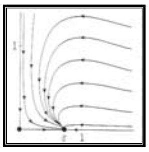
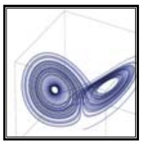
數學模式與科學研究

主講人：郭鴻基 教授



（掠食者的滅絕）

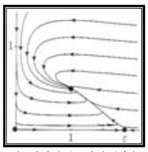


（Lorenz 吸子）

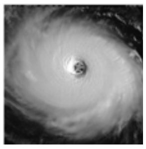
內容：

科學研究是探索未知知識的建構過程，而數學是科學的語言。隨著電腦的進步，資料大幅度的數位化，科學計算更成為打開非線性科學研究的敲門磚。數學建模、科學計算、分析詮釋與驗證等過程，更是現今數學科學的典範。演講將簡介一個最簡單的氣候模式，並討論數學建模的過程，以及相關因素。

（感謝中央氣象局鄭明典博士提供部分投影片）



（掠食者與被掠食者共存）



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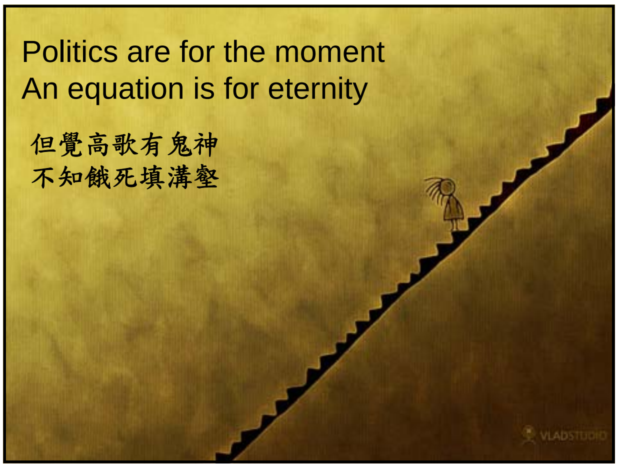
2009.03.03 建國中學

動 力 與 模 擬 研 究 室

郭鴻基 教授

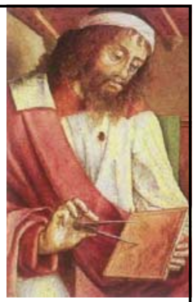
- 颱風與渦旋動力
  Typhoon and Vortex Dynamics
- 數學建模科學計算
  Mathematics Modeling/Scientific computations
- 地球物理流體力學
  Geophysical Fluid Dynamics
- 兩度空間亂流
  Two-Dimensional Turbulence

2



讀 算 寫

幾何
代數
微積分
電腦計算繪圖



+ -	x /	
加、減	乘、除	數量化、數位化
線性	非線性	數學化--模式--動力系統
大題大作	小題大作	

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17世紀	力學
18世紀	力學、流體力學
19世紀	熱力學、統計力學、熱力學、電磁學、生物學 （小獵犬號船長是一個氣象學家）
20世紀	輻射學、量子力學、原子物理、電腦、氣象科學 生命科學

5

Icons of Knowledge

$$\nabla \cdot B = 0$$
$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\frac{\partial^2 B}{\partial t^2} = c^2 \nabla^2 B$$

流力非線性方程

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0$$
$$\nabla \cdot g = -4\pi G \rho$$
$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi = E\psi$$

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi$$
$$\frac{\partial T}{\partial t} = k \nabla^2 T$$

$$\nabla \psi = \lim_{\int dv \to 0} \frac{\int \psi\, dA}{\int dV}$$
$$\nabla \cdot V = \lim_{\int dv \to 0} \frac{\int V \cdot ndA}{\int dV}$$
$$\nabla \times V = \lim_{\int dv \to 0} \frac{\int ndA \times V}{\int dV}$$
$$\nabla^2 \psi = \lim_{\int dv \to 0} \frac{\int \nabla \psi \cdot ndA}{\int dV}$$

Divergent (Gauss) theorem

Stokes theorem

$$\int_A B \cdot ndA = \int_V \nabla \cdot BdV$$
$$\oint B \cdot dl = \int_A \nabla \times B \cdot ndA$$

6

數學科學

Formulation 數學建模（微分、差分方程式）

Solution / Analysis 分析、求解

Interpretation 科學詮釋

中階課程：微分方程(ODE,PDE)
統計、線性代數
程式、計算與繪圖

數量化、數位化
數學化--模式--動力系統

7

"Six monkeys, set to strum unintelligently on typewriters for millions of years, would be bound in time to write all the books in the British Museum." Huxley

君子致用在乎經邦，經邦在乎立事，立事在乎師古，師古在乎隨時。必參古今之宜，窮終始之要，始可以度其古，中可以行於今。通典

共49個字，假設中文常用字爲1000字，共有10^147個選擇

地球歷史 10^18 sec
10^10 一百億隻猴子在打字，假設每秒鐘打一萬字 10^4，

10^10*10^18*10^4 = 10^32

10^32/10^147=10^(-115) ~ 0 機率爲零 ，不可能的巧合！

研究學問是苦心孤詣的事業！ 不要人云亦云！

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Bode's Law of Astronomy 1778

0 3 6 12 24 48 96 192 384

4 7 10 16 28 52 100 196 388

0.4 0.7 1.0 1.6 2.8 5.2 10 19.6 38.8

Mercury	0.4 (0.39)	Venus	0.7 (0.72)	Earth	1.0 (1.0)
Mars	1.6 (1.52)	Asteroids	2.8 (2.77)	Jupiter	5.2 (5.2)
Saturn	10 (9.54)	Uranus	19.6 (19.19)	Neptune	38.8 (30.07)
Pluto	fails (39.60)				

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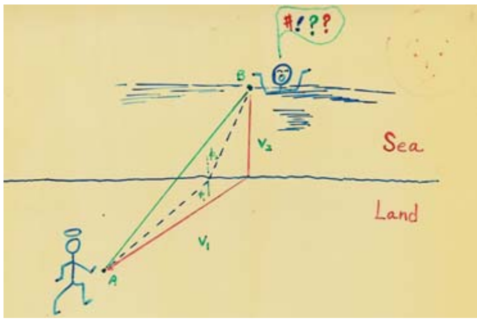

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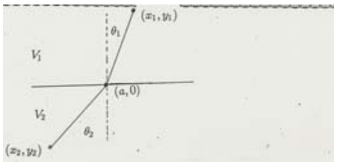

$$T(a) = \frac{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}{V_1} + \frac{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}{V_2}$$

$$\frac{\partial T}{\partial a} = 0$$

$$\frac{-(x_1-a)}{[(x_1-a)^2+y_1^2]^{\frac{1}{2}}}\frac{1}{V_1} + \frac{(a-x_2)}{[(x_2-a)^2+y_2^2]^{\frac{1}{2}}}\frac{1}{V_2} = 0$$

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2}}$$

This is the Snell's Law.

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一樣觀魚多樣情！

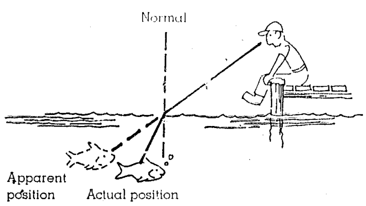

FIGURE 5.13 The refraction of light as it passes from the water into the less-dense air causes a fish to appear closer to the surface than it actually is.

（1）魚快樂嗎？
（2）熱血沸騰，立志革命！
（3）折射定律，最小原理。

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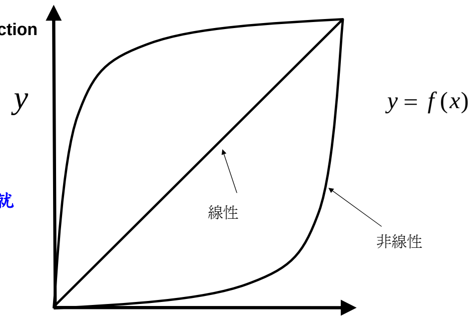
一分耕耘 ， 一分收穫？？

函數 Function

(因果）

y：成就



$$\frac{dy}{dx} > 0 \Rightarrow \frac{d^2y}{dx^2} = ? \qquad x$$

x：努力

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蝴蝶效應 Butterfly Effect Chaos 混沌

混沌 非線性 $y = x^{30}$

精確度有限
非線性

$$0.02 \begin{cases} x = 0.99 \\ x = 1.01 \end{cases} \qquad 0.61 \begin{cases} y \approx 0.74 \\ y \approx 1.35 \end{cases}$$

預報能力的喪失！！

"Sensitivity dependence on initial condition."

H Poincare

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Thomas Robert Malthus

Wikipedia

(1766～1834)

English demographer and political economist

人口學家
政治經濟學家

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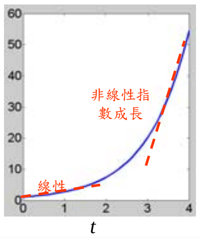
Malthusian Model

Population Growth

$$\lim_{\Delta \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \frac{dp}{dt} = \alpha p \;\Rightarrow\; p = p_0 e^{\alpha t}$$



線性 非線性

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

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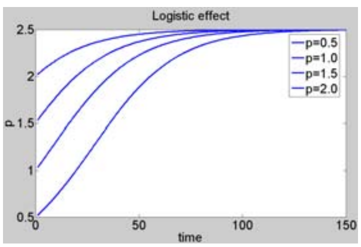
Logistic Effect

Time Series (時間序列)

$$\frac{dp}{dt} = \alpha p - \beta p^2 = \alpha p(1 - \frac{p}{\frac{\alpha}{\beta}})$$

其中 $\frac{\alpha}{\beta}$ → 環境之承載效應



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動力系統

$$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}, \gamma_i)$$

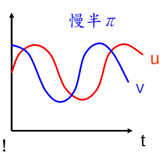
時間變化謂之動力 變數 許多外在及內在控制參數

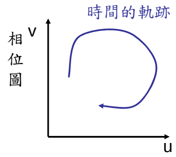
$$\int_0^{2\pi} \cos t \sin t dt = 0$$

$$u \quad v$$

$$\overline{uv} = 0$$

Cos 和 Sin 零相關、不來電！





推背圖：前知三百年，後知三百年
可以解釋〝已知〞，可以預測〝未來〞

相位圖

18

Periodic phenomena are actually everywhere in the biological world.

What else can you think of?

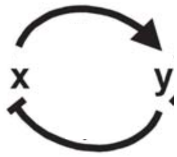
19

Negative Feedback Oscillators

負回饋　恩將仇報　以德報怨

反者道之動　常

天之道其猶張弓　損有餘　補不足



$$\frac{dy}{dt} = x$$

$$\frac{dx}{dt} = -y$$

What can X and Y be?　**X Cost**　**Y Sin t**

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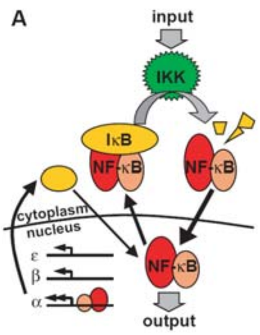
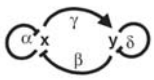
NF- κ B and I κ B Model





X: nucleus NF- κ B

Y: I κ B

$$\frac{d}{dt}x = S - \alpha x - \beta y$$

$$\frac{d}{dt}y = \gamma x - \delta y$$

Science **298:** 1241-1245.

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Lotka-Volterra Model

A Predator-Prey Model

Humberto D'Ancona
1926

World War One　掠食者比例變大

	Port	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923
Predator	**Fiume**	12%	21%	22%	21%	36%	27%	16%	16%	15%	11%
Prey	**Trieste**	14%	7%	16%	15%	-	18%	15%	13%	11%	10%

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

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Vito Volterra

Wikipedia

(1860～1940)

Italian
Mathematician & Physicist

Contribution:

1) Mathematical biology

2) Volterra-Lotka equations

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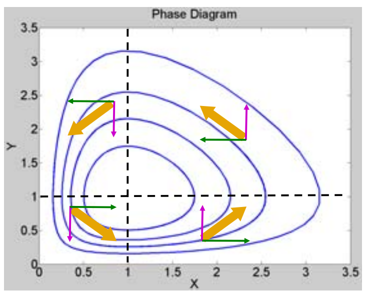
動力系統



dx/dt = 　x(1-y)

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

相位圖

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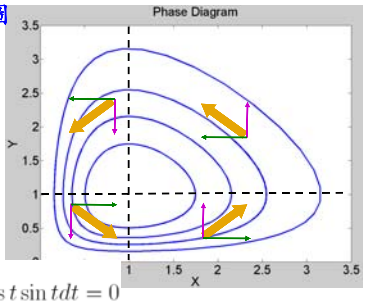
愛情動力系統

相位圖



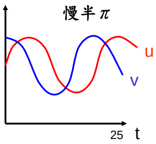
dx/dt = x(1-y)

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

dy/dt = y (x-1)

慢半 π



$$\int_0^{2\pi} \cos t \sin t dt = 0$$

$u \quad v$

Cos 和 Sin 零相關、不來電！

動力系統

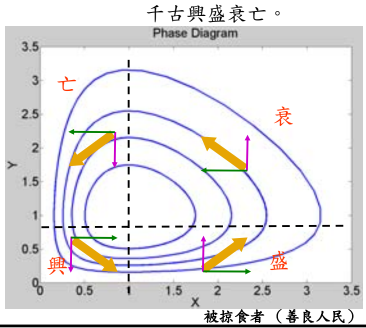
鴛鴦蝴蝶夢，一段愛情故事；

千古興盛衰亡。

$$\frac{dx}{dt} = x - xy$$

$$\frac{dy}{dt} = -y + xy$$

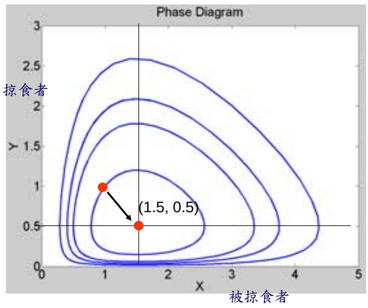
掠食者（官僚）



被掠食者（善良人民）

$$\begin{cases} \frac{dx}{dt} = x - xy - hx \\ \frac{dy}{dt} = -y + xy - hy \end{cases}$$

h=0.5



Harvest effect
對掠食者與被掠食者同時進行獵取殺害，可以使生態往對被掠食者有利方向移動。

1960 DDT 事件

無間道警察
水滸傳哲學

微積分數學

$$u = u(x, y)$$

Chain Rule(連鎖律)

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

偏微分

例如：$\frac{\partial x^2 y}{\partial y} = x^2$

只對y變數微分，不改變x變數

你快樂嗎？一個簡單的生涯規劃動力系統

u：快樂指數
x:考試作業量
y:玩魔獸的時間

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

天縱英明的資優生 <0 >0 <0 <0 >0

考試越少越不快樂，
玩魔獸的時間越多越不快樂

$\frac{\partial u}{\partial x}$>0 考試越多越快樂
$\frac{\partial u}{\partial y}$<0 電動越玩越不快樂
→ 人的個性

$\frac{dx}{dt}$, $\frac{dy}{dt}$ → 人的境遇

個性+境遇=人生
相形不如論心
論心不如則術
形不勝心
心不勝術 荀子非相

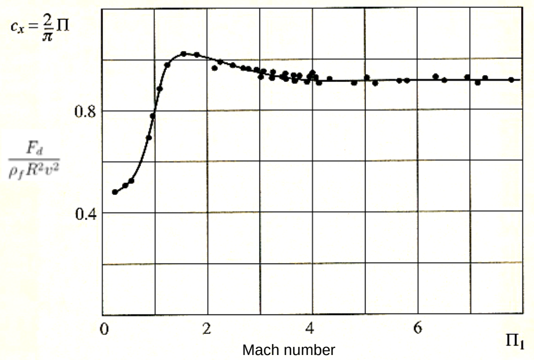
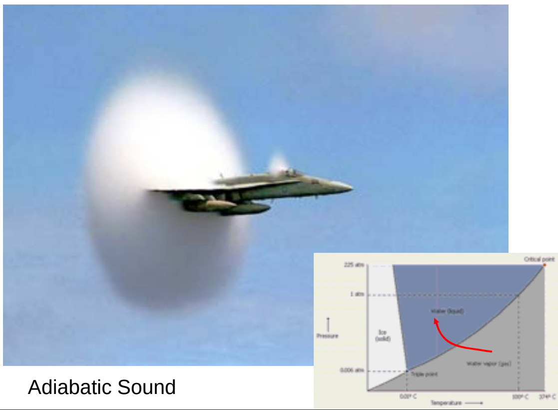


Figure 1.11. The dimensionless drag on a sphere, Π (times $2/\pi$), as a function of the dimensionless governing parameter $\Pi_1 = U/c$, the Mach number (Chernyi 1961). The quantity Π approaches a constant for large values of Π_1.

Sonic Boom



Adiabatic Sound

Dead-water near the coast

Wave drag problem

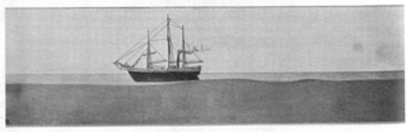
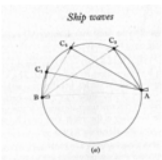


Fig. 6.2. (a) Surface "slicks" showing the presence of internal waves in the wake of a ship in Bute Inlet, British Columbia. The vessel was traveling at 0.5 m s^{-1} in a surface layer of almost fresh water only slightly deeper than its 3.4 m draft. The internal waves caused horizontal motion at the surface that affects the ripple pattern and so renders the internal wave pattern visible at the surface during calm conditions. [Photo courtesy of Defence Research Establishment Pacific, Victoria, British Columbia.] (b) A laboratory experiment [from Ekman (1904)], showing internal waves being generated by a model ship. The tank is filled with two fluids of different density, the heavier one being dyed to make the interface clearly visible. The model ship (the superstructure of the "Fram" has been drawn in subsequently) is towed from right to left, causing a wake of waves on the interface.



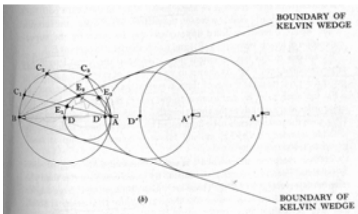


Figure 68. Waves generated in deep water by a ship B. Case (a): positions C_1, C_2, C_3 of any waves generated t_2 seconds ago (when the ship was at A) if their energy had travelled a distance ct_2. Case (b): the real positions E_1, E_2, E_3 of the same waves, taking into account that their energy has only travelled a distance $\frac{1}{2}ct_2$. At each, the dependence of wavelength on direction of emission, as inferred from equation (183), is shown. The circle with diameter AD is the locus of all such waves. Other such circles, with diameters A′D′ and A″D″, are where waves generated when the ship was at A′ and A″ are now to be found. All such circles lie within the Kelvin wedge of semi-angle (184).

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Kelvin edge deep water waves

< Ship-wave pattern >

$$2 * \sin^{-1}\left(\tfrac{1}{3}\right) = 2 * 19.5^{o}$$

[*Courtesy of Aerofilms Ltd.*]

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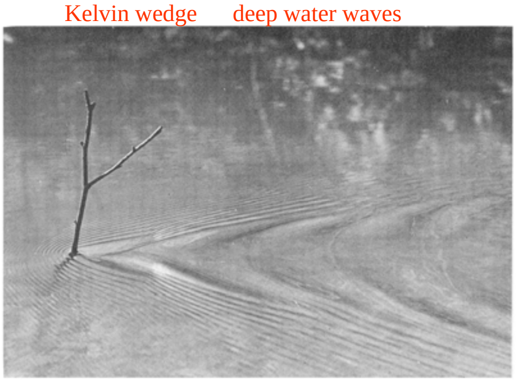
Kelvin wedge deep water waves



[*From V. A. Tucker, "Waves and Water Beetles,"* Physics Teacher **9**, 10-14 19 (1971), Fig.3 *(Copyright 1971 by American Assoc. Phys. Teachers)*]

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Pictures-6

< Capillary Waves >

Beetle making waves on a water-air interface

[appeared on the cover of Science 166(Nov.14, 1969,Copyright 1969 by the American Association for the Advancement of Science),in connection with an article by V. A. Tucker, "Wave-Making by Whirligig Beetles(Gyrinidae)," pp. 897-99.]

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Buckingham's Pi Theorem

n variables can always be combined to form
Exactly (n- r) independent nondimensional variables, where
R is the rank of the dimensional matrix.

Eg. T, M, L, g are the 4 variable in the simple pendulum problem

kg, m, s are the rank 3 dimensional unit

4-3 =1 nondimensional variable $T \sim \sqrt{\frac{l}{g}}$

Similar result can be derived from Newton's mechanics or
from the fact that the difference of potential energy and kinetic energy over
a period will be minimized; the Hamilton principle.

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$$\Pi = \frac{r_f}{E^{1/5} t^{2/5} \rho_0^{-1/5}}$$

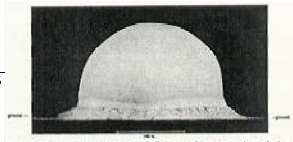
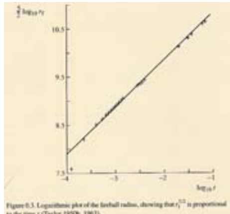


Figure 1.5. A photograph of a fireball 15 ms after an atomic explosion on the ground illustrates the spherical symmetry of the phenomenon and the sharp boundary of the perturbed region (Taylor, 1950a, b, 1963).

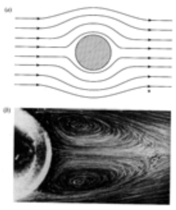
G.I. Taylor 1950

Symbol	*Definition*	*Representative value or first guess*
R	radius of wavefront	10^2 m
t	time	10^{-2} s
p_0	ambient pressure	10^5 Pa
ρ_0	ambient density	1 kg m^{-3}
E	energy released	10^{14} J



原子彈能量～ 10^{14} J

38



$$\frac{F_d}{\rho_f R^2 v^2}$$

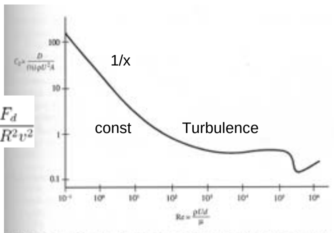


Figure 8.2 Drag coefficient for a sphere. The characteristic area is taken as $A = \pi d^2/4$. The reason for the sudden drop of C_D at Re ~ 5 × 10[5] is the transition of the laminar boundary layer to a turbulent one, as explained in Chapter 10.

$$\frac{vR}{\nu} = \mathrm{Re}$$

F_d $[M][L][T]^{-2}$ drag force;
ρ_f $[M][L]^{-3}$ density of fluid
R $[L]$ radius of sphere
v $[L][T]^{-1}$ speed
ν $[L]^2[T]^{-1}$ kinematic viscosity.

$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathrm{Re} << 1.$$

$$F = C_L\ \rho_{air}\ v^2\ S$$

$$W = \rho_f\ V\ g = \rho_f\ S^{1.5}\ g$$

$$F = W$$

$$S^{0.5} = \frac{C_L\ \rho_{air}\ v^2}{\rho_f\ g}$$

$$W = \rho_f\ g\ S^{1.5} = \frac{C_L{}^3\ \rho_{air}^3\ v^6}{\rho_f^2\ g^2}$$

v: speed
V: volumn
S: area

40

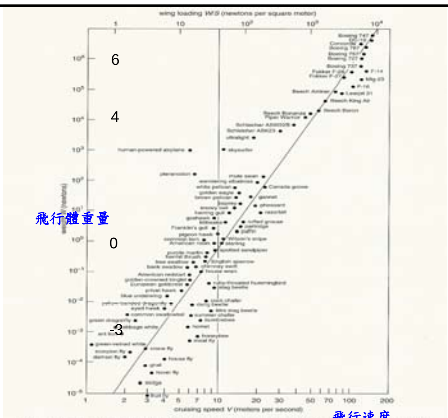
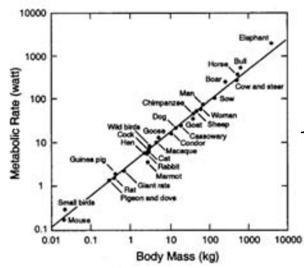


Fig. 2.2 The weight of many flying objects (vertical axis) against their cruising speed (horizontal axis) on a log–log plot. This figure is reproduced from reference [106] with permission from MIT Press.

41

Metabolic rate vs size



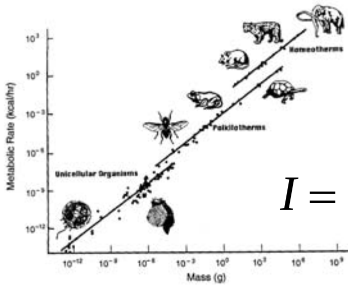
$$I = I_0 M^{3/4}$$

FIGURE 1 Metabolic rate (in watts) for a series of mammals and birds as a function of mass (in kg); the scale is logarithmic and exemplifies the 3/4-power scaling law discovered by Kleiber [2, 22, 27, 29].

42

Kleiber (1932) Body size and metabolism. *Hilgardia* **6**, 315-353.

“Metabolic rate vs size” extended



$$I = I_0 M^{3/4}$$

FIGURE 2 Metabolic rate (in kcal/hr) for a series of organisms ranging from the smallest microbes to the largest mammals as a function of mass (in g), exemplifying the persistence of the 3/4-power scaling law (the solid lines) over 20 orders of magnitude (Hemmingsen [12]).

Hemmingsen (1960) *Reports of the Steno Memorial Hospital and Nordisk Insulin Laboratorium* **9**, 6-110

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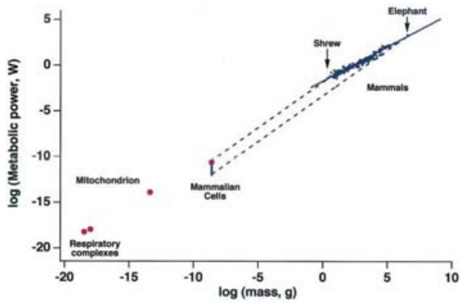
“Metabolic rate vs size” down to molecule



West, Woodruff, Brown (2002) PNAS **99**, 2473-2478

44

45

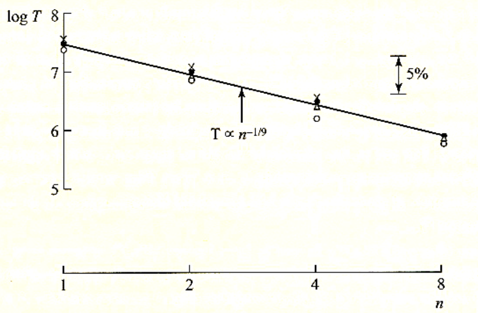


Figure 1.13. The $-1/9$ power-law dependence of the rowing time T on the number of oarsmen n (solid line). This may be compared with racing times over 2000 m, all at calm or near calm conditions: Δ, 1964 Olympics, Tokyo; ●, 1968 Olympics, Mexico City; ×, 1970 World Rowing Championships, Ontario; ○, 1970 Lucerne International Championships. After McMahon (1971).

$$F \sim \rho v^2 l^2$$

假設： G, A 為常數

A:strength of oarsperson

$$P \sim \rho v^3 l^2$$

$$G \sim \frac{l^3}{n}$$

$$l^2 \sim G^{2/3} n^{2/3}$$

$$P = nA \sim \rho v^3 l^2 \sim \rho v^3 G^{2/3} n^{2/3}$$

$$v \sim n^{1/9}$$

47

Isaac Newton

Principia 1687

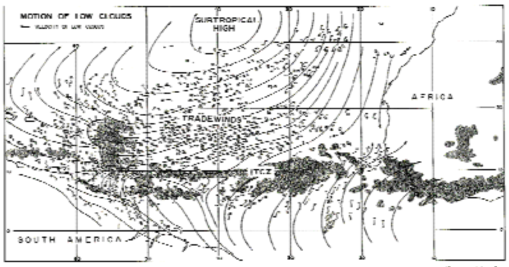
Nature and nature’s law
lay hid in night,
God said,
Let Newton be,
and all was light. **A. Pope**

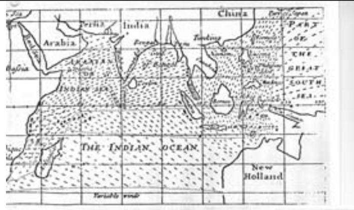
48

Halley 1686

First proposed the atmospheric motion is connected with the distribution of sun heat (follow the sun in the daily scale; thus wind is westward.)

Fujita, 1971

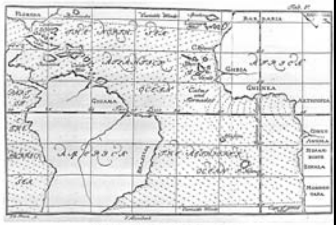




Halley's World Map of Wind Circulations
Fig. 8.1

Halley 1686

D'Alembert 1746



D'Alembert's Map of the Winds in the Lower Latitudes
Fig. 8.2

Euler's Equations for Fluid Flow

Leonhard Euler, born on 15 April, 1707 in Basel. Died on 18 September, 1783 in St Petersburg.
Euler formulated the equations for incompressible, inviscid fluid flow:

$$\frac{\partial \mathbf{V}}{\partial t} + \boxed{\mathbf{V}\cdot\nabla\mathbf{V}} + \frac{1}{\rho}\nabla p = \mathbf{g}.$$

$$\nabla\cdot\mathbf{V} = 0$$

流體力學之父

Partial Differential Equations
偏微分方程式 PDE

非線性

Euler 18 century 流體動量、質量守恒

Fluid Dynamics

- Pressure gradient force 壓力梯度力

$$-\frac{1}{\rho}\nabla P$$

- Eulerian-Lagrangian transformation 座標轉換

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

- Mass conservation (continuity equation) 質量守恒

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

Isothermal sound speed (same mistake as Newton)

$$\left(\frac{\partial P}{\partial \rho}\right)_T \quad v.s. \quad \left(\frac{\partial P}{\partial \rho}\right)_\theta$$

fv: E-W Coriolis force
Conservation of angular momentum, 角動量守恒
Hadley 1735

旋轉力學

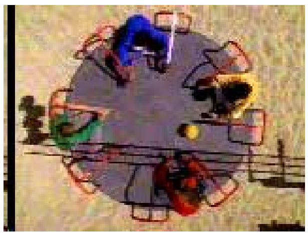
■ **fu: N-S Coriolis force**
Centrifugal force, thermal wind balance 向心力
Ferrel, 1859

■ **Coriolis force, Coriolis, 1835** 科氏力
▲ Falkland ship battle in WW I

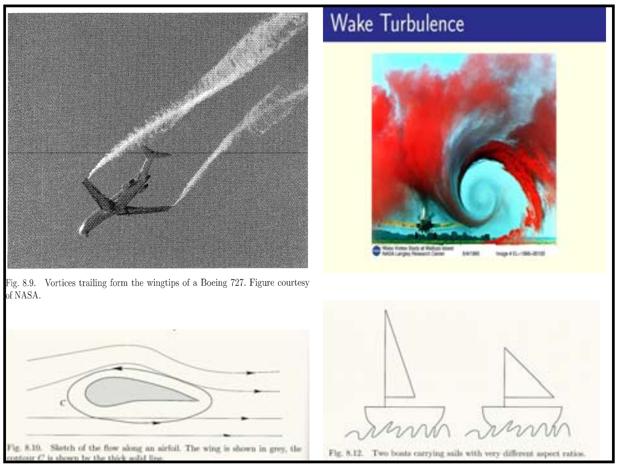
■ **Laplace,** (1740-1827)

Atmospheric Observational net work (1800-1815)
Hydrostatic balance approximation
Tidal wave equation,
Laplacian
Adiabatic sound speed

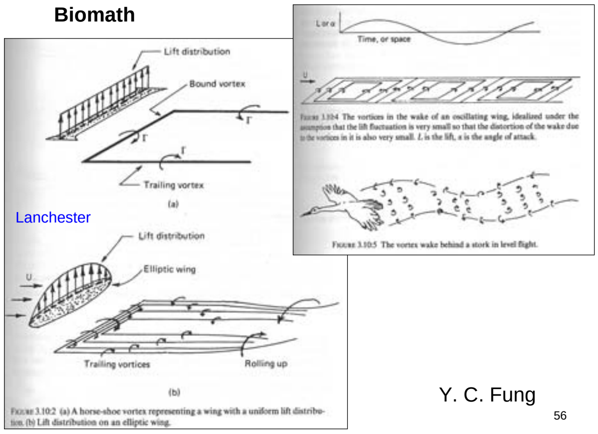
Coriolis Force



Non-inertial Frame



Biomath



Y. C. Fung

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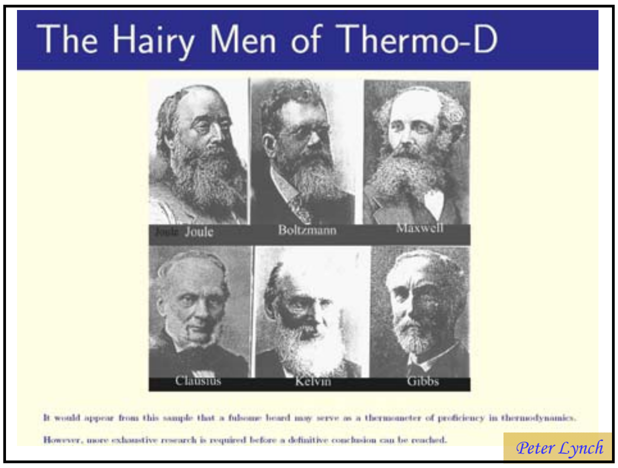

Ideal Gas Law Equation of State 理想氣體方程

- **1662, Boyle law, PV = c when T = c.**
- **1787, Charles law, V/T = c when P = c.**
- **1803, Gay-Lussac law, P/T = c when V = c.**
- **1811, Avagardro, 1 mole gas is 22.4 l in volume.**

Universal Gas Constant
R* = 8314.3 J /(deg.n kmol)

PV = n R* T
PV = m/M R* T P = m/V R*/M T
P = ρ R T, R = R*/M
R_d = 287 J/deg.kg (R*/M_d)
R_v = 461 J/deg.kg (R*/M_v)

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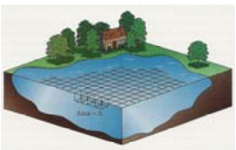
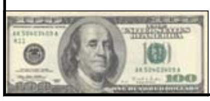
Estimate Avogadro's Number

Benjamin Franklin (1773)



Oil spreads on water
⟶ molecular size
⟶ Avogadro's number



(1) Molecular size

$$l = \frac{V}{A} = \frac{4.9\ \text{cm}^3}{2.0\times10^7\ \text{cm}^2} = 2.4\times10^{-7}\ \text{cm}$$

(2) Number of molecules

$$N = \frac{A}{l^2} = \frac{2.0\times10^7\ \text{cm}^2}{(2.4\times10^{-7}\ \text{cm})^2} = 3.5\times10^{20}\ \text{molecules}$$

(3) Mass of the oil

$$m = V\times D = 4.9\ \text{cm}^3\times 0.95\ \frac{\text{g}}{\text{cm}^3} = 4.7\ \text{g}$$

(4) Number of moles of oil

$$\text{Moles of oil} = \frac{4.7\ \text{g}}{200\ \text{g/mol}} = 0.024\ \text{mol}$$

(5) Avogadro's number

$$\text{Avogadro's number} = \frac{3.5\times10^{20}\ \text{molecules}}{0.024\ \text{mol}} = 1.5\times10^{22}$$

Now we know: $N_A = 6.022142\times10^{23}$ /mol

59

Development of Thermodynamics 熱力學
19 century
第一定律 能量作功，能量守恆
First law: Energy is what makes it go and energy is conserved.
△Q = △U + WORK
Second law: Entropy tells it where to go!
第二定律 時間之矢，自然單向
Joule, Rudolf Clausius, Lord Kelvin and others
宏觀 微觀
Macro --- Micro
Classical and Statistical Thermodynamics
統計熱力學
Ludwig Boltzmann, 1844-1906, whose work led to an understanding of the macroscopic world on the basis of molecular dynamics.
S = k Log W

雲微物理
Precipitation

Enthalpy
Entropy
Gibbs Free energy

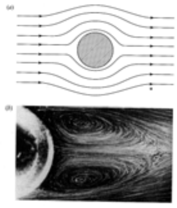
Leonardo da Vinci (1452-1519)

"Observe the movement of the surface of the water which resembles that of hair which has two motions, of which one depends on the weight of the hair and the other on the direction of the curls. Thus water forms eddying whirlpools of which one part depends on the predominant current and the other on the incidental motion and the return flow."

Multiple Scale Interactions
Steady and Turbulent Flows
多重尺度交互作用

1717-1783

D'Alembert Paradox



$$t_v >> t_\nu, \quad \frac{R}{v} >> \frac{R^2}{\nu}, \quad or \quad \frac{vR}{\nu} = \mathrm{Re} << 1$$

D'Alembert Solution of the Wave Equation
[f(x + ct) and f(x-ct)]

Re small viscosity important
Re large viscosity unimportant

Atmospheric Motion first expressed mathematically
(Won the 1746 Berlin Academy's Award; aqua-planet Endorsement of Euler)

Solar and Lunar Force Cause the Atmopheric Motion

62

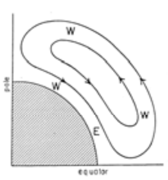


Fig. 2. A schematic view of a meridional cross-section of the general circulation as visualized by Hadley (1735). Streamlines indicate the meridional and vertical flow, while letters E and W indicate regions of easterly and westerly flow.

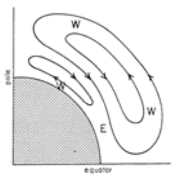


Fig. 3. The same as Fig. 2, but for the general circulation as visualized by Thomson (1857) and Ferrel (1859).

Hadley (1685-1758)

Distribution of sun heating (north and south; seasonal scale)

Earth rotation (conservation of angular momentum)

Thomson (1857)
Ferrel (1859)

Centrifugal force

Coriolis **1835**

D'Alembert **1746**

Math. Model for Atmospheric Motion in aqua-planet
(Won the 1746 Berlin Academy's Award; Euler's endorsement)

Solar and Lunar Force

Fourier **1768-1830**

Why the earth not heating up when receive sun energy continuously?

Heat emission or diffusion (by IR)

His calculations showed a very cold surface (No green house effect)

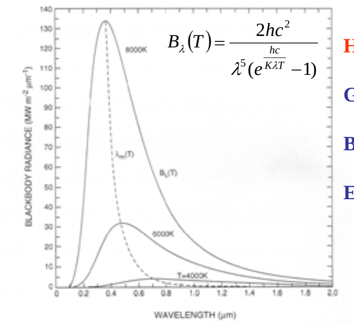
Arrhenius **1896**

CO_2 green house effect, but were dismissed by scientists [WHY??]

63

Planck, Unwilling Revolutionary: the idea of quantization

1900



$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{K\lambda T}} - 1)}$$

Figure 8.7 Spectra of emitted intensity $B_\lambda(T)$ for blackbodies at several temperatures, with wavelength of maximum emission $\lambda_m(T)$ indicated.

Hall of Fame in Science

Gravitational Law

Blackbody Radiation

E= MC^2

64

科氏力(18 19)

Momentum Conservation (18)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \nu\nabla^2 w$$

Mass conservation (18)

$$\frac{\partial \rho}{\partial t} + \frac{\partial u\rho}{\partial x} + \frac{\partial v\rho}{\partial y} + \frac{\partial w\rho}{\partial z} = 0$$

Energy conservation (19)

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = Q$$

Equation of State(17,18,19)

$$p = \rho R_a T, \quad \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_a}{c_p}}$$

(19)

Radiation
大氣輻射
(19,20)
Moisture
Latent heat

雲物理
(19,20)

問蒼茫大氣，誰主浮沈？
質量、動量、能量與大氣狀態方程式

65

Lewis Fry Richardson, 1881–1953.

During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

His 'forecast' was a catastrophic failure:

$\Delta p = 145$ hPa in 6 hours

His method was unimpeachable.

So, what went wrong?

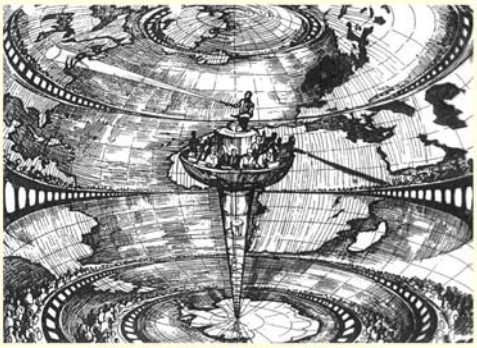
$$\frac{dQ}{dt} \rightarrow \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

$$\frac{df}{dx} \rightarrow \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}.$$

13x13=169個ODE

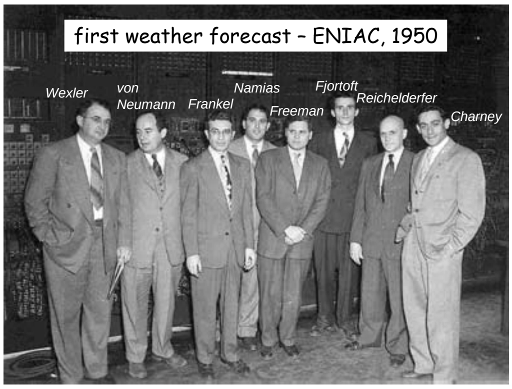
169 自由度

Richardson's Dream



Richardson's Forecast Factory (A. Lannerback).
Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, ECMWF, 1984

64,000 Computers: The first Massively Parallel Processor



In front of the Eniac, Aberdeen Proving Ground, April 4, 1950, on the occasion of the first numerical weather computations carried out with the aid of a high-speed computer. 68

The ENIAC Electronic Numerical Integrator and Computer

18000 vacuum tubes
70000 resistors
10000 capacitor
6000 switches

140 K Watts power

No high-level language
Assembly language

500 Flops
Function Table 0.001 s

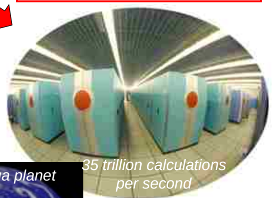
3,700,000,000 times slower than current day large computer

第一部電腦　氣象預報

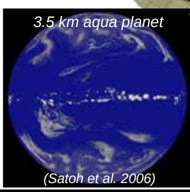
69

ENIAC – late 40s

Earth Simulator -- 2002



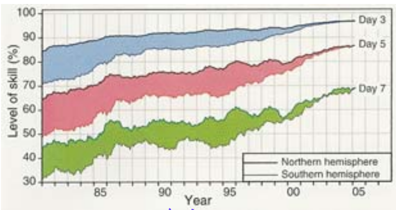
NASDA, JAERI, JAMSTEC



70

南北半球 對於3，5，7天之預報能力隨時間的進展

預報能力指標



年代

南北差異日漸減少主要是由於近年來衛星觀測以及資料同化技術日漸成熟

7天預報一年進步約1.5%，3天預報一年進步0.3%

71

凝結尾　人為

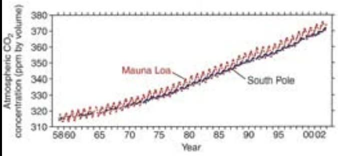
Fig. 10.43 Condensation trails. [Photograph courtesy of Art Rangno.]

All commercial aircraft flights in U.S. were stopped for 3 days after the 911 attack.

Ground diurnal Temperature is larger by 1.1K in these 3 days as compared to the climate mean.

72

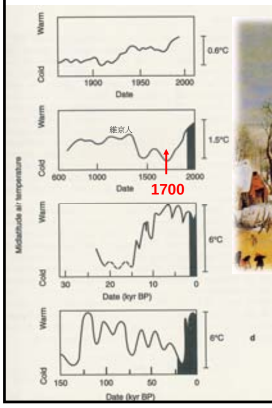
Keeling Curve (1958-)



Charles Keeling (left) at the dedication of the Keeling Building at the Mauna Loa Observatory, Hawaii (1997)

73

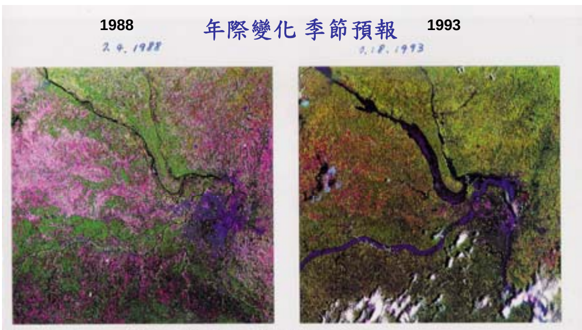
氣候變遷 Warming trend begins 1700A.D.



Bruegel, Pieter, the Younger
< Winter Landscape (1601) >

74

風雨之不時，是無世而不常有之。 荀子天論



Heavy rains in the summer of 1993 produced floods along most of the Mississippi River in the central United States, as shown in these Earth satellite photographs of St. Louis, Missouri on July 4, 1988 (left) and July 18, 1993 (right). Extreme climatic events may be increasing in frequency as a consequence of added radiation absorbing gases in the atmosphere.

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SPECIAL REPORT
Business & Media Institute
ADVANCING THE CULTURE OF FREE ENTERPRISE IN AMERICA

MAY 17, 2006

FIRE AND ICE

Journalists have warned of climate change for 100 years, but can't decide weather we face an ice age or warming

BY R. WARREN ANDERSON
RESEARCH ANALYST

DAN GAINOR
THE BOONE PICKENS FREE MARKET FELLOW

A Time Magazine Time-line

"The discoveries of changes in the sun's heat and the southward advance of glaciers in recent years have given rise to conjectures of the possible advent of a new ice age." — Sept. 10, 1923 (-)

"Gaffers who claim that winters were harder when they were boys are quite right... weather men have no doubt that the world at least for the time being is growing warmer." — Jan. 2, 1939 (+)

"Climatological Cassandras are becoming increasingly apprehensive, for the weather aberrations they are studying may be the harbinger of another ice age." — June 24, 1974 (-)

"[S]cientists no longer doubt that global warming is happening, and almost nobody questions the fact that humans are at least partly responsible." — April 9, 2001 (+)

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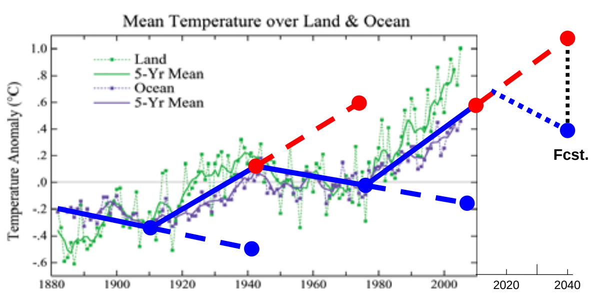


77

JANUARY 31

1977

78

Science Digest

February, 1973

Reports that the world's ***climatologists are agreed*** that "we must prepare for the next ice age."

Time magazine's June 24, 1974, story showed how Arctic snow and ice had grown from 1968 to 1974.

Newsweek

April 28, 1975

In an article titled "The Cooling World" said that meteorologists are almost **unanimous** that catastrophic famines might result from global cooling.

80

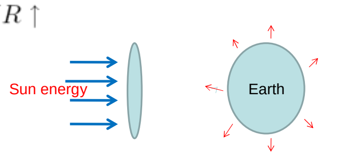
A P R I L 3

2 0 0 6

81

$$C\frac{dT}{dt} = S\downarrow - IR\uparrow$$

比熱 specific heat



$S\downarrow = \pi a^2\, s(1-\alpha)$ $\quad IR\uparrow = 4\pi a^2\, \epsilon\sigma T^4$

反照率 albedo

比熱　　海水 深層海水

反照率　　冰雪　　雲（IPCC沒討論的因素，氣象最大的挑戰）

太陽常數　天文因素　太陽物理

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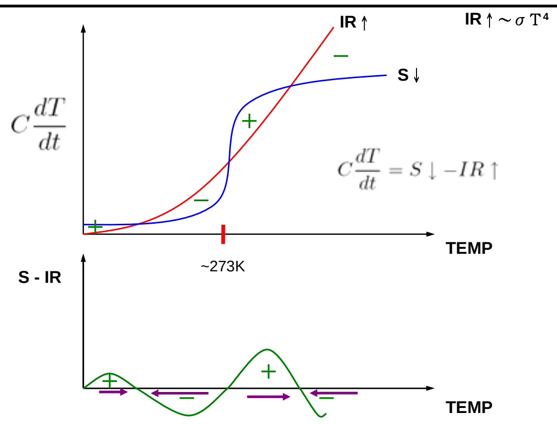

2 stable multiple equilibria

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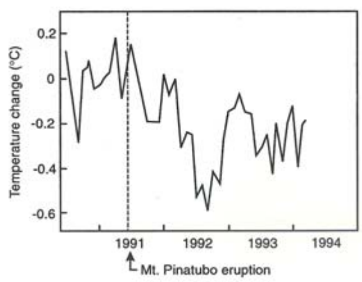
Evidence

Volcano Eruption



A year without summer

Memory effect

Fig. 8-6 Observed change of the Earth's global mean surface temperature following the Mount Pinatubo eruption (June 1991). Adapted from Intergovernmental Panel on Climate Change. *Climate Change 1994*. New York: Cambridge University Press, 1995.

Jacob (1999)

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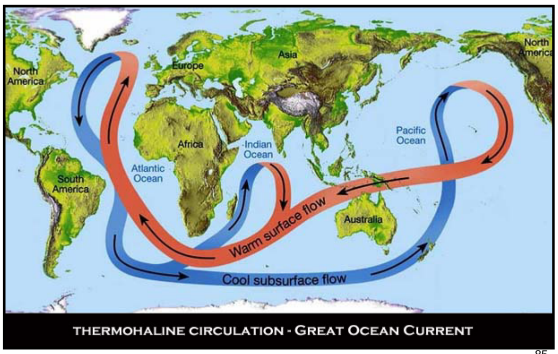
洋流—深海循環



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Uncertainty over weakening circulation

Petr Chylek

Barbara Goss Levi's Search and Discovery story (PHYSICS TODAY, April 2006, page 26) discusses evidence of weakening ocean circulation and its possible connection to global warming. The Atlantic Ocean circulation across 25° N latitude has been used as a benchmark

Bryden and Longworth *Nature* 2005

1957	$22.9 \pm 6\ SV$
2004	$14.8 \pm 6\ SV$
Net	$8.1 \pm 6\ SV$

$1\ SV = 10^6 m^3 s^{-1}$

correct result. It is a mystery how such an error was missed by Levi and by the editors and reviewers of the original paper. The observed change of 8.1 Sv is well within the uncertainty of the measurement. The correct conclusion from

$8.1 \pm 12\ SV$

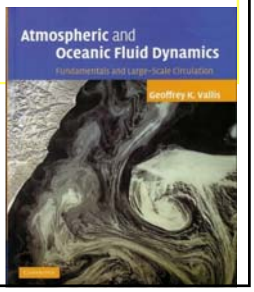
20th Century

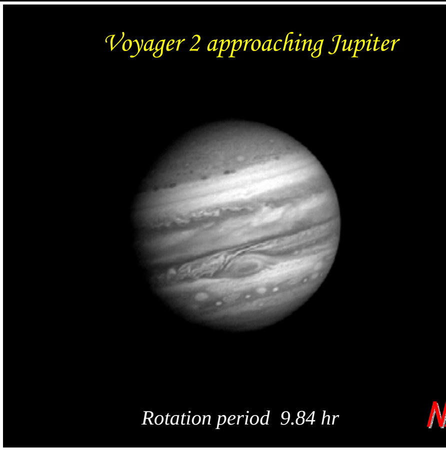
Geophysical Fluid Dynamics (GFD)

Atmospheric Oceanic Fluid Dynamics (AOFD)

is for those interested in doing research in the physics, chemistry, and/or biology of Earth fluid environment.

Fig. 9.2 Karman vortex streets in (a) the laboratory, for water flowing past a cylinder [From M. Van Dyke, An Album of Fluid Motion, Parabolic Press, Stanford, Calif. (1982) p. 56.], and (b) in the atmosphere, for a cumulus-topped boundary layer flowing past an island [NASA MODIS imagery].





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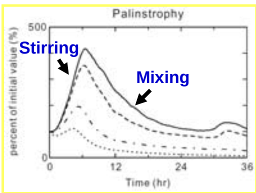
$$\frac{D\theta}{Dt} = \frac{\partial \theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

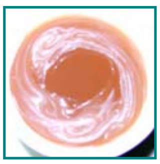
$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta\, dV$$

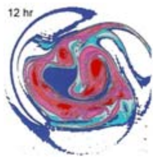
$$\frac{dC}{dt} = \int(\vec{V}\cdot\nabla\theta)\nabla^2\theta\, dV - \nu\int(\nabla^2\theta)\, dV$$

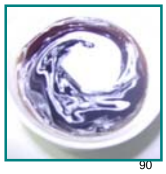
Stirring Mixing



The best part of waking up, is to see vortex in your cup!!







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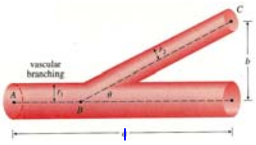
$$-2\pi r l \mu \frac{dv}{dr} = \triangle p \pi r^2$$

$$v(r) = \frac{\triangle p}{4 l \mu}\left(r_0^2 - r^2\right)$$

$$I = \int_0^{r_0} v\, 2\pi r dr = \frac{\pi}{8}\frac{\triangle p}{\mu}\frac{r_0^4}{l}$$

$$\triangle p = I\left(\frac{8}{\pi}\frac{\mu l}{r_0^4}\right)$$

$$V = IR, \quad R = f\left(\frac{cl}{r_0^2}\right)$$



$$\triangle p \pi r^2 = \rho l \pi r^2 \frac{\triangle v}{\triangle t}$$

$$\triangle v \pi r^2 = \frac{\triangle p}{\rho l}\triangle t$$

$$\triangle t \sim \frac{r^2}{\nu}$$

$$I \sim \pi \frac{\triangle p}{\mu}\frac{r_0^4}{l}$$

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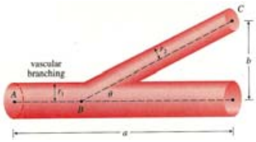

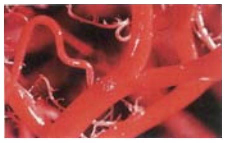

Resistance: $R = C\frac{L}{r^4}$

Total Resistance: $R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$

The Resistance is Minimized when:

$$\cos\theta = \frac{r_2^4}{r_1^4}$$

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$$\begin{cases} \frac{dp}{dt} = K_1 p(1 - \frac{p}{c_1}) - \alpha_1 pq \\ \frac{dq}{dt} = K_2 q(1 - \frac{q}{c_2}) - \alpha_2 pq \end{cases}$$

原方程式變數： K_1 c_1 α_1 / K_2 c_2 α_2

令 $x = \frac{p}{c_1}$, $y = \frac{q}{c_2}$, $\frac{\alpha_1 c_2}{K_1} = a$, $\frac{\alpha_2 c_1}{K_2} = b$

$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$

強弱參數＝攻擊力/修護力， 攻擊力＝戰鬥力＊族群數目

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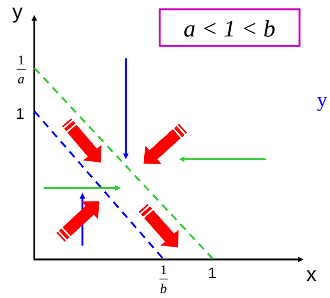
$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$



$a < 1 < b$ X 比 Y強

y 滅絕

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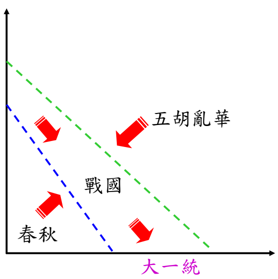
強弱之爭



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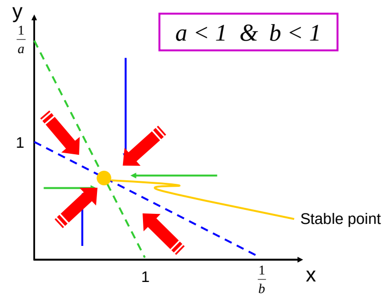
$$\begin{cases} \frac{dx}{dt} = K_1 x(1 - x - ay) \\ \frac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$

$a < 1$ & $b < 1$



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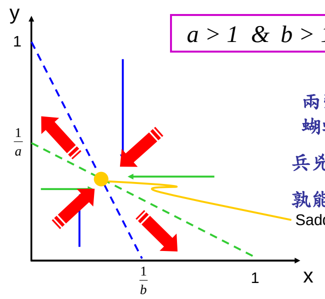
$$\begin{cases} \dfrac{dx}{dt} = K_1 x(1 - x - ay) \\ \dfrac{dy}{dt} = K_2 y(1 - y - bx) \end{cases}$$

$a > 1 \ \& \ b > 1$



兩強相爭
蝴蝶效應

兵凶戰危！

孰能必勝？

Saddle point

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- Colorado State University, USA 博士後研究員 1987-1988

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Epidemics

- **Epidemics: *epi* "upon" and *demos* "the people" , i.e., "upon the people"**
- **An epidemic is the occurrence in a community or region of cases of an illness, specified health behavior, or other health-related events clearly in excess of normal expectancy; the community or region, and the time period in which cases occur, are specified precisely** (Last JM, ed. A Dictionary of Epidemiology. New York: Oxford University Press, 1995)

The "*Black Death*" of 1347–51

SIR Model

$$\frac{dS}{dt} = -\beta SI$$

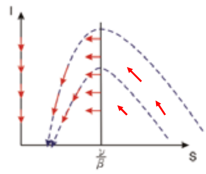
$$\frac{dI}{dt} = +\beta SI - \nu I = (\beta S - \nu)I$$

$$\frac{dR}{dt} = +\nu I$$

ν Recovery Rate

β Infection Rate

No Death in the model



$S = \frac{\nu}{\beta}$ 有 null cline

Forecast and control of epidemics in a globalized world **PNAS vol.101 no.42**

Hufnagel†, Brockmann, and Geisel

演講者: 陳怡文　日期: 2007/12/18 指導教授: 郭鴻基 老師

Use the SIR model with the stochastic forcing from international aviation network to simulate the spread of the SARS, and to explore the strategy for the disease control.

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HIV Modeling

Perelson and Nelson (1999)

$$\frac{dV}{dt} = P - cV, \quad \text{藥物治療}$$

$$\frac{dT}{dt} = kT_0 V - \alpha T,$$

$$P = N\alpha T.$$

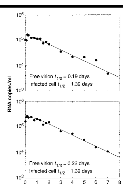

Figure 4.5. Log of plasma concentrations (copies per mL) of HIV-1 RNA (circles) for two representative patients (upper panel, patient 104; lower panel, patient 107) after *ritonavir* treatment was begun on day 0. The solid line is a nonlinear least square fit to the data. HIV-1 RNA level is an easier measure of HIV virions since each HIV virion contains two RNA molecules. (See exercise 5 for more details.) (From Perelson et al. [1996], used by permission of Alan S. Perelson.)

觀察病人服藥後反應決定C

$$P(t_0) \cong cV(t_0) \sim 2 \times 3 \times 10^5 \ \ (1/(day \cdot ml))$$

V: number of virions
p: rate of production of new HIV virions
c: clearance rate for the virions in the plasma
T: infected target cells in unit volumn
: non-infected cells in unit volumn
N: 被感染細胞在其生命期內產生的病毒數目
k: 正常細胞被病毒感染率

Early and aggressive therapeutic intervention is necessary if a marked clinical impact is to be achieved.

何大一雞尾酒療法

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$$\frac{dx}{dt} = x(1-x) - h$$

h: constant rate of population harvested.

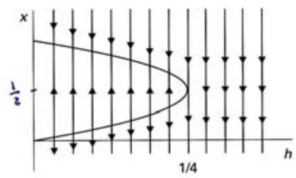

Figure 1.7 The bifurcation diagram for $f_h(x) = x(1-x) - h$.

Small changes in harvesting rate can lead to disastrous changes in population has been observed many times in real situations on earth.

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$$\frac{dP}{dt} = \gamma P \left(1 - \frac{P}{N}\right) - h$$

N 環境承載

$$P^* = \frac{P}{N} \quad t^* = \gamma t \quad h^* = \frac{h}{N\gamma}$$

γ 成長率

h 捕獲率

$$\frac{dP^*}{dt^*} = P^* \left(1 - P^*\right) - h^*$$

$$h < \frac{1/4\ N}{1/\gamma}$$

補獲率在一個生長期內不可以超過環境承載1/4

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