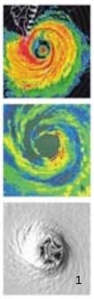
Two Dimensional Turbulence
and
Typhoon Concentric Eyewall Formation

郭鴻基
教育部國家講座教授
臺大終身職特聘教授

6/24/2009 中研院環變中心



The two-dimensional turbulence, which may be stemmed either from the fluid rotation or the fluid internal stratification, has been a paradigm for geophysical fluid dynamics. It is a remarkable fact that for any type of random initial state or external forcing, a two-dimensional fluid will rapidly organize itself into a system of coherent, interacting vortices swimming through a sea of passive filamentary structure produced from earlier vortex interactions. This discipline has also been instrumental in the development of single charge plasma physics, galaxy spiral, the Great Red spot, the ozone-hole problem and the typhoon dynamics. A brief review of 2D and 3D turbulence will be given. The typhoon merger dynamics, the eyewall rotation and mixing, the successive formation of tropical vortices and the concentric eyewall formation dynamics will be presented. The essence of the scientific computing in 2-D turbulence will be addressed. An efficient domain decomposed Chebyshev spectral method will be discussed.

1

一杯咖啡，古今往事盡付笑談中。

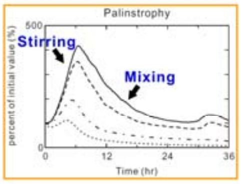
The best part of waking up, is the vortex in your cup!

$$\frac{D\theta}{Dt}=\frac{\partial\theta}{\partial t}+\vec{V}\cdot\nabla\theta=\nu\nabla^2\theta$$

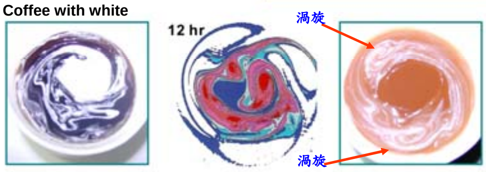
$$C=\frac{1}{2}\int\nabla\theta\cdot\nabla\theta\, dV$$

$$\frac{dC}{dt}=\int(\vec{V}\cdot\nabla\theta)\nabla^2\theta\, dV-\nu\int(\nabla^2\theta)\, dV$$

Stirring　　Mixing



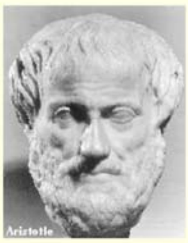
Coffee with white



2

Politics are for the moment
An equation is for eternity

Aristotle's *Meteorologia*



Aristotle (384-322 BC) was a past master at asking questions.

He wrote the first book on Meteorology, the *Μετεωρολογια* (*μετεωρον*: Something in the air)

This work dealt with the causes of various weather phenomena and with the origin of comets.

While a masterly speculator, Aristotle was a poor observer: for example, he believed that the lightning followed the thunder!

形而上學：憑藉第一原因，一切事物方能知曉，但其本身是自明的。

5

Galileo's Thermometer

Heat and Buoyancy

溫度與浮力

Potential temperature

The Galileo Thermometer is a popular modern *collectable* and an attractive decoration.

As temperature rises, the fluid expands and its density decreases.

The reduced buoyancy causes the glass baubles to sink, indicating temperature changes.

$\rho' = -\alpha T' \quad \alpha = -\frac{\partial\rho}{\partial T}$ >0 in general

6

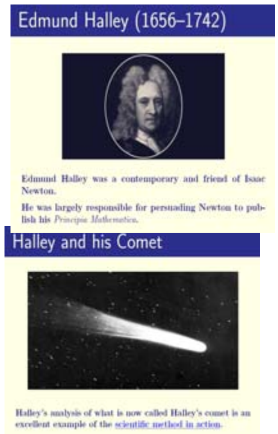
Peter Lynch

Sir Isaac Newton (1642-1727)

Isaac Newton

Principia 1687

Nature and nature's law
lay hid in night,
God said,
Let Newton be,
and all was light. A. Pope

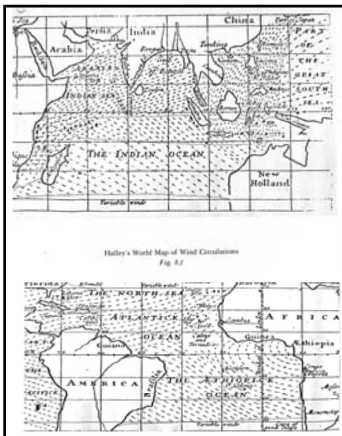


A Tricky Question??

If the astronomers can make
Accurate 76-year forecasts,
Why can't the meteorologists do
the same?

Size of the problem
大氣海洋自由度無限 ＋ 熱力學

Order versus chaos
大氣海洋的混沌、蝴蝶效應



Halley 1686

First proposed the atmospheric motion is connected with the distribution of sun heat (follow the sun in the daily scale; thus wind is westward.)

Halley 1686

Sun heat

夸父逐日
東風逐日

環流在東西方向均勻

航海時不同經度判定

Y-Z Profile

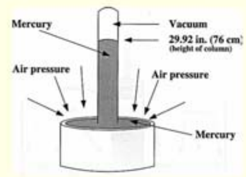
Torricelli's Barometer

Galileo's "suction vacuum"
Torricelli's air pressure "pushing"

Air weigh 1000kg/m^2

Evangelista Torricelli (1608–1647), a student of Galileo, devised the first accurate barometer.

Barometric Pressure



The relationship between the height of the mercury column and the character of the weather was soon noticed.

P = force/area = ML/t^2/L^2
= M/Lt^2
1Nt/m^2 = 1 Pascal
1 millibar = 100 Pa = 1 hPa

Material volume; Fluid parcel

Euler and Bernoulli regarded the pressure as a field acting upon an imaginary closed boundary and mechanically equivalent to the action of the fluid exterior to the boundary upon that interior to it.

Previous scientists had regarded the pressure as the weight of a unit column of fluid.

Figure 21 A cartoon by Sidney Harris, a noted chronicler of humor in science. ("Hydrodynamics" Bernoulli is our man.)

流體力學之父

Euler's Equations 18世紀

D/Dt= $\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = -\frac{1}{\rho}\nabla p - \nabla\Phi$

Momentum equation

$\frac{d\rho}{dt} + \rho\nabla\cdot\vec{v} = 0$ Continuity equation

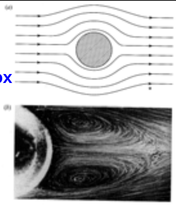
4 equations 5 unknows

$p = p(\rho)$ Constitutive equation
Equation of State

流體塊
壓力梯度力
座標轉換
質量守恒

Nonlinear Partial Differential Equations
非線性偏微分方程式 PDE

D'Alembert Paradox



1717-1783
D'Alembert Wave Solution

$t_v >> t_\nu,\quad \frac{R}{v} >> \frac{R^2}{\nu},\quad or\quad \frac{vR}{\nu} = \mathrm{Re} << 1$

Re small viscosity important
Re large viscosity unimportant

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x),\, u_t(x,0) = g(x)$$

$$u(x,t) = \frac{1}{2}[f(x-ct) + f(x+ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(\xi)d\xi$$

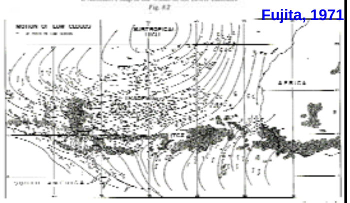
1746

D'Alembert 1746

Solar and Lunar Force

Math. Model for Atmospheric Motion in aqua-planet (Won the 1746 Berlin Academy's Award; Euler's endorsement)

Fujita, 1971



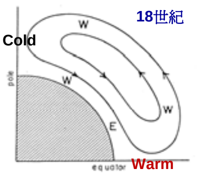


Fig. 2. A schematic view of a meridional cross-section of the general circulation as visualized by Hadley (1735). Streamlines indicate the meridional and vertical flow, while letters E and W indicate regions of easterly and westerly flow.

Hadley (1685-1758)

Distribution of **sun** heating (north and south; seasonal scale)

Earth **rotation** (conservation of angular momentum) 角動量守恒

Energy transport and balance

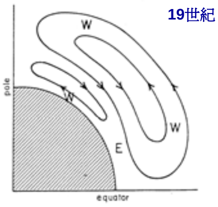


Fig. 3. The same as Fig. 2, but for the general circulation as visualized by Thomson (1857) and Ferrel (1859).

Ferrel (1859), Thomson (1857)

Centrifugal force 離心力
Thermal wind balance 熱力風平衡
Surface friction

Coriolis 1835

Pierre-Simon Laplace

1740-1827

Observational net work 1800-1815
氣象觀測網

Tidal equations

Laplacian $\nabla^2\psi = f$

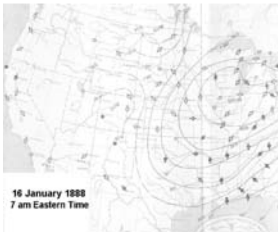
Hydrostatic approximation $\frac{\partial p}{\partial z} = -\rho g$

Adiabatic sound speed $\left(\frac{\partial p}{\partial \rho}\right)_\theta = c_s^2$
絕熱聲速

Laplace Transform

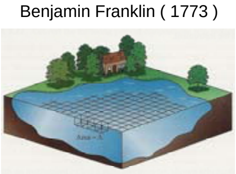
Astronomer

Black hole and gravitational collapse

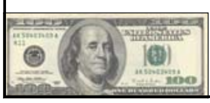


Estimate Avogadro's Number

Benjamin Franklin (1773)



Oil spreads on water
⟶ molecular size
⟶ Avogadro's number



(1) Molecular size

$l = \frac{V}{A} = \frac{4.9\ cm^3}{2.0\times10^7\ cm^2} = 2.4\times10^{-7}\ cm$

(2) Number of molecules

$N = \frac{A}{l^2} = \frac{2.0\times10^7\ cm^2}{(2.4\times10^{-7}\ cm)^2} = 3.5\times10^{20}$ molecules

(3) Mass of the oil

$m = V\times D = 4.9\ cm^3\times0.95\frac{g}{cm^3} = 4.7\ g$

(4) Number of moles of oil

Moles of oil $= \frac{4.7\ g}{200\ g/mol} = 0.024$ mol

(5) Avogadro's number

Avogadro's number $= \frac{3.5\times10^{20}\ \text{molecules}}{0.024\ \text{mol}} = 1.5\times10^{22}$

Now we know: $N_A = 6.022142\times10^{23}$ /mol

Ideal Gas Law **Equation of State 理想氣體方程**

- **1662, Boyle law, PV = c when T = c.**
- **1787, Charles law, V/T = c when P = c.**
- **1803, Gay-Lussac law, P/T = c when V = c.**
- **1811, Avagardro, 1 mole gas is 22.4 l in volume.**

Universal Gas Constant
R* = 8314.3 J /(deg^{-1} kmol)

P = f(V, T)
V = h(P, T)
溫度可以改變壓力或體積，熱可以做功

PV = n R* T
PV = m/M R* T P = m/V R*/M T
P = ρ R T, R = R*/M
R_d = 287 J/deg.kg (R*/M_d)
R_v = 461 J/deg.kg (R*/M_v)

Development of Thermodynamics 熱力學
19 century
第一定律　能量作功，能量守恆
First law: Energy is what makes it go and energy is conserved.
△Q = △U + WORK
Second law: Entropy tells it where to go!
第二定律　時間之矢，自然單向
Joule, Rudolf Clausius, Lord Kelvin and others
宏觀　微觀
Macro --- Micro
Classical and Statistical Thermodynamics
統計熱力學
Ludwig Boltzmann, 1844-1906, whose work led to an understanding of the macroscopic world on the basis of molecular dynamics.
S = k Log W

雲微物理
Precipitation

Enthalpy
Entropy
Gibbs Free energy

Fourier 1768-1830

1824 Why the earth not heating up when receive sun energy continuously?
Heat emission or diffusion (by IR)
His calculations showed a very cold surface (No **green house effect**)

Tyndall 1864
H_2O is a green house gas

Arrhenius 1896
CO_2 green house effect, but were dismissed by scientists [WHY??]

Lord Rayleigh 1881
Rayleigh scattering
Why the sky is blue??

Chandrasekhara Raman 1928
Raman scattering
Color of the sea

Fovell, Taipei, 2008

Jame Chappuis 1881
Chappuis band
dismisseed by scientist

Edward Olson Hulburt 1956
Twilight blue
O_3 layer

20

The profound study of nature is the most fertile source of mathematical discoveries.

$$f(x) = \sum \hat{f}_k e^{ikx}$$

$$\hat{f}_k = \frac{1}{2\pi}\int_0^{2\pi} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}_k e^{ikx} dx$$

$$\hat{f}_k = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

Fourier, Jean Baptiste Joseph
1768-1830

1807 at age 39; argued with Lagrange and Laplace on the representation of a triangle wave with cosine and sine functions.

Heat emission or diffusion (by IR)

His calculations showed **a very cold surface** (No green house effect)

f(x) does not have to be analytical;
f(x) does not have to be periodic.

21

Planck, Unwilling Revolutionary: the idea of quantization

1900

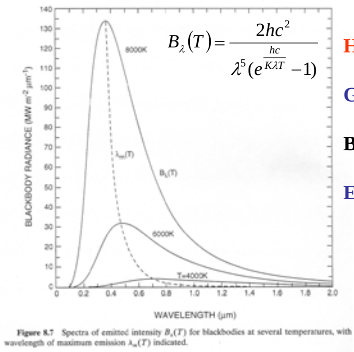


Figure 8.7 Spectra of emitted intensity $B_\lambda(T)$ for blackbodies at several temperatures, with wavelength of maximum emission $\lambda_m(T)$ indicated.

Hall of Fame in Science

Gravitational Law

Blackbody Radiation

E= MC^2

1858-1947

22

Multiple Scale Interactions

多重尺度交互作用
Current vortices

Leonardo da Vinci 1452-1519

*Observe the movement of the surface of the water which resembles that of hair which has **two motions**, of which one depends on the weight of the hair and the other on the direction of the **curls**. Thus water forms eddying whirlpools of which one part depends on the predominant **current** and the other on the incidental motion and the return flow."*

Brief History of Fluid Dynamics

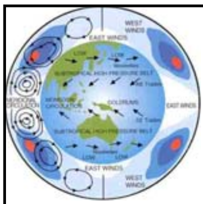
Newton	**1700s**	Viscosity Law of motion for a particle	
Euler / **D. Bernoulli**	**1750s**	Equations for inviscid flow, Law of motion applied to fluids	18世紀
Navier / **Stokes**	**1827** / **1845**	Equations for **viscous** fluid flow	
Boussinesq	**1877**	Turbulent mixing, **eddy viscosity**	19世紀
Reynolds	**1880**	Transition to turbulence, Reynolds Number, **Reynolds stress**	
G. I. Taylor	**1915-1970**	Geophysical flows, rotating flows	20世紀
Prandtle	**1904**	Boundary Layer	

$$\tau_{ij} = -\rho_0 \overline{u_i' u_j'}$$

Reynolds stress

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial \bar{p}}{\partial x_i} + \frac{g}{\theta_0}\bar{\theta}\delta_{i3} - \frac{\partial}{\partial x_j}\left(\overline{u_i' u_j'}\right) + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2}$$

24



大氣受熱力驅使， 熱力受大氣控制

Atmosphere is thermally forced, the heating mechanism are very complicated and motion dependent.

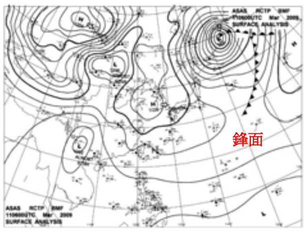
20 Century

3 symmetric cells

多重尺度擾動--平均流交互作用

The dynamical response to the heating is also complicated because of eddies-mean flow interaction. The eddies are neither purely random nor purely regular.

25



$\overline{v'T'} > 0$

Dove **1837** weather is contributing; but was dismissed.

Bigelow 1902
Defant 1921
heat down-gradient transport

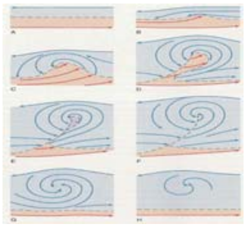
$\overline{u'v'} > 0$

Jefferey 1926 momentum up-gradient transport

Confirmed by **Starr** and **J. Bjerkness** 1948
Starr: Physics of negative viscosity phenomena

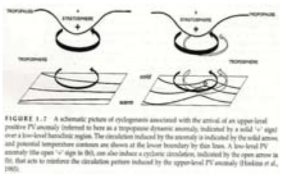
Origin of the eddies?? Baroclinic Instability
Eddy 1950, Charney 1959

26



Dove **1837** weather is contributing; but was dismissed.

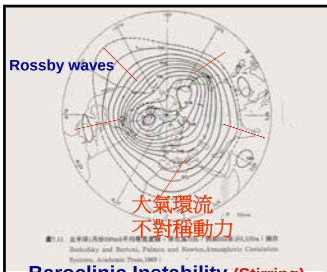
V. Bjerkness 1937
Symmetric component is unstable to the asymmetries .



Baroclinic Instability
Eady 1950, Charney 1959

高層低壓槽
和低層低壓
垂直交互作用

27



Baroclinic Instability (Stirring)

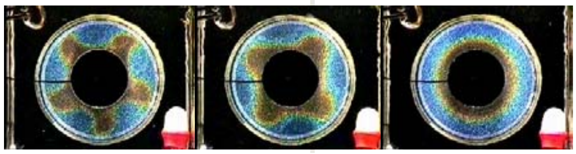
Wave-mean flow interaction

$\overline{v'T'} > 0$

Heat down-gradient

$\overline{u'v'} > 0$

Momentum up-gradient

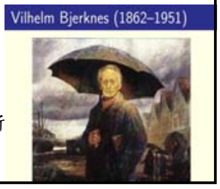


The Ultimate Problem in Meteorology Bjerknes 1911
氣象的終極問題

I The Present state of the atmosphere must be characterized as accurately as possible. 正確的觀測大氣現狀
[多重時空尺度]

II The intrinsic laws, according to which the subsequent states develop out of the preceding ones, must be known.
正確的大氣運作規律

Vilhelm Bjerknes (1862–1951)



Numerical Weather Prediction 數值天氣預報
[第一部電腦ENIAC, EBV model, 1950]
The Observation component 觀測
The diagnostic or analysis component 診斷分析
The prognostic component 預報

科氏力(18 19)

Momentum Conservation (18)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \nu\nabla^2 w$$

Mass conservation (18)

$$\frac{\partial \rho}{\partial t} + \frac{\partial u\rho}{\partial x} + \frac{\partial v\rho}{\partial y} + \frac{\partial w\rho}{\partial z} = 0$$

Energy conservation (19)

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = Q$$

Radiation
大氣輻射
(19,20)
Moisture
Latent heat

Equation of State(17,18,19)

$$p = \rho R_a T, \quad \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_a}{C_p}}$$

(19)

雲物理
(19,20)

問蒼茫大氣，誰主浮沈？
質量、動量、能量與大氣狀態方程式

30

Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

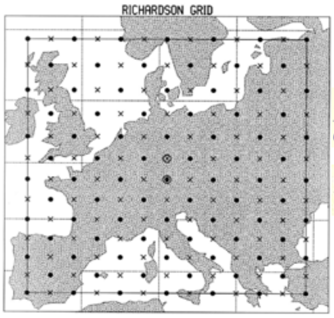
His 'forecast' was a catastrophic failure:

$\Delta p = 145$ hPa in 6 hours

His method was unimpeachable.

So, *what went wrong?*

Peter Lynch



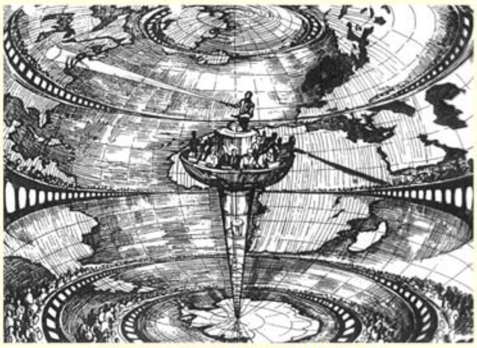
$$\frac{df}{dx} \rightarrow \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$$\frac{dQ}{dt} \rightarrow \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

13x13=169個ODE

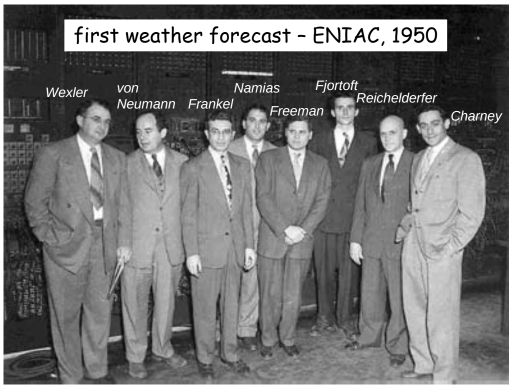
169 自由度

Richardson's Dream



Richardson's Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, ECMWF, 1984

64,000 Computers: The first Massively Parallel Processor



In front of the Eniac, Aberdeen Proving Ground, April 4, 1950, on the occasion of the first numerical weather computations carried out with the aid of a high-speed computer.

The ENIAC Electronic Numerical Integrator and Computer

18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

No high-level language
Assembly language

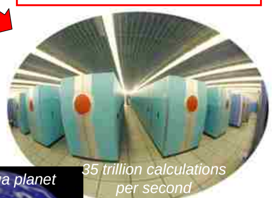
500 Flops
Function Table 0.001 s

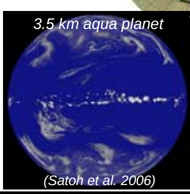
3,700,000,000 times slower than current day large computer

第一部電腦　氣象預報

ENIAC – late 40s

Earth Simulator -- 2002





NASDA, JAERI, JAMSTEC

mid 50
- Observational study of eddy transport UCLA,MIT
- Haboratory study U of Chicago, MIT
- NWP －Charney, Fjortoft and Neumann (1950) E Barotropic
  Charney and Phillips (1953) 2 level QG model

Phillips' numerical experiment

Phillips (1956)
two-level
Quasi-Geostrophic model

Corresponding to (IV.3.1) for ℓ = 1, 2 and (IV.3.2) for ℓ = 1 but with a ew vertical index as shown in Fig. IV.8, the equations of his model are

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_{g1}\cdot\nabla\right)(\zeta_{g1}+f)-f_0\frac{\omega_2}{\Delta p}=A\nabla^2\zeta_{g1}, \quad \text{(IV.4.1)}$$

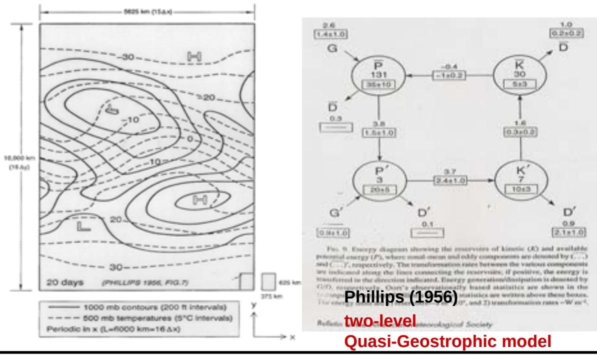
$$\left(\frac{\partial}{\partial t}+\mathbf{v}_{g3}\cdot\nabla\right)(\zeta_{g3}+f)+f_0\frac{\omega_2}{\Delta p}=A\nabla^2\zeta_{g3}-k\zeta_{g4}, \quad \text{(IV.4.2)}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_{g2}\cdot\nabla\right)(\psi_1-\psi_3)-\frac{1}{\mu^2}\frac{f_0}{\Delta p}\omega_2=A\nabla^2(\psi_1-\psi_3)+\frac{R}{c_p}\frac{1}{f_0}Q, \quad \text{(IV.4.3)}$$

to study meteorology as an experimental science

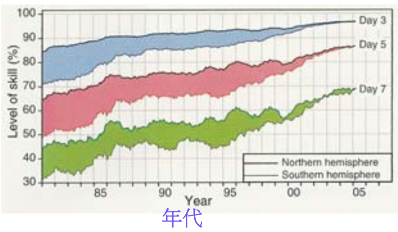
37

To study meteorology as an experimental science



Phillips (1956)
two-level
Quasi-Geostrophic model

南北半球 對於3，5，7天之預報能力隨時間的進展



預報能力指標

年代

南北差異日漸減少主要是由於近年來衛星觀測以及資料同化技術日漸成熟

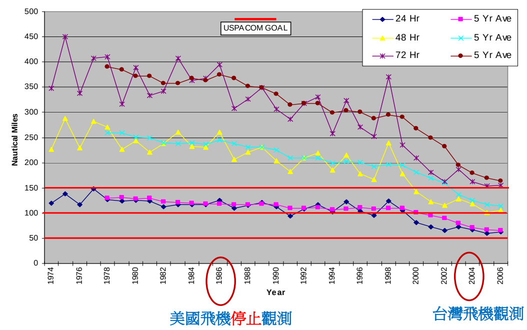
7天預報一年進步約1.5%，3天預報一年進步0.3%

39

West Pac Track Errors

Edward Fukada
JTWC

Error cut in half since 1990 **Total Forecast Error**

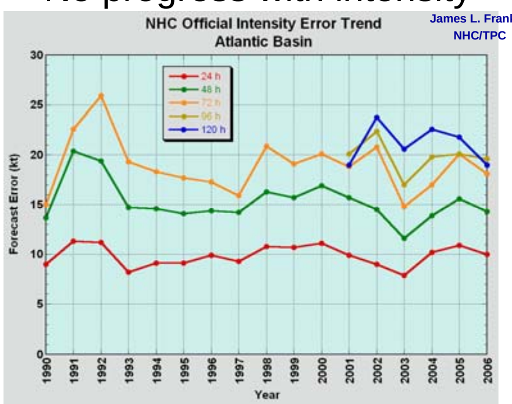


美國飛機停止觀測

台灣飛機觀測

40

No progress with intensity

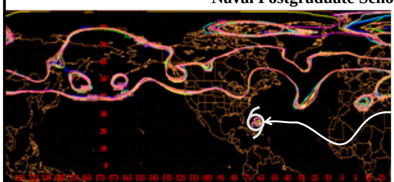


James L. Franklin
NHC/TPC

41

The Downstream Influences of the Extratropical Transition of Tropical Cyclones

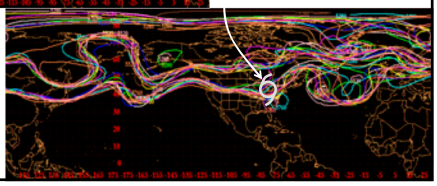
Patrick Harr
Naval Postgraduate School



0000 UTC 16 Sep 2003
GFS Ensembles +00

Hurricane Isabel

GFS 500 hPa Ensembles
+108 h
VT 1200 UTC 20 Sep 03



Acknowledgment: Office of Naval Research, Marine Meteorology Program

颱風潛熱與其它能量的比較

賀伯颱風的全台灣平均總雨量爲400*mm*

$400\ mm = 0.4\ m$

$0.4\ m * 1000\ kg\ m^{-3} * 2.5\times10^6\ J\ kg^{-1}$

$= 10^9\ J\ m^2$

$10^9\ J\ m^2 * 3.5\times10^{10}\ m^2$

$= 3.5\times10^{19}\ J \sim 10^{20} J$

$^1_0n + ^{235}_{92}U \rightarrow ^{142}_{56}Ba + ^{91}_{36}Kr + 3^1_0n$

$1.68 * m * 10^{13} J/mol$

$\Rightarrow 1.46\times10^6\ kg\ U^{235}\ (6*10^6\ mol)$

能量估計値		備註
賀伯颱風降雨總潛熱能量	10^{20} J	可使台灣整層大氣增溫100度
台灣一年用電量	$5*10^{17}$ J	需數百年用電量才相當
全世界核子彈爆炸釋放能量	$2*10^{19}$ ~$2*10^{20}$ J	與賀伯颱風同等級
核戰後燃燒釋放能量	$2*10^{20}$ J	與賀伯颱風同等級
地球一天接受的太陽能量	$1.5*10^{22}$ J	數百個賀伯颱風
Tunguska隕石撞地球 （西元1908年，西伯利亞）	10^{16} J	賀伯颱風的萬分之一
火流星撞地球（恐龍滅絕？）	$4*10^{23}$ J	數千個賀伯颱風

43

Quote from 1970 Tropical Conference by Bill Gray:

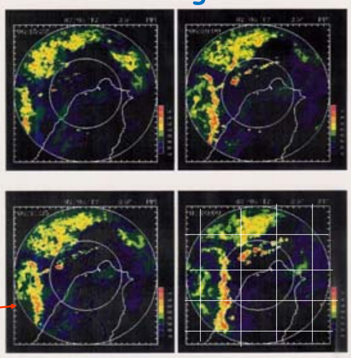
“If little cumulus weren’t important, God wouldn’t have made so may of them”

44

Mesoscale Organization

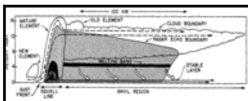
狂風不終朝
暴雨不終日
孰為此者
天地
天地尚不能久
而況人乎

颱線
Squall line



1987 年 5 月 17 日鋒前颱線之雷達回波圖

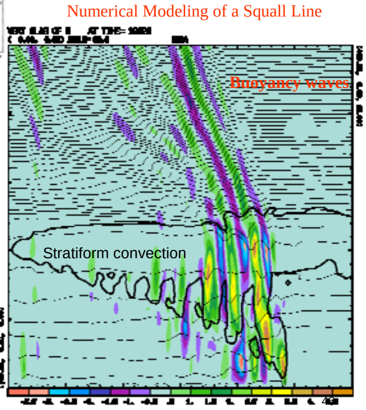
模式解析？



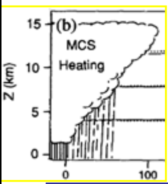
(Houze 1977)

Mesoscale Organization

Numerical Modeling of a Squall Line



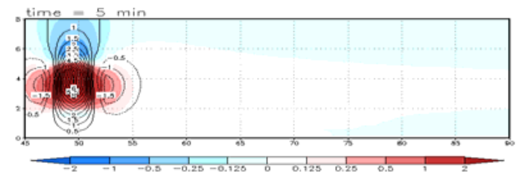
Coupling Between Convection and Its Environment

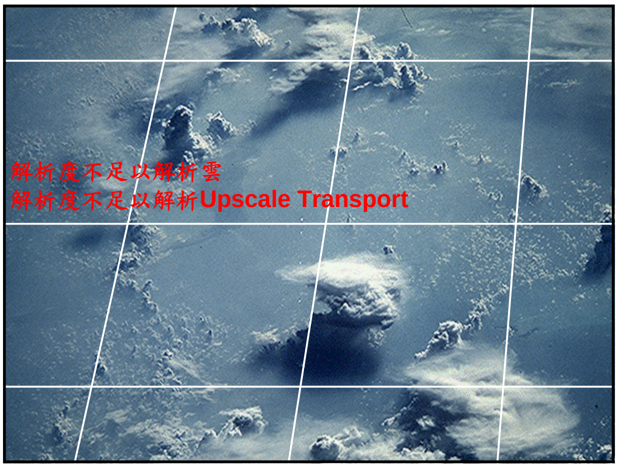


“Gregarious Tropical Convection”
(Mapes 1993; Nicholls et al. 1991)

Dispersion of gravity waves from MCS heating; low-level ascent associated with “stratiform” component can lead to new convective growth near previous cells

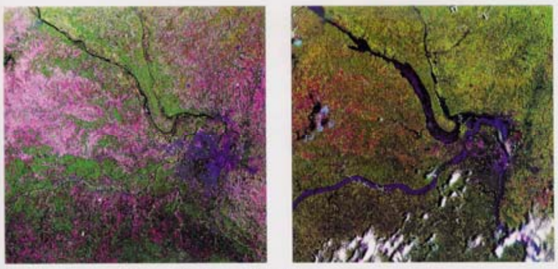
Simulation of gravity waves from a “top-heavy” heat source, Fovell, UCLA





風雨之不時，是無世而不常有之。 荀子天論

1988 年際變化 季節預報 1993



Heavy rains in the summer of 1993 produced floods along most of the Mississippi River in the central United States, as shown in these Earth satellite photographs of St. Louis, Missouri on July 4, 1988 (left) and July 18, 1993 (right). Extreme climatic events may be increasing in frequency as a consequence of added radiation absorbing gases in the atmosphere.

49

The Atmosphere is Moist 濕的大氣

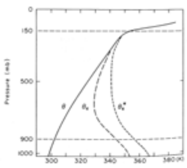
Water vapor is an efficient absorber and emitter of Long-wave radiation. [Green House Effect.]
溫室效應 大氣輻射

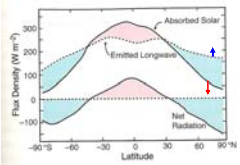
Water vapor stores energy in the form of "latent hea
[Evaporative cooling of surface.]
地球表面蒸發冷卻

Water vapor can condense release latent heat.
[A variety of cloud and associated processes.]
雲過程

50

Conditional unstable atm.

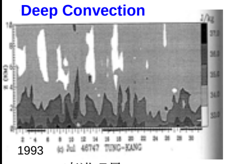




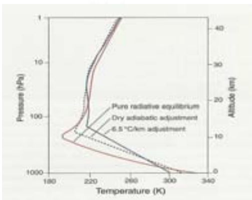
Solar + IR
(Green house gas)

Finite amplitude forcing
Rooted in BL
High h* for instability

Deep Convection



1993 東港 7月

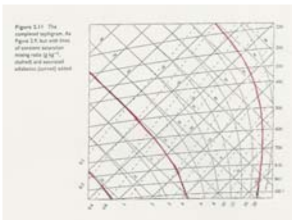


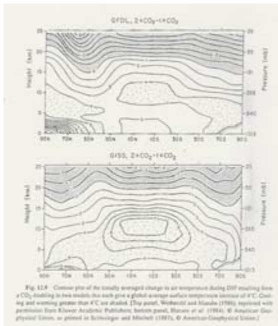
Radiative Convective Adjustment
水文循環

1K/day cooling
1m/year precipitation

51

大氣浮力 阿基米得原理
濕絕熱降溫率 Moist adiabatic lapse rate





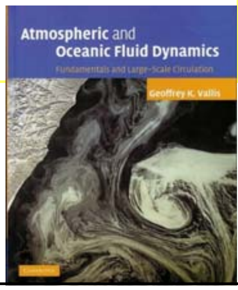
52

Harteman, Climate text book

20th Century

Geophysical Fluid Dynamics (GFD)
Atmospheric Oceanic Fluid Dynamics (AOFD)
is for those interested in doing research in the physics, chemistry, and/or biology of Earth fluid environment.

Fig. 9.2 Karman vortex streets in (a) the laboratory, for water flowing past a cylinder [From M. Van Dyke, An Album of Fluid Motion, Parabolic Press, Stanford, Calif. (1982) p. 56.], and (b) in the atmosphere, for a cumulus-topped boundary layer flowing past an island [NASA MODIS imagery].



熱力學 ＋ 流體力學

Euler 1755

$$\frac{d}{dt}\int_{v_m} \rho\vec{v}\,dv = -\int_{\partial v_m} p\,d\vec{s}$$

$$\int_{v_m} \rho\frac{d\vec{v}}{dt}\,dv = -\int_{v_m} \nabla p\,dv$$

$$\rho\frac{d\vec{v}}{dt} = -\nabla p$$

Lagrange 1781

$$\frac{\partial \vec{u}}{\partial t} + \vec{\zeta}\times\vec{u} = -\frac{1}{\rho}\nabla p - \nabla K - \nabla\Phi$$

Rotation , Vortex

Lorentz Force Law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$$\mathbf{F} = q(-\nabla V + \mathbf{v}\times\mathbf{B})$$

Helmholtz 1858

$$\frac{\partial\vec{\zeta}}{\partial t} + \vec{v}\cdot\nabla\vec{\zeta} + \vec{\zeta}\,\nabla\cdot\vec{v} = \vec{\zeta}\cdot\nabla\vec{v} + \vec{B}$$

$$\vec{B} = \nabla\times(-\frac{1}{\rho}\nabla p)$$

19世紀煙圈表演

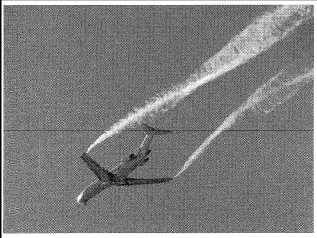


Fig. 8.9. Vortices trailing form the wingtips of a Boeing 727. Figure courtesy of NASA.

Wake Turbulence

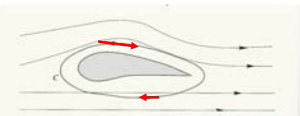


Fig. 8.10. Sketch of the flow along an airfoil. The wing is shown in grey, the contour C is shown by the thick solid line.

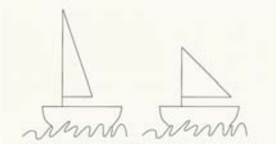
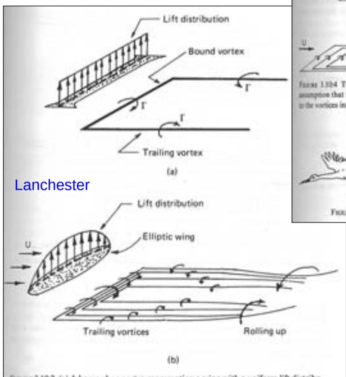


Fig. 8.12. Two boats carrying sails with very different aspect ratios.

Biomath



Lanchester

FIGURE 3.10:2 (a) A horse-shoe vortex representing a wing with a uniform lift distribution. (b) Lift distribution on an elliptic wing.

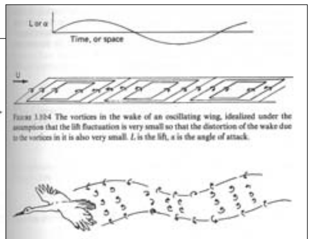
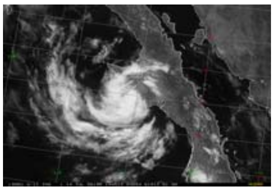
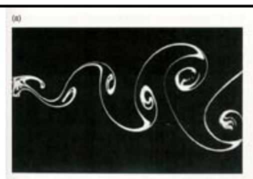
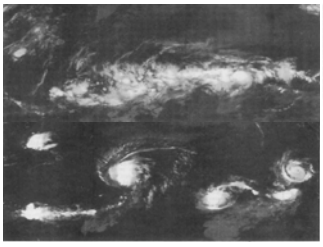


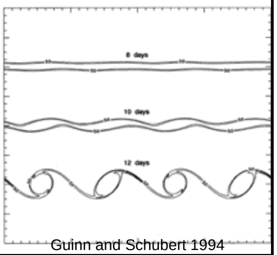
FIGURE 3.10:5 The vortex wake behind a stork in level flight.

Y. C. Fung









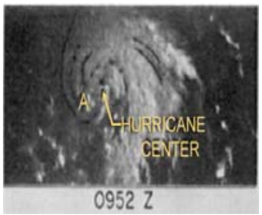
Guinn and Schubert 1994

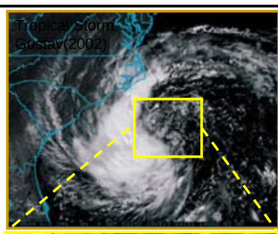
Hendricks et al. (2004)

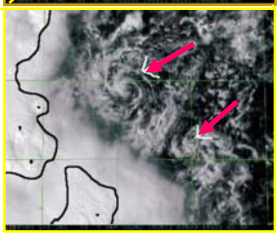
Vortical Hot Towers

- **10km in diameter on average**

1969/08/16 Debbie
A : increase its diameters from 50 to 100 km in ~2 hours.
C, D : grew into circular clouds with about a 20 km diameter in less than 15 min.







$$\frac{DP}{Dt}=\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}+v\frac{\partial P}{\partial y}+w\frac{\partial P}{\partial z} \qquad P=\frac{\vec{\zeta}\cdot\nabla\theta}{\rho}$$

Potential Vorticity

$\frac{\partial}{\partial z}$ $\quad P\to\zeta$

$$\frac{D\zeta}{Dt}=\frac{\partial \zeta}{\partial t}+u\frac{\partial \zeta}{\partial x}+v\frac{\partial \zeta}{\partial y}$$

Barotropic Vorticity

$$\frac{D\zeta_g}{Dt}=\frac{\partial \zeta_g}{\partial t}+u_g\frac{\partial \zeta_g}{\partial x}+v_g\frac{\partial \zeta_g}{\partial y}$$

$$\zeta_g=\nabla^2\psi \quad u_g=-\frac{\partial\psi}{\partial y} \quad v_g=\frac{\partial\psi}{\partial x}$$

Quasi-Geostrophic Vorticity

(Strong Rotation)

$\nabla\cdot\vec{V}=0$

$$\frac{\partial \zeta}{\partial t}+\frac{\partial(\psi,\zeta)}{\partial(x,y)}=\nu\nabla^2\zeta$$

Nondivergent Barotropic Equation

2D Turbulence

Stratification and/or Rotation
Vortex Waves Turbulence

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}=\nu\nabla^2\zeta$$

$$u=-\frac{\partial\psi}{\partial y},\quad v=\frac{\partial\psi}{\partial x}.$$

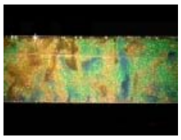
$$\frac{\partial \zeta}{\partial t}+\frac{\partial(\psi,\zeta)}{\partial(x,y)}=\nu\nabla^2\zeta$$

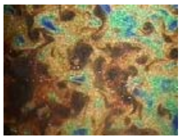
旋轉
Rotation

Coriolis Force

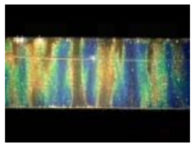
Non-inertial Frame

3D

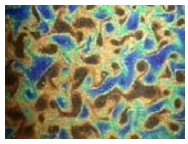




2D (strong rotation)



Taylor columns Vortex Tubes



Vortices with sharp edge

62

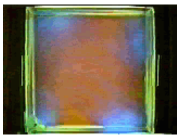
Kyoto Univ. GFD group

Waves with zero potential vorticity

Non-rotation

rotation

rotation

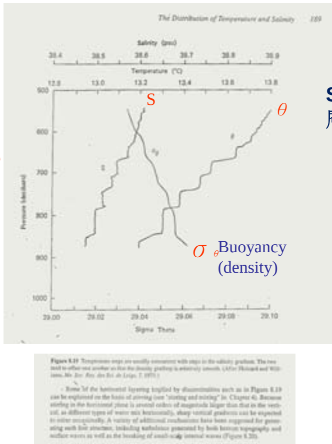


Gravity waves

Kelvin Waves
Edge waves

63

Ocean Spice



Buoyancy
(density)

Stratification
層化

64

Kelvin-Helmholtz Instability

Turbulent Mixing 晴空亂流

Intermediate
wavelength

Multiple Scale Interactions in Vortex

Wave mean flow interaction in stable stratified fluid
Turbulent feed back to the vortex mean flow

2D turbulence

66

$$\frac{d}{dt}\int E(k)dk = 0. \qquad \frac{d}{dt}\left(\int k^2E(k)dk\right) = \frac{d}{dt}\int Z(k)dk = 0$$

$$\frac{d}{dt}\left(\int (k-k_1)^2E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\int k^2E(k)dk + k_1^2\int E(k)dk - 2k_1\int kE(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\frac{\int kE(k)dk}{\int E(k)dk}\right) < 0,$$

Kinetic energy moves toward large scales

$$\frac{d}{dt}\left(\int (k^2-k_1^2)^2E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\int k^2Z(k)dk + k_1^4\int E(k)dk - 2k_1^2\int k^2E(k)dk\right) > 0$$

$$\frac{d}{dt}\left(\frac{\int k^2Z(k)dk}{\int Z(k)dk}\right) > 0,$$

Enstrophy moves toward small scales

67

Non-divergent barotropic model (Nearly Inviscid Fluid)

$$\frac{\partial}{\partial t}\zeta + J(\psi,\zeta) = \upsilon\nabla^2\zeta \qquad \boxed{\nabla^2\psi = \zeta}$$

The energy and enstrophy relations

$$\frac{dE}{dt} = -2\upsilon Z \qquad E = \iint \frac{1}{2}(u^2+v^2)\,dx\,dy \quad \text{kinetic energy}$$

$$Z = \iint \frac{1}{2}\zeta^2\,dx\,dy \quad \text{enstrophy}$$

$$\frac{dZ}{dt} = -2\upsilon P \qquad P = \iint \frac{1}{2}\nabla\zeta\cdot\nabla\zeta\,dx\,dy \quad \text{palinstrophy}$$

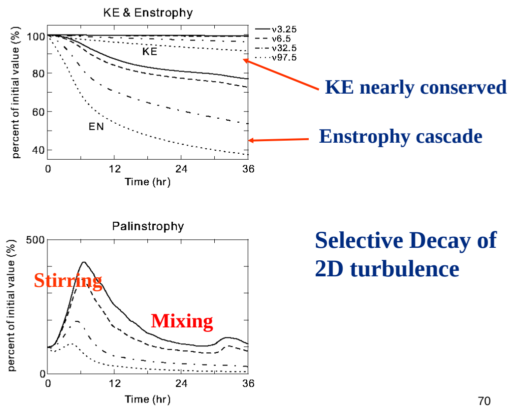
Batchelor 1969

68

Small viscosity led to large palinstrophy and the large enstrophy cascade

Stirring

69



Selective Decay of 2D turbulence

70

E ~ p'[2] / L[2] (KE) geostrophy

Z ~ p'[2] / L[4] (Enstrophy)

KE nearly conserved L ~ p'

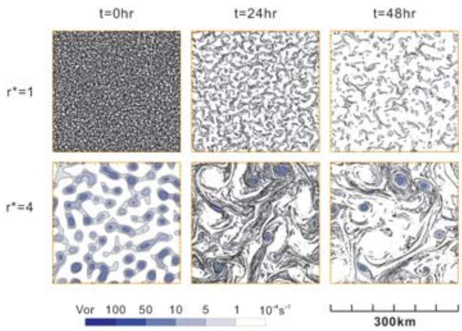
Entrophy cascade L↑ (L increase Z decrease)

Selective Decay of 2D turbulence

The vortices become, on the average, larger, stronger, and fewer.

Merger and Axisymmetrization Dynamics

71



小尺度變大尺度

Fewer and stronger vortices !!!

Coherent structure with filamentations in 2-D turbulence

72

Weiss(1981,1991), Rozoff et al. (2004)

$$\frac{D}{Dt}(\nabla\zeta) = -J(\nabla\psi,\zeta)$$

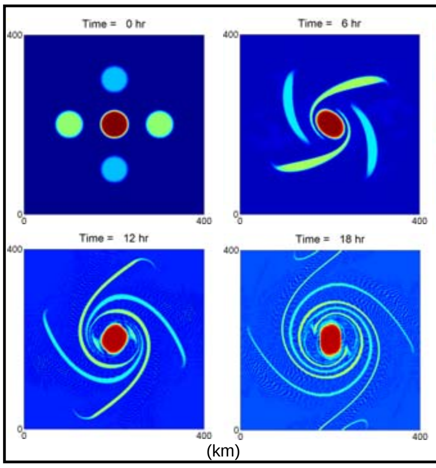
$$\rightarrow \nabla\zeta(t) \propto \exp(\lambda t) \quad \lambda = \pm\frac{1}{2}\sqrt{Q} = \pm\frac{1}{2}\sqrt{S_1^{\,2}+S_2^{\,2}-\zeta^2}$$

$$S_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \quad \text{(stretch deformation)}$$

$$S_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad \text{(shear deformation)}$$

Q > 0 (strain dominates)
→vorticity gradient will be stretched
Q < 0 (vorticity dominates)
→vortex is stable (survival of eyewall meso-vortices)

73

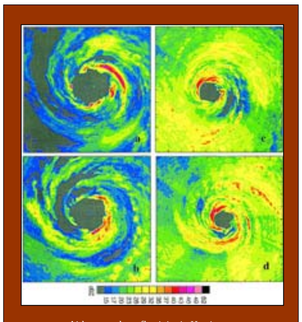


Conserves the angular impulse
$\iint (x^2+y^2)\zeta\, dxdy$

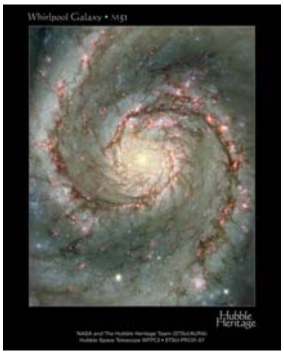
Melander et al. 1986

74

Spiral Band in Hurricane and Galaxy



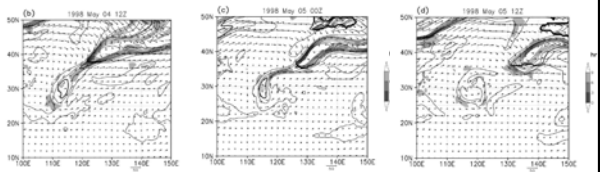
Airborne-radar reflectivity in Hurricanes
Guillermo (1997) (left panels) and Bret (1999) (right panels).



Kossin and Schubert 2001

75

Filamentation Time Diagnosis of Thinning Troughs and Cutoff Lows



$$\tau_{fil} = 2\left\{\delta + \left[S_n^2 + S_s^2 - \left(\zeta - \frac{2u\tan\phi}{a}\right)^2\right]^{1/2}\right\}^{-1} \quad \text{for } S_n^2 + S_s^2 > \left(\zeta - \frac{2u\tan\phi}{a}\right)^2$$

Tsai et al. 2009

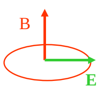
76

Electron density redistribution in experimental plasma physics

single sign charge
+
axial magnetic field confinement

$\mathrm{E} = -\nabla\psi$

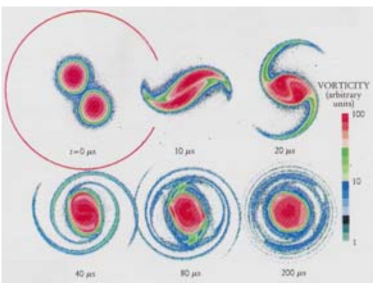
$\nabla\cdot\mathrm{E} = -\nabla^2\psi = \frac{\rho}{\epsilon}$



$\vec{E}\times\vec{B}$ drift

Coriolis force

Axisymmetrization 軸對稱化



Core is protected, thin filaments from edges

77

Huang and Robinson 1998

Forcing added

Inverse energy cascade to zonal Harmonics

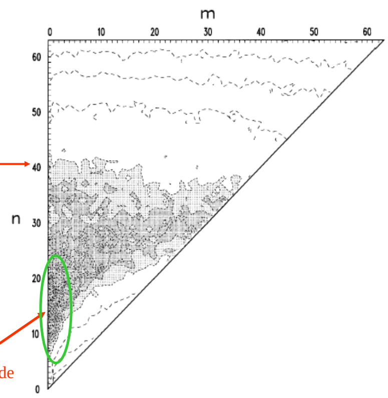


FIG. 2. Energy spectrum $E(m, n)$ of an ensemble mean at day 80 of 10 decaying turbulence experiments. The magnitude of the spectrum is normalized by the maximum value on the map. Contour levels are 0.0001, 0.001, 0.01, 0.1–0.9 with increment 0.1. Area with $E(m, n) > 0.1$ is lightly shaded, $E(m, n) > 0.2$ heavily shaded.

Jupiter Rotational period 9.84hr **Huang and Robinson 1998**

The Great Red Spot

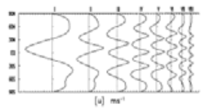
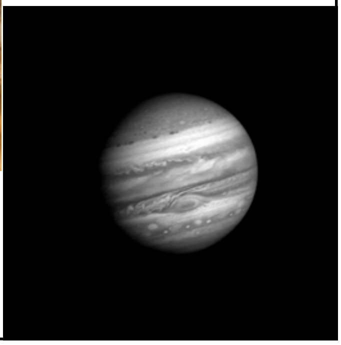


Fig. 4. Time-mean zonal-mean zonal wind profiles for cases I-VIII in Table 1 (the eight open circles in Fig. 3). Each grid on the abscissa represents 1 m s^{-1}.



Bowmen and Mangus (1993)

臭氧洞衛星觀測

Observations of deformation and mixing of the total ozone field in the Antarctic polar vortex

核心空氣被渦旋鎖住

細絲帶

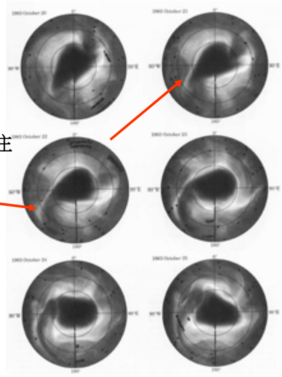
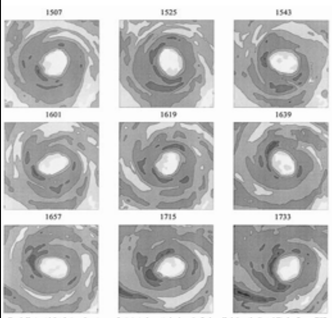


Fig.1: Daily TOMS images of total ozone in the Southern Hemisphere for six consecutive days in October 1983. Latitude circles are drawn at 40˚,60˚, and 80 ˚S. The outermost latitude is 20 ˚S.

80

賀伯颱風的橢圓形眼 Kuo et al. 1999 JAS



144 min rotation period

Lamb, 1932

Kirchhoff vortex (nonlinear)

$$\zeta \frac{ab}{(a+b)^2} = \omega$$

rotating period **P** = $\frac{2\pi}{\zeta}\frac{(a+b)^2}{ab}$

Kelvin PV wave (linear)

$$c = V_{max}(1-\frac{1}{m}) \qquad m=2$$

$$\sim \frac{2\pi}{\omega} = \frac{2\pi}{2\frac{V_{max}}{r}} * 4 = \frac{2\pi}{\zeta} * 4$$

Vortex Rossby Waves

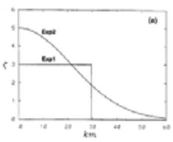
Deep convections rotation 144 min

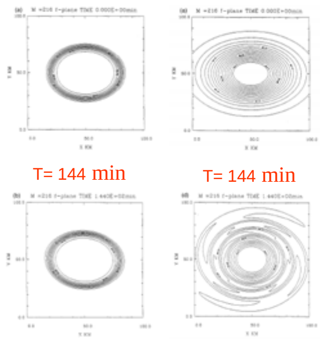
81

Eye rotation is nonlinear $J(\psi,\zeta)$ >> $J(\psi,\zeta)$

EXP 1 EXP 2

T= 0 min T= 0 min



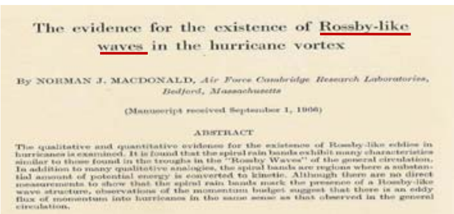


T= 144 min T= 144 min

$$\frac{\partial}{\partial t}\zeta + J(\psi,\zeta) = 0$$

Coherent vortex
Dispersion resistant
Vorticity Dynamics
Nonlinear Dynamics

1968
Tellus



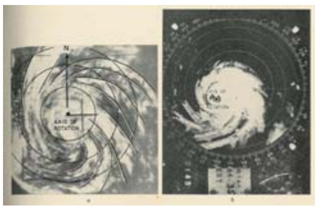
The evidence for the existence of Rossby-like waves in the hurricane vortex

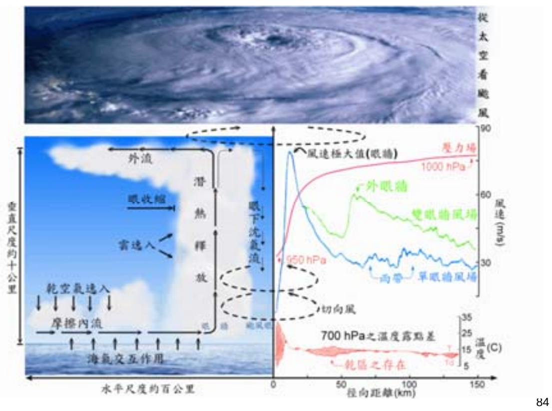
By NORMAN J. MACDONALD, *Air Force Cambridge Research Laboratories, Bedford, Massachusetts*

(Manuscript received September 1, 1966)

ABSTRACT

The qualitative and quantitative evidence for the existence of Rossby-like eddies in hurricanes is examined. It is found that the spiral rain bands exhibit many characteristics similar to those found in the troughs in the "Rossby Waves" of the general circulation. In addition to many qualitative analogies, the spiral bands are regions where a substantial amount of potential energy is converted to kinetic. Although there are no direct measurements to show that the spiral rain bands mark the presence of a Rossby-like wave structure, observations of the momentum budget suggest that there is an eddy flux of momentum into hurricanes in the same sense as that observed in the general circulation.





84

The important of *air-sea interaction* to the maintenance of tropical cyclone.

"Tritium measurement indicates 60%-80% of the water in the eyewall could have evaporated from the ocean not so long ago."

"Tropical cyclone consumes a great deal more latent heat than it can collect from the pre-existing atmospheric vapor."

Huge CAPE without air-sea interaction make NO tropical cyclone.

Ooyama 1969

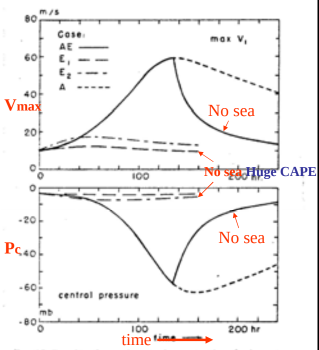
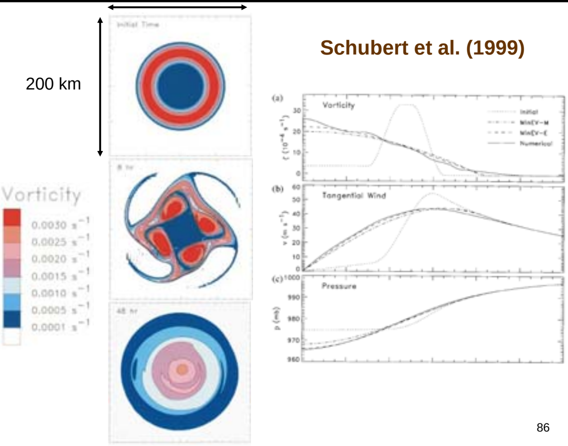


FIG. 15. Results of experiments demonstrating the importance of direct energy supply from the ocean to tropical cyclones. In Case AE, the energy supply to the Case A cyclone is suddenly stopped ($C_E=0$) at $t=134$ hr, to simulate landfall. In Cases E_1 and E_2, in which η is initially assumed to be 2.0 and 3.0, respectively, there is no energy supply from the ocean.

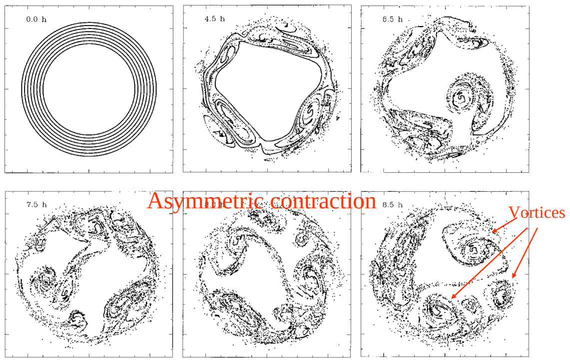
Schubert et al. (1999)



86

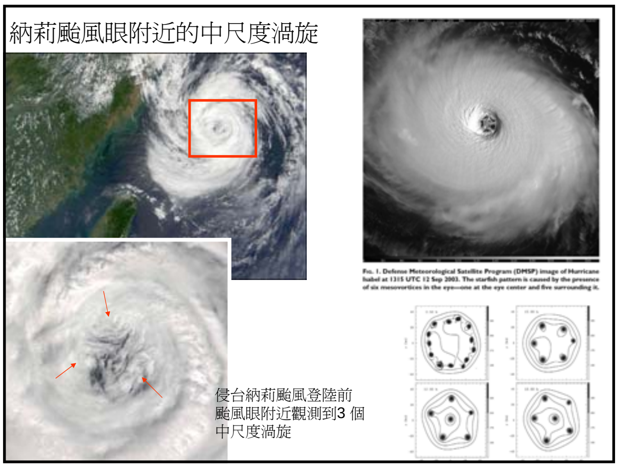
Kossin and Schubert (2002)

Mixing due to Barotropic Instability (Stirring)

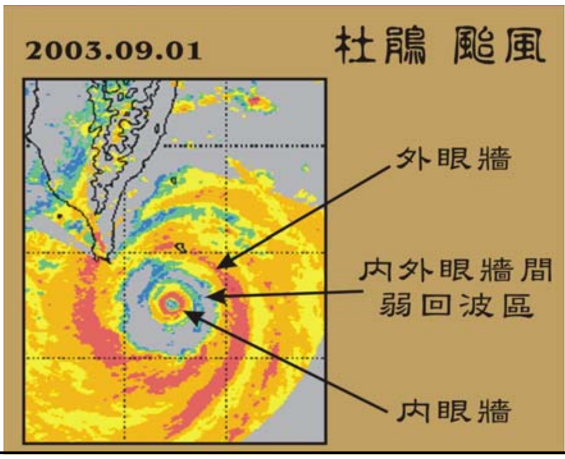


Advective rearrangement is different from the down-gradient diffusion 87

納莉颱風眼附近的中尺度渦旋

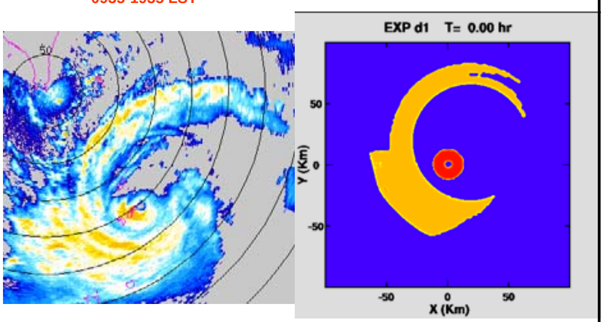


侵台納莉颱風登陸前颱風眼附近觀測到3 個中尺度渦旋

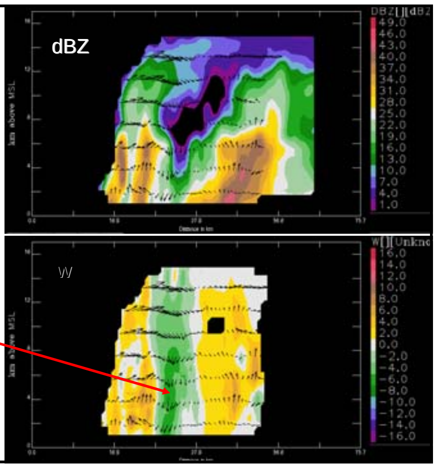


Typhoon Lekima (2001)

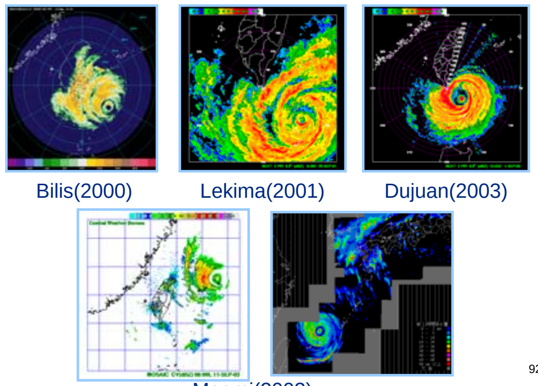
0935-1935 LST



ELDORA data show downward motion between the two eyewalls



Concentric eyewalls near Taiwan

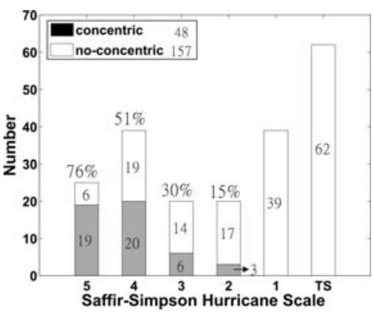


92

Kuo et al. (2009)

WNPAC

- 1997~2005
- 221 tropical cyclones
- 205 tropical cyclones examined
- 48 concentric tropical cyclones (23%)
- higher intensity, higher percentage of concentric eyewall

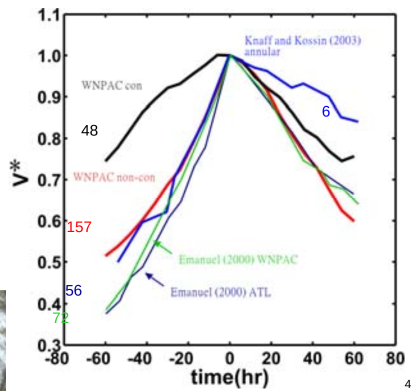


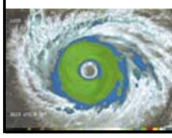
category 5 (135+ kts)	category 4 (114-135 kts)
category 3 (96-113 kts)	category 2 (83-95 kts)
category 1 (64-82 kts)	TS(63- kts)

93

Composite Time Series of the Normalized Intensity

Kuo et al. (2009)



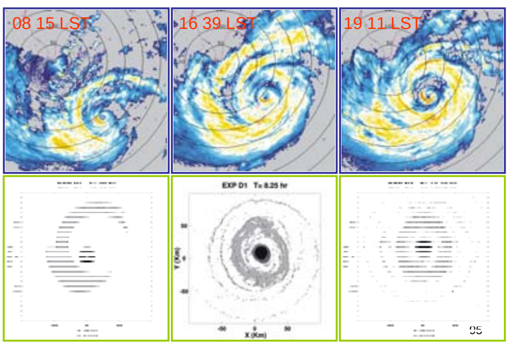


annular

4

雙颱風眼生成動力研究

Kuo, H.-C., L.-Y. Lin, C.-P. Chang, and R. T. Williams, 2004: The formation of concentric vorticity structure in typhoons. *J. Atmos. Sci.*, **61**, 2722-2734.



95

Binary vortex interaction

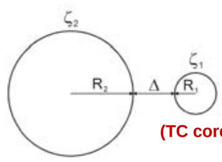
Kuo et al. (2004)

【Variables】

$R_1, R_2; \Delta; \zeta_1, \zeta_2$

【Parameters】

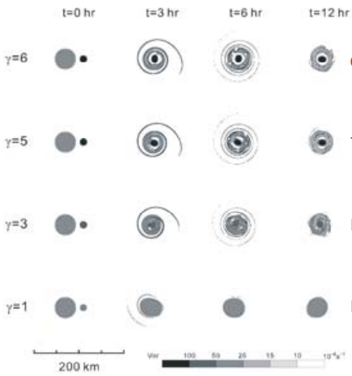
- Vortex radius ratio $(r) = \frac{R_1}{R_2}$
- Dimensionless gap $(\frac{\Delta}{R_1})$
- Vortex strength ratio $(\gamma) = \frac{\zeta_1}{\zeta_2}$



- An extension of Dritschel and Waugh's (1992) work.
- In addition to the radii ratio and the normalized distance between the two vortices, the vorticity ratio is added as a third external parameters.

96

$$\gamma = \frac{\zeta_1}{\zeta_2}$$



Concentric

Tripole

Merger

Merger

Figure 3: The sensitivity of the vorticity field in the binary vortex experiments with respect to the vorticity strength ratio (γ) at hour 0, 3, 6 and 12 with the dimensionless gap Δ/R1=1, and the vortex radius ratio r=1/3.

97

Same strain outside the RMW but different core vortex strength

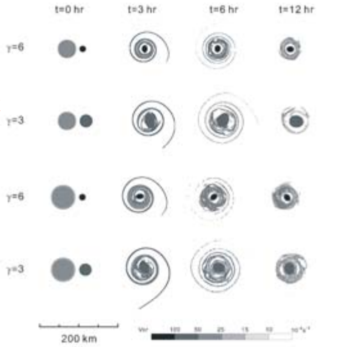


Figure 9: The sensitivity of the vorticity field in the binary vortex experiments with the core vortices process the same maximum wind but different radius of vorticity field. Two core vortices considered have the vorticity and radius of (1.8 × 10^{-3} s^{-1}, 10 km) and (0.9 × 10^{-3} s^{-1}, 20 km) respectively. The dimensionless gap is 1 in the experiments. The outer vortices considered have the radius of 30 km and 40 km respectively.

98

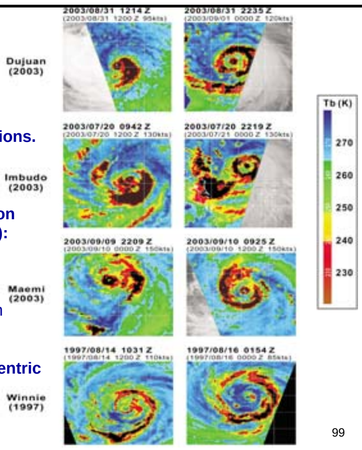
Examples of asymmetric → symmetric concentric formations.

~ 12 hours.

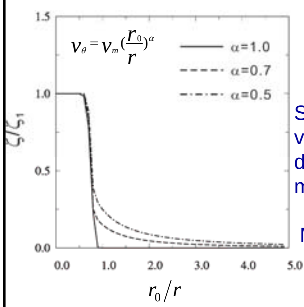
Initial Δ (outer deep convection region - vortex core distance):

Typhoon Dujuan: nearly 0 km
Typhoon Imbudo nearly 50 km
Typhoon Maemi: nearly 100 km
Typhoon Winnie nearly 260km

A wide range of radii of concentric eyewalls



99



Skirt of significant cyclonic relative vorticity (slow tangential wind decrease outside the radius of maximum wind).

Mallen et al. 2005

Radial profile of the vorticity for the core vortex with the skirt parameter (α) 1.0, 0.7 and 0.5.

100

Examples of the vorticity field at hour 12, showing different classifications of binary vortex interactions involving a skirted core vortex.

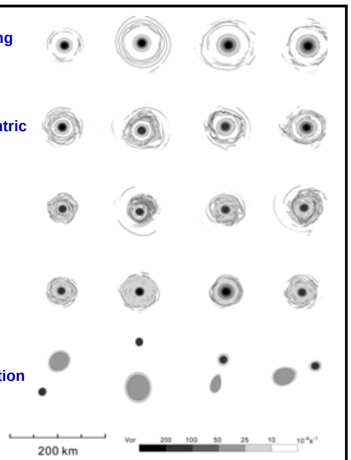
Straining out

Concentric

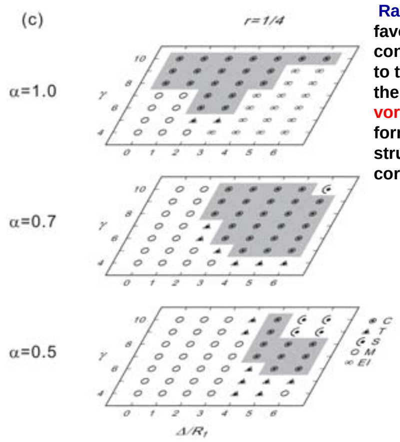
Tripole

Merger

Elastic Interaction



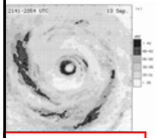
Rankine vortex (α = 1.0) favors the formation of a concentric structure closer to the core vortex, while the α = 0.7 and α = 0.5 vortices favor the formation of concentric structures farther from the core vortex.

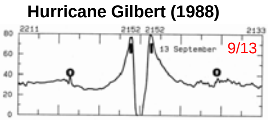


102

A major issue in understanding changes in typhoon intensity

Black and Willoughby (1992)
Hurricane Gilbert (1988)





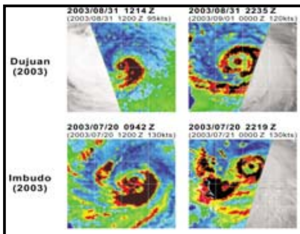
Development of symmetric structure from asymmetric convection in 12 hours

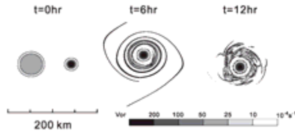
The contraction of the Outer tangential wind maximum

Core vortex intensity remains approximately the same during the contraction period

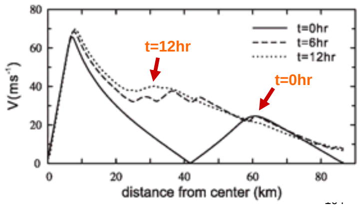
Inner core dissipate, TC weakens

103





The contraction and the increase of the secondary wind maximum by nonlinear advection dynamics.



Terwey and Montgomery, June JGR 2008

D12112 TERWEY AND MONTGOMERY: MODELED SECONDARY EYEWALL FORMATION D12112

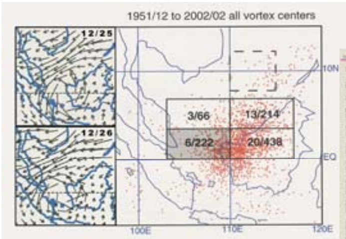
Table 1. List of Secondary Eyewall Formation Hypotheses With Summary of Relevance to our Modeled Hurricanes[a]

Authors	Hypothesis Summary	Relevance to Current Model Results	Type
Willoughby et al. [1982] borrowing from the squall line research of *Zipser* [1977]	Downdrafts from the primary eyewall force a ring of convective updrafts.	Few downdraft-forced updrafts during this time in the simulations.	O
Willoughby [1979]	Internal resonance between local inertia period and asymmetric friction due to storm motion.	No systematic storm motion in the simulated storms.	A
Hawkins [1983]	Topographic effects	No topographic forcing in the simulations.	O
Willoughby et al. [1984]	Ice microphysics	"Warm-rain" (no-ice) sensitivity case also produces secondary eyewall.	A
Molinari and Skubis [1985] and *Molinari and Vallaro* [1989]	Synoptic-scale forcings (e.g., inflow surges, upper-level momentum fluxes)	No synoptic-scale forcings in the simulations	O
Montgomery and Kallenbach [1997], *Camp and Montgomery* [2001] and *Terwey and Montgomery* [2003]	Internal dynamics-axisymmetrization via sheared vortex Rossby wave processes; collection of wave energy near stagnation or critical radii	Possible explanation	N
Nong and Emanuel [2003]	Sustained eddy momentum fluxes and WISHE feedback	Possible explanation	A
Kuo et al. [2004, 2008]	Axisymmetrization of positive vorticity perturbations around a strong and tight core of vorticity	Possible explanation	N

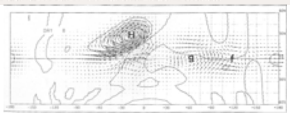
[a]The type column refers to the type of model or observations that were used to formulate the hypothesis. O stands for observationally-based; A stands for axisymmetric model; N stands for nonaxisymmetric model.

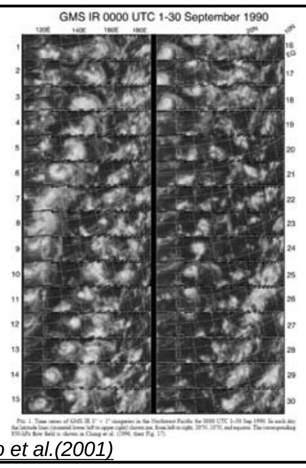
105

畫眉颱風



左側圖為畫眉颱風生成時間因寒潮南下之大尺度風場；右側圖為過去51年冬天婆羅洲外海小尺度雷雨系統中心位置分佈圖。方塊內數字為雷雨系統持續96小時以上之頻率；畫眉颱風生成區之頻率為6/222。雷雨系統及寒潮風場持續4天之交互作用產生了畫眉颱風。





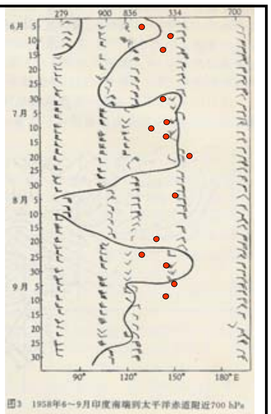
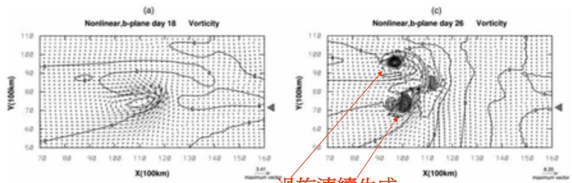


图3 1958年6～9月印度南端到太平洋赤道附近700 hPa东西风分界线与台风发生的关系

Kuo et al.(2001)

謝與陳(1963)

107



Beta Rossby number (V/ β r ^2) ~ 1

渦旋連續生成

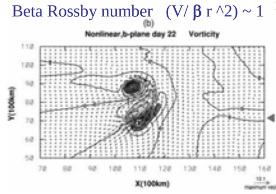
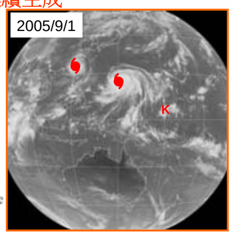
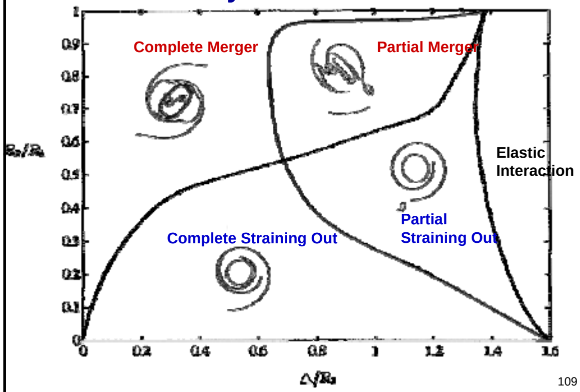


FIG. 11. Same as in Fig. 10 except for the nonlinear β-plane



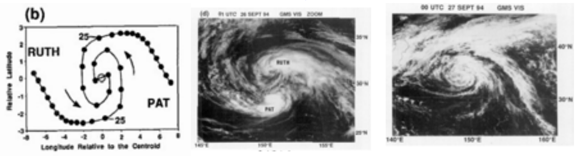
Kuo et al. 2001

Binary vortex interaction



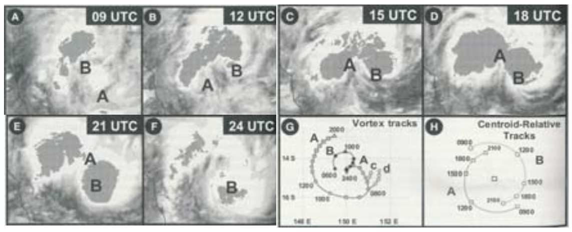
(Adapted from Dritschel and Waugh 1992.)

雙颱風互繞（藤原效應）合併 ---- 颱風 PAT 與 RUTH



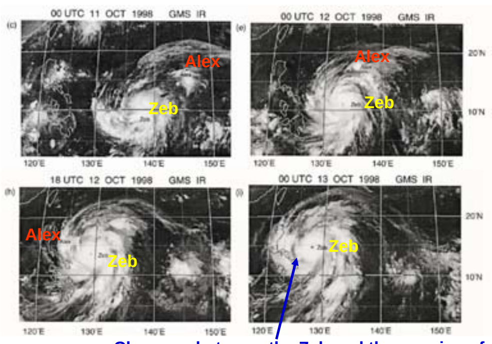
中尺度對流系統互繞

Simpson et al. 1998



颱風渦旋合併動力探討研究

Kuo, H.-C., G. T.-J. Chen, and C.-H. Lin, 2000: Merging processes of tropical cyclone Zeb and Alex. *Mon. Wea. Rev.*, **128**, 2967-2975.



Clear gap between the Zeb and the remains of Alex

Multiple Scale Interactions!

- **Waves, Turbulence, and Vortices**
  波動、亂流 、 渦旋 **(2D turbulence)**
- **Cumulus Parameterization**
  積雲合成作用 **(3D turbulence)**

Concerning turbulence, Sir Horace Lamb is quoted
in an address to the British Association for the Advancement of Science as follow:

I am an old man now, and when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. About the former I am rather optimistic.

Before too long, climate modelers will have much more in common with turbulence modelers and micrometeorologists than either group now seems to realize.

Tennekes 1978, Turbulent Flow in Two and Three Dimension

(Tennekes and Lumley, A first course in turbulence)

Spectral Method $f = \sum a_n \phi_n$

1. Completeness（完整性） → Sufficient condition
2. Orthogonality（正交性）
3. Speed of convergence（收斂速度）
4. Fast Transform（快速轉換）

(2–4) → Application

Sturm-Liouville equation

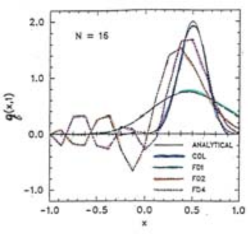
$$L\phi(x) = -\frac{d}{dx}(p(x)\phi'(x)) + q(x)\phi(x) = \lambda W(x)\phi(x)$$

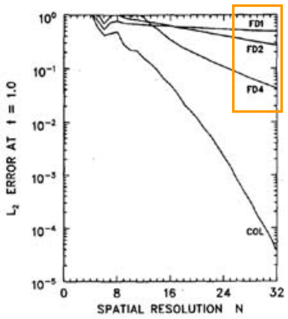
with suitable boundary conditions and restrictions on functions $p(x)$, $q(x)$, $W(x)$, we have a countably infinite set of solutions $\{\phi_n(x)\}_{n=0}^{\infty}$ corresponding to discrete eigenvalues $\{\lambda_n\}_{n=0}^{\infty}$

Performance of Chebyshev Collocation Method

Fulton & Schubert (1987 a)

Linear Advection Equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$





COL error exponential convergence

FD error algebraic convergence

115

Shallow Water Equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + \frac{\partial h}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + \frac{\partial h}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} + (\bar{h} + h)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = Q \qquad (3)$$

where $Q(x,y,t) = q_0 \exp\left[-\left(\frac{x-x_c}{x_0}\right)^2 - \left(\frac{y-y_c}{y_0}\right)^2\right] 4t^2 t_0^{-2} e^{-2t/t_0}$

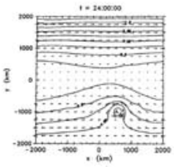
$q_0 = 6250 m^2 s^{-2}$, $x_0 = y_0 = 200 km$,

$t_0 = 6$ hours $= 21600$ sec, $(x_c, y_c) = (1000, -1000)$

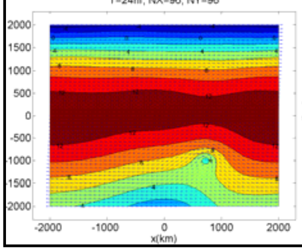
$\bar{h} = c^2 = 2500 \frac{m^2}{s^2}$

116

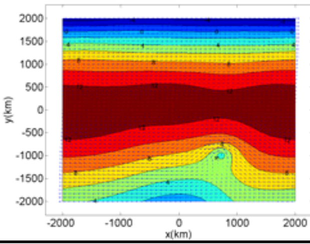
Domain Decomposition Numerical Results



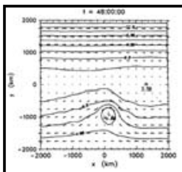
Single Domain



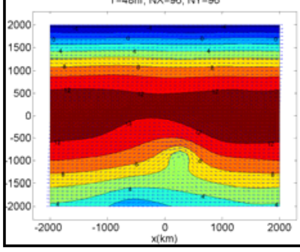
Domain Decomposition(2*2)



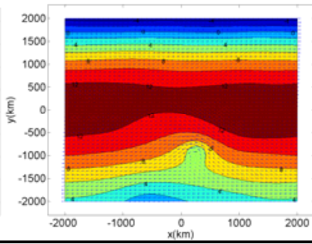
Domain Decomposition Numerical Results



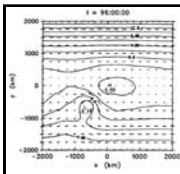
Single Domain



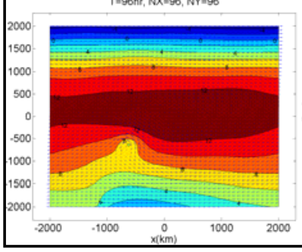
Domain Decomposition(2*2)



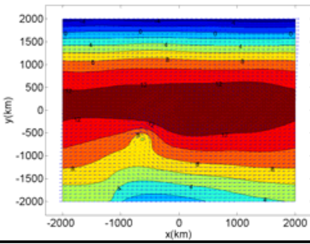
Domain Decomposition Numerical Results



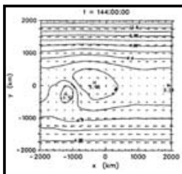
Single Domain



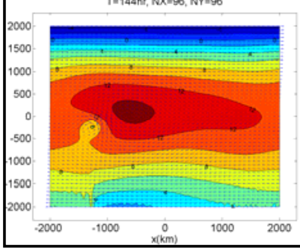
Domain Decomposition(2*2)



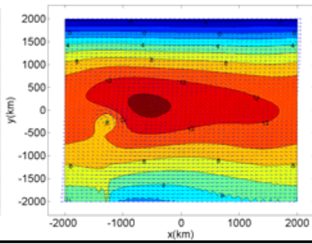
Domain Decomposition Numerical Results



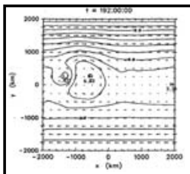
Single Domain

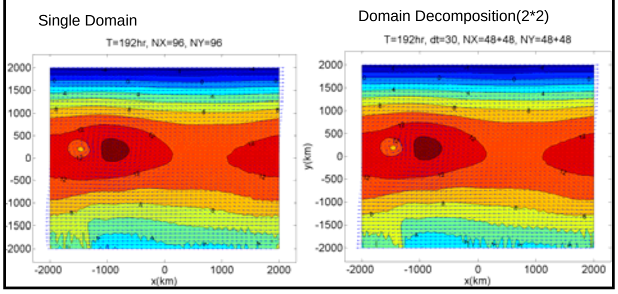


Domain Decomposition(2*2)



Domain Decomposition Numerical Results





問世間颱風是何物？

- 夫大塊噫氣、其名為風 -- 莊子齊物論
- 「颱」-- 明末清初17世紀首見於漢文
- 康熙年間台灣府奏摺：今年亢旱之後、繼以颱風
  今歲入秋亢旱、繼又颶風大作
  台澎海外地方、每直秋潮節候、颱颶時有
- Typhoon -- 英文字根為16世紀阿拉伯語Touffon（旋轉）
- Typhon -- 希臘神話的風神
- Kamikaze -- 神風（？）13世紀末蒙古征日

渦旋，旋轉流體

Vortex, Rotational Dynamics

Vortex radius ratio

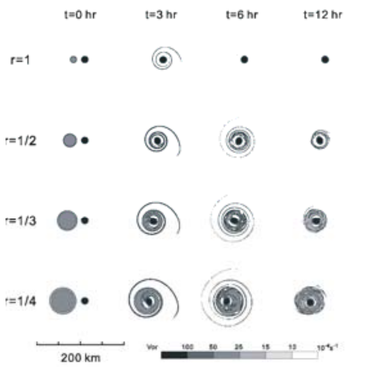


Figure 5: The sensitivity of the vorticity field in the binary vortex experiments with respect to the vortex radius ratio r at hour 0, 3, 6 and 12 with the vorticity strength ratio γ=5, and the dimensionless gap Δ/R_1=1.

$\gamma = 10$

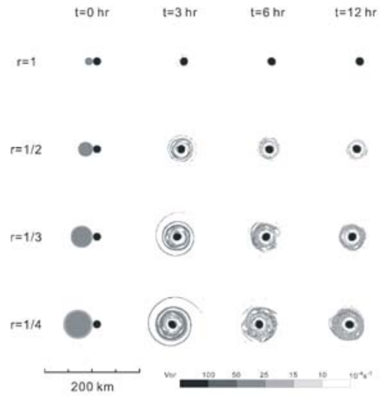
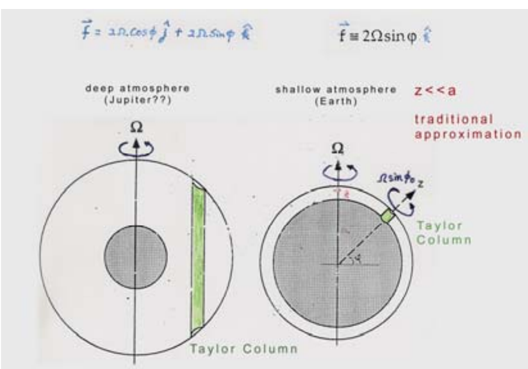


Figure 6: Similar to Figure 5 except that the dimensionless gap Δ/R_1=0 and the vorticity strength ratio γ=10.

124



5

Cascade (Nonlinear Dynamics)

So, nat'ralists observe, a flea
Hath smaller fleas that on him prey;
And these have smaller yet to bite 'em,
And so proceed *ad infinitum*.
--------- Jonathan Swift

Big whirls have little whirls
that feed on their velocity,
and little whirls have lesser whirls,
and so on, to viscosity.
----- Lewis Fry Richardson 1922

126

Highly Nonlinear System

Turbulence, Order and Chaos
Multiple scale interaction
Cascade of kinetic energy and enstrophy
Deterministic and statistical dynamics

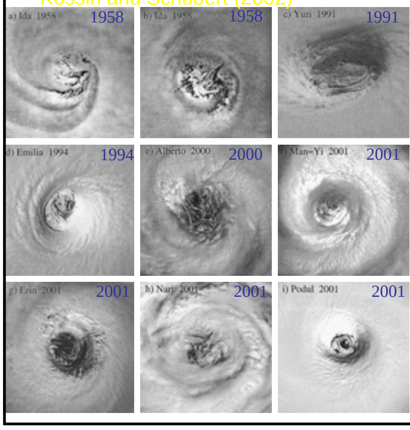
Laminar yields turbulence

Order (i.e. turbulence) emerges from chaos

Coherent structures emerge from chaos, under the action of an external constraint

127

Kossin and Schubert (2002)



Vortical Swirls in Hurricane Eye Clouds

Eye Dynamics!
Vortex Mixing!

128

Symmetric Model

Ooyama 的貢獻 (1969)—三層軸對稱模式

Ooyama 的實驗有一半以上在討論海氣交互作用

- 「主環流」與「次環流」的交互作用
  — "積雲尺度"、"氣旋尺度"二尺度的"合作"
- 組織性對流
  — 非絕熱效應、積雲參數化的問題
- 非線性效應 **linear theory is wrong**
  $\frac{dP}{dt} = -\frac{P}{h}Q$, $\frac{c^2}{f^2}$ V.S. $\frac{c^2}{\left(f+\frac{\partial rV}{r\partial r}\right)\left(f+\frac{2V}{r}\right)}$
- 海、氣交互作用 C_D, C_E 之重要性
  — SST, 初始的 CAPE 不重要
  (颱風的維持不能只靠大氣的 CAPE)
- 中層的「內流」
  — Hurricane **spin up**
- PBL 的乾空氣「進入」
  — frictional driven **divergence!!!**

Cumulus Parameterization
- reduced the multiple scale problems to one scale problem
(Be careful not to play with a loaded dice !)
-Buoyancy not exactly zero

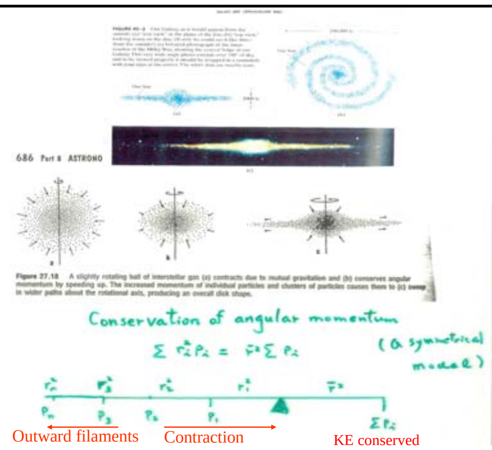
Nonlinear Dynamics

Importance of air sea interaction
(Initial CAPE NOT important)

Mid-level entrainment

PBL and subsidence
Frictional driven divergence and convergence

129



130

- Kirchhoff vortex (nonlinear) **Lamb, 1932**

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

angular velocity = $\zeta\frac{ab}{(a+b)^2}=\omega$

rotating period **P** = $\frac{2\pi}{\zeta}\frac{(a+b)^2}{ab}$

a ~ 30 km b ~ 20 km

P ~ $(2\pi/\zeta)*4$ ζ ~ $3*10^{-3}$ s^{-1}

Vmax ~ 50 ms^{-1}

- Kelvin PV wave (linear)

$c = V_{max}(1-\frac{1}{m})$ $m=2$

angular velocity = $c/r = \frac{V_{max}}{2r}$

rotating period **P**

$$\sim \frac{2\pi}{\omega} = \frac{2\pi}{2\frac{V_{max}}{r}}*4 = \frac{2\pi}{\zeta}*4$$

Same as Kirchhoff vortex !!

131