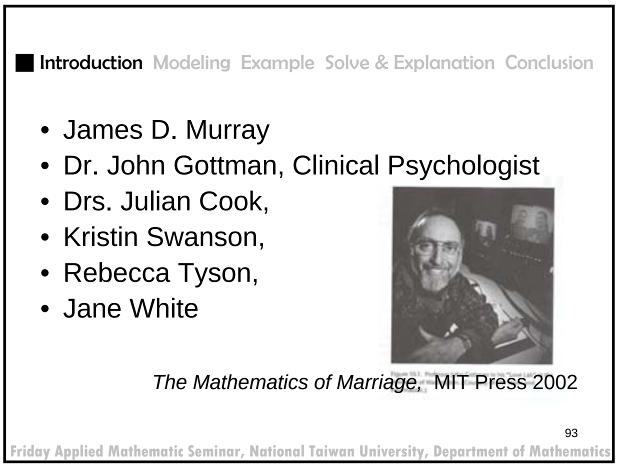
[*Courtesy of Aerofilms Ltd.*]

$$2*\sin^{-1}\left(\tfrac{1}{3}\right)=2*19.5^{\circ}$$

$$\boxed{u\cos\phi = c}$$

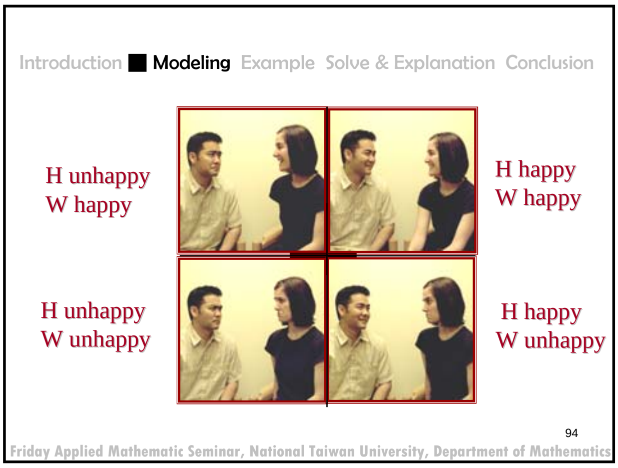
Stationary waves

Figure 68. Waves generated in deep water by a ship B. Case (a): positions C_1, C_2, C_3 of any waves generated t_0 seconds ago (when the ship was at A) if their energy had travelled a distance ct_0. Case (b): the real positions E_1, E_2, E_3 of the same waves, taking into account that their energy has only travelled a distance $\frac{1}{2}ct_0$. At each, the dependence of wavelength on direction of emission, as inferred from equation (183), is shown. The circle with diameter AD is the locus of all such waves. Other such circles, with diameters A′D′ and A″D″, are where waves generated when the ship was at A′ and A″ are now to be found. All such circles lie within the Kelvin wedge of semi-angle (184).



- James D. Murray
- Dr. John Gottman, Clinical Psychologist
- Drs. Julian Cook,
- Kristin Swanson,
- Rebecca Tyson,
- Jane White

The Mathematics of Marriage, MIT Press 2002



H unhappy
W happy

H happy
W happy

H unhappy
W unhappy

H happy
W unhappy

- First Application of Mathematical Modeling in Social-Sciences.
- Problem: The divorce rate for second marriages is even higher! Why? Don't they become wiser from their first experience?
- Based on mismatches in the couples personality or modes of communication seems NOT be too successful.

- "Men are from Mars, women are from Venus"— a lack of understanding of gender differences in communication styles is at the root of marital problems.
- Another way approaching.

- Self-Interaction (uninfluenced steady state) 本性

$$\frac{dx}{dt} = r_1(x_0 - x), \qquad u(t) = u(0)e^{-r_1 t}$$

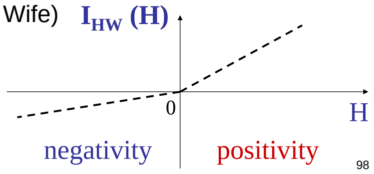
$$\frac{dy}{dt} = r_2(y_0 - y). \qquad v(t) = v(0)e^{-r_2 t}$$

- Marital Interactions (influenced function) 影響對方

$$\frac{dx}{dt} = r_1(x_0 - x) + I_1(y), \qquad I_i(z) = \begin{cases} a_i z & if\ z > 0 \\ b_i z & if\ z < 0 \end{cases}$$

$$\frac{dy}{dt} = r_2(y_0 - y) + I_2(x).$$

- **Five marriage types** – different interaction styles – related to interaction functions (I) in the model
- Basic model fits empirical data to a two-slope linear interaction function I (one each for the Husband and Wife)



- + I validating
0 + II avoider
- 0 III volatile

- Characteristics (stable and unstable types):

Volatile (S) – romantic, passionate, have heated arguments with cycles of fights and sex.

Validating (S) – calmer, intimate, value companionate marriage, shared experience rather than individuality.

Avoiders (S) – avoid confrontation and conflict, interact only in positive range of their emotions.

Hostile (U) – (mixed) conflict-avoiding wife, validating husband.

Hostile-Detached (U) – (mixed) volatile wife, validating husband

---

An example of a Validating Couple

- For a validating couple, take for simplicity, we have

$$\frac{dx}{dt} = r_1(x_0 - x) + a_1 y,$$

$$\frac{dy}{dt} = r_2(y_0 - y) + a_2 x.$$

Let (x^*, y^*) denote the equilibrium solution, then

An example of a Validating Couple

$$r_1(x_0 - x^*) + a_1 y^* = 0,$$

$$r_2(y_0 - y^*) + a_2 x^* = 0.$$

The solution is

$$x^* = [x_0 + \frac{a_1}{y_1} y_0]/[1 - \frac{a_1 a_2}{r_1 r_2}],$$

$$y^* = [y_0 + \frac{a_2}{y_2} x_o]/[1 - \frac{a_1 a_2}{r_1 r_2}].$$

$$\boxed{(x^*, y^*) > (x_0, y_0)}$$

Stable Marriage

- New language for describing marital interaction and social influence and rationale for the marital experiments
- Concept that marriages can be classified into one of 5 types of marriage depending on the couple’s interaction style:
- Stable marriages have matched interaction styles. Unstable marriages have mismatched interaction styles
- Couple’s interaction data suggest specific therapy

Catastrophe

The Spruce budworm problem

$$\frac{du}{dt} = f(u) = \left[ru\left(1-\frac{u}{q}\right)\right] - \left[\frac{u^2}{(1+u^2)}\right] \quad (6.1)$$

$$\left[ru\left(1-\frac{u}{q}\right)\right] - \left[\frac{u^2}{1+u^2}\right] = 0 \quad (6.2)$$

$$\left[r\left(1-\frac{u}{q}\right)\right] - \left[\frac{u}{(1+u^2)}\right] = 0 \quad (6.3)$$

$$g(u) = r\left(1-\frac{u}{q}\right) \quad (6.4)$$

$$h(u) = \frac{u}{1+u^2} \quad (6.5)$$

$$g(u) = r\left(1-\frac{u}{q}\right) \qquad h(u) = \frac{u}{1+u^2}$$

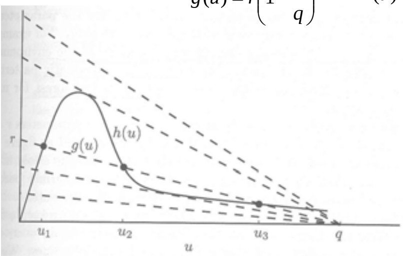


Figure 6.1: Budworm problem: the three potential points of intersection of the two bracketed functions in (6.3).

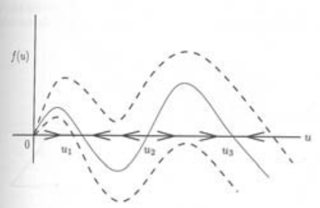


Figure 6.2: Qualitative form of $f(u)$ in equation (6.1) with r and q in the ranges where there are one or three nonzero solutions. The dotted lines are given by a large enough or small enough r such that there is only one stable steady-state solution. For the highest dotted line, the only stable steady-state solution is u_3, and for the lowest dotted line we are left with u_1.

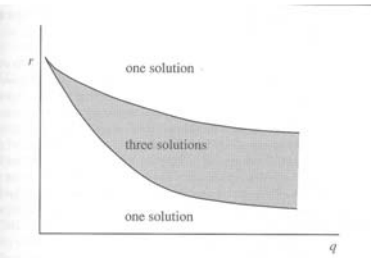


Figure 6.4: A cusp is formed when the surface in figure 6.3 is projected onto the $r-q$ plane.

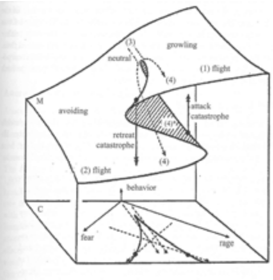


Figure 6.6: The cusp catastrophe illustrating fear and rage as conflicting factors influencing aggression.

1


2


3

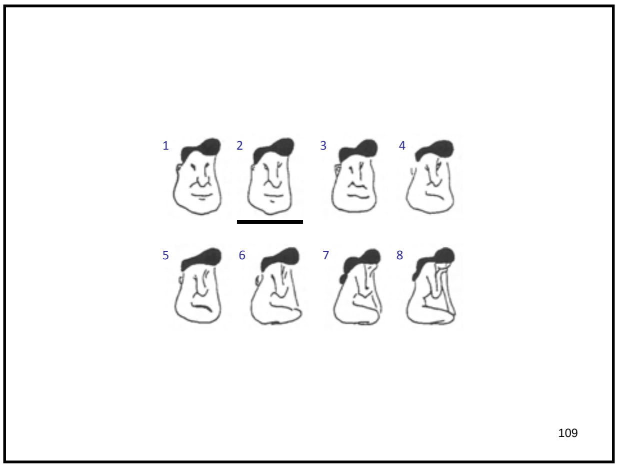

4


5


6


7


8

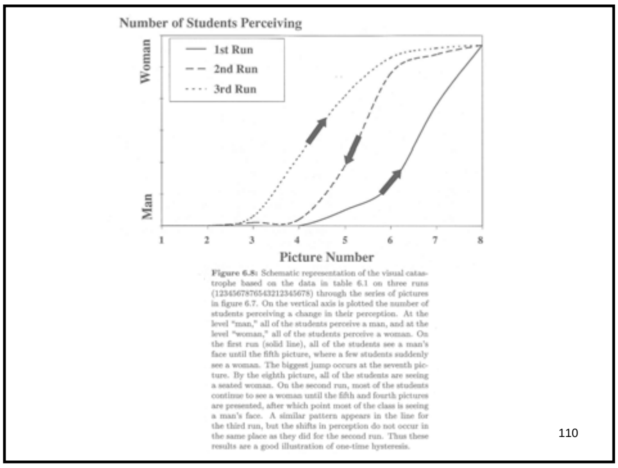
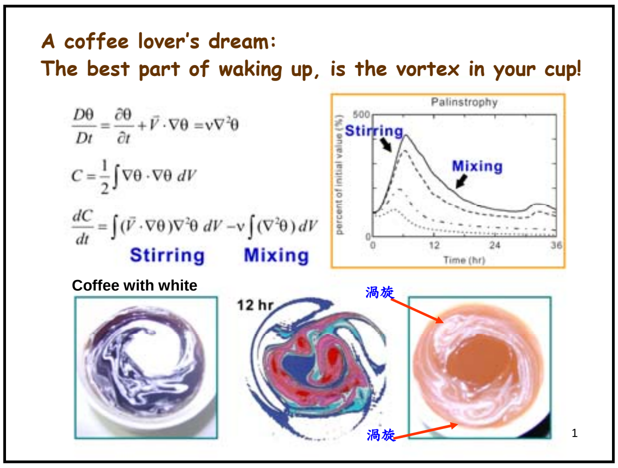


Figure 6.8: Schematic representation of the visual catastrophe based on the data in table 6.1 on three runs (123456787654321234 5678) through the series of pictures in figure 6.7. On the vertical axis is plotted the number of students perceiving a change in their perception. At the level "man," all of the students perceive a man, and at the level "woman," all of the students perceive a woman. On the first run (solid line), all of the students see a man's face until the fifth picture, where a few students suddenly see a woman. The biggest jump occurs at the seventh picture. By the eighth picture, all of the students are seeing a seated woman. On the second run, most of the students continue to see a woman until the fifth and fourth pictures are presented, after which point most of the class is seeing a man's face. A similar pattern appears in the line for the third run, but the shifts in perception do not occur in the same place as they did for the second run. Thus these results are a good illustration of one-time hysteresis.



A coffee lover's dream:
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt}=\frac{\partial\theta}{\partial t}+\vec{V}\cdot\nabla\theta=\nu\nabla^2\theta$$

$$C=\frac{1}{2}\int\nabla\theta\cdot\nabla\theta\,dV$$

$$\frac{dC}{dt}=\int(\vec{V}\cdot\nabla\theta)\nabla^2\theta\,dV-\nu\int(\nabla^2\theta)\,dV$$

Stirring **Mixing**

Coffee with white

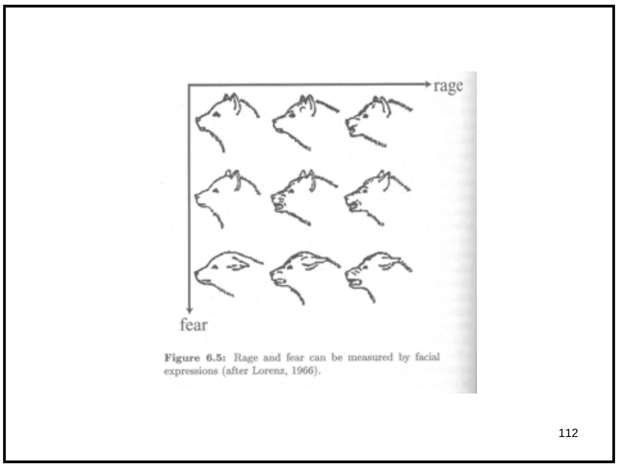


Figure 6.5: Rage and fear can be measured by facial expressions (after Lorenz, 1966).

Leonardo da Vinci (1452-1519)

Multiple Scale Interactions
Steady and Turbulent Flows
多重尺度交互作用

"*Observe the movement of the surface of the water which resembles that of hair which has two motions, of which one depends on the weight of the hair and the other on the direction of the curls. Thus water forms eddying whirlpools of which one part depends on the predominant current and the other on the incidental motion and the return flow.*"

$$\frac{d\mathbf{v}}{dt}+A\mathbf{v}=\mathbf{f}$$

$$\frac{dv}{dt}+\lambda v=f_0e^{i\Omega t}$$

$$t=[\frac{1}{\Omega}]t^*$$

$$v=[\frac{f_0}{\lambda}]v^*$$

$$\frac{\Omega}{\lambda}\frac{dv^*}{dt^*}+v^*=e^{it^*}$$

$$\epsilon\frac{dv^*}{dt^*}+v^*=e^{it^*}$$

$$\epsilon=\frac{\frac{1}{\lambda}}{\frac{1}{\Omega}}$$

$$\boxed{\epsilon\frac{d\mathbf{v}^*}{dt^*}+A\mathbf{v}^*=\mathbf{f}^*}$$

Multiple time scales

Fast time/Slow time

Quasi-equilibrium Dynamics

Fast transient

Boundary layer Dynamics

$$\frac{D\mathbf{V}}{Dt} + 2\mathbf{\Omega} \times \mathbf{V} = -\frac{1}{\rho}\nabla_{\mathbf{z}}\mathbf{p} + \nu\nabla^{2}\mathbf{V}.$$

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} = -\nabla_{\mathbf{p}}\phi + \nu\nabla^{2}\mathbf{V}.$$

$$\epsilon\frac{D\mathbf{V}^*}{Dt^*} + \boxed{\mathbf{k} \times \mathbf{V}^* = -\nabla^*_{\mathbf{p}}\phi^*} + \frac{\epsilon}{\mathbf{Re}}\nabla^{*2}\mathbf{V}^*.$$

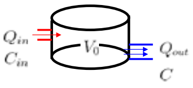
Geostrophy
Rotation Dynamics

Singular Perturbation Problems
Quasi-balanced Dynamics

Boundary Layer Dynamics
Nearly Invicid

$$\epsilon = \frac{1/f}{L/U}$$ **Rotation time scale / Advective time scale**

$$Re = \frac{L^2/\nu}{L/U}$$ **Diffusion time scale / Advective time scale**



$$\boxed{C = C_{in}\left(1-(1-\frac{\triangle Q}{V_0}t)^{Q_{in}/\triangle Q}\right)}$$

$$t = \frac{V_0}{Q_{in}}t^*, \quad C = C_{in}C^* \quad n = \frac{Q_{in}}{\triangle Q}$$

$$\boxed{\begin{aligned} \frac{dV}{dt} &= Q_{in} - Q_{out} = -\triangle Q, \\ \frac{dVC}{dt} &= Q_{in}C_{in} - Q_{out}C, \\ C(0) = 0, &\quad V(0) = V_0. \end{aligned}}$$

$$\frac{dC^*}{dt^*} = \frac{1-C^*}{1-t^*/n}$$

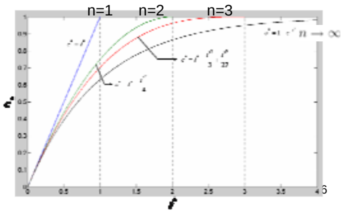
$$\triangle Q = 0,\ n \to \infty$$

$$\boxed{\begin{aligned} C^* &= 1-(1-\frac{t^*}{n})^n. \\ C^* &= 1-\exp(-t^*) \end{aligned}}$$

$$V(t) = V_0 - \triangle Qt.$$

$$V\frac{dC}{dt} = Q_{in}(C_{in} - C),$$

$$\frac{dC}{dt} = \frac{Q_{in}(C_{in} - C)}{V_0 - \triangle Qt}.$$



$$\frac{dP}{dt} = \gamma P\left(1 - \frac{P}{N}\right) - h$$

N 環境承載

$$P^* = \frac{P}{N} \quad t^* = \gamma t \quad h^* = \frac{h}{N\gamma}$$

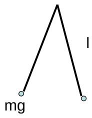
γ 成長率

h 捕獲率

$$\frac{dP^*}{dt^*} = P^*(1-P^*) - h^*$$

$$h < \frac{1/4\ N}{1/\gamma}$$

補獲率在一個生長期內不可以超過環境承載1/4



$$T \sim \sqrt{\frac{l}{g}} \qquad V \sim \frac{l}{T} \sim \sqrt{l}$$

Walking speed

$$T \sim \sqrt{\frac{I}{mgr}} \sim \sqrt{\frac{ml^2}{mgl}} \sim \sqrt{l}$$

$$T \sim \sqrt{\frac{I}{L_{max}}} \qquad T \sim \sqrt{\frac{l^3 l^2}{l^2 l}} \sim l$$

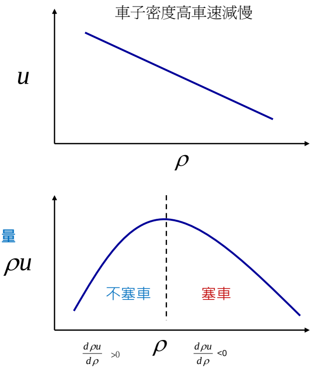
Runner speed is independent of leg length

$I = l^2 m/3$ the moment of inertia

Force is proportional to muscle area
Mass m is proportional to volume

L_{max} is the torque (force * length)

$$L_{max} \sim F_m l \sim l^2 l,$$



$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial \rho u}{\partial x} = 0 \\ u = u(\rho) \end{cases}$$

$$\frac{\partial \rho}{\partial t} + \frac{d\rho u}{d\rho}\frac{\partial \rho}{\partial x} = 0 \qquad c = \frac{d\rho u}{d\rho}$$

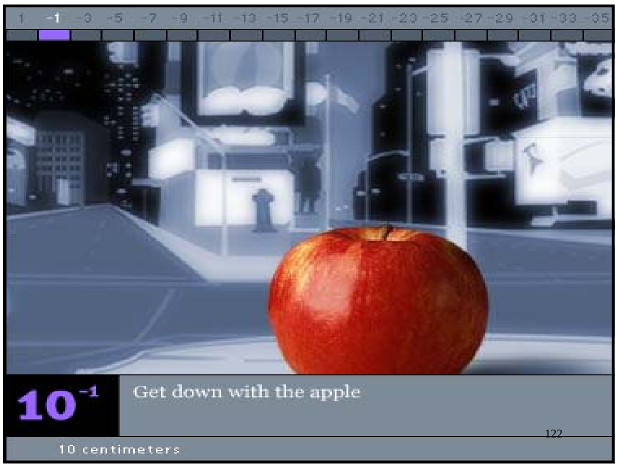
$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} = 0 \qquad \rho = f\left(x - ct\right)$$

密度向下游傳送　密度向上游傳送

A beautiful woman !

10^{1} An apple in the Big Apple

10 meters

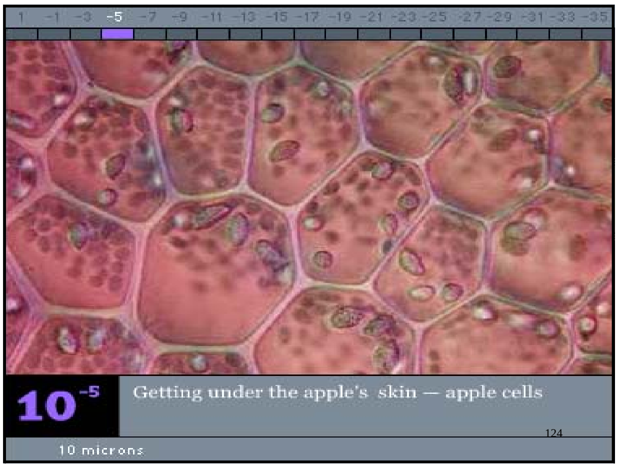


10^{-1} Get down with the apple

10 centimeters

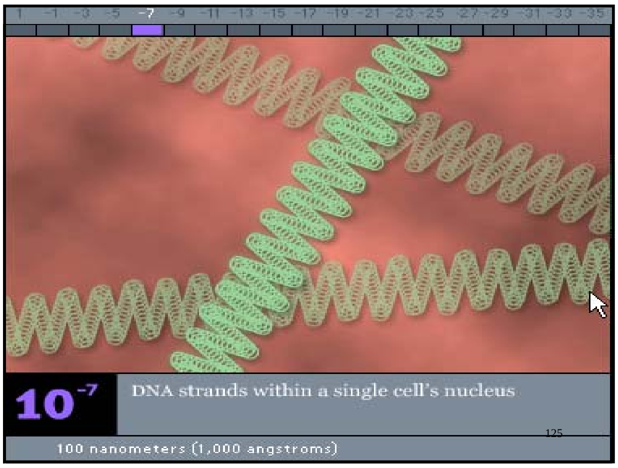
10^{-3} Touch the apple

1 millimeter



10^{-5} Getting under the apple's skin — apple cells

10 microns



10^{-7} DNA strands within a single cell's nucleus

100 nanometers (1,000 angstroms)

10^{-9} The double helix within a strand of DNA

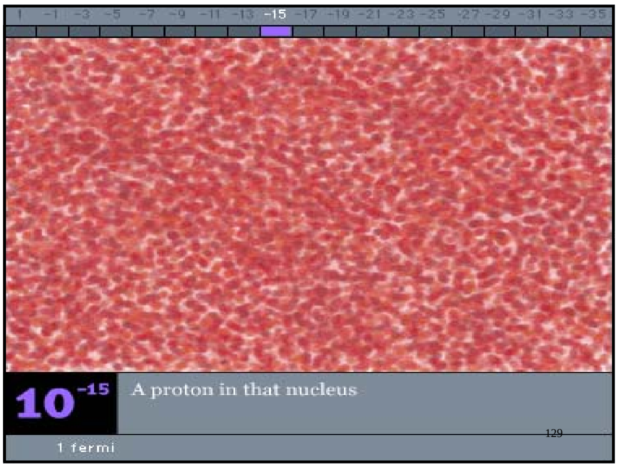
1 nanometer (10 angstroms)

10^{-11} Electrons within a carbon atom

10 picometers

10^{-13} The nucleus of that carbon atom

100 fermis

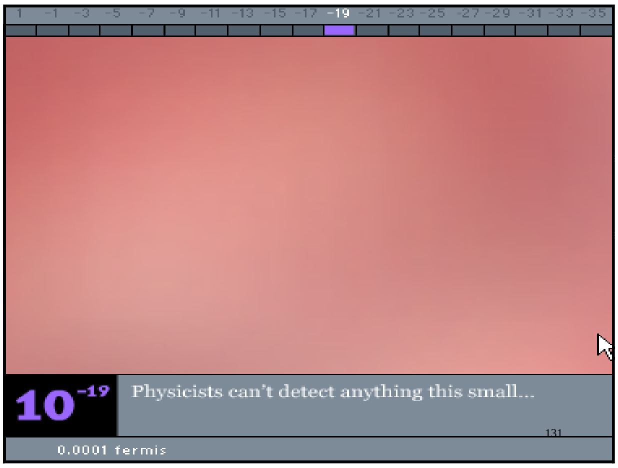


10^{-15} A proton in that nucleus

1 fermi

10^{-17} Quarks in that proton

0.01 fermis

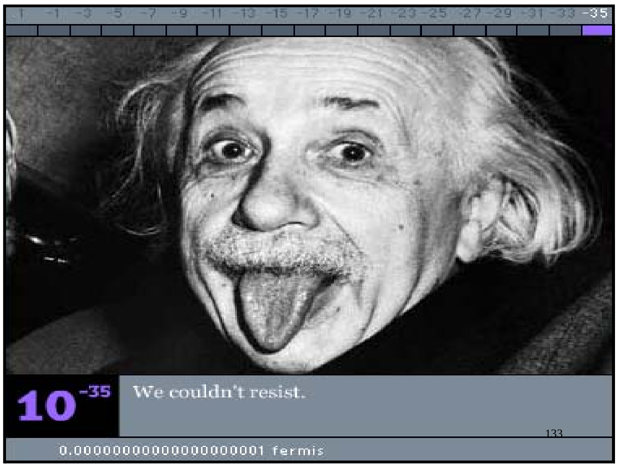


10^{-19} Physicists can't detect anything this small...

0.0001 fermis

10^{-33} We're there! A string!

0.000000000000000001 fermis



10^{-35}

We couldn't resist.

0.0000000000000000001 fermis