Isaac Newton

Principia 1687

Nature and nature's law
lay hid in night,
God said,
Let Newton be,
and all was light.

Euler **18 century** 流體動量、質量守恒

Fluid Dynamics

- **Pressure gradient force** 壓力梯度力

$$-\frac{1}{\rho}\nabla P$$

- **Eulerian-Lagrangian transformation** 座標轉換

$$\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$$

- **Mass conservation (continuity equation)** 質量守恒

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z}=0$$

Isothermal sound speed (same mistake as Newton)

$$\left(\frac{\partial P}{\partial \rho}\right)_T \quad v.s. \quad \left(\frac{\partial P}{\partial \rho}\right)_\theta$$

fv: E-W Coriolis force

Conservation of angular momentum, 角動量守恒

Hadley 1735

旋轉力學

■ **fu: N-S Coriolis force**

Centrifugal force, thermal wind balance 向心力

Ferrel, 1859

■ **Coriolis force, Coriolis, 1835** 科氏力

▲ **Falkland ship battle in WW I**

■ **Laplace, (1740-1827)**

Atmospheric Observational net work (1800-1815)

Hydrostatic balance approximation

Tidal wave equation,

Laplacian

Adiabatic sound speed

Brief History of Fluid Dynamics

Newton	**1700s**	Viscosity Law of motion for a particle	18世紀
Euler **D. Bernoulli**	**1750s**	Equations for inviscid flow, Law of motion applied to fluids	
Navier **Stokes**	**1827** **1845**	Equations for viscous fluid flow	19世紀
Boussinesq	**1877**	Turbulent mixing, eddy viscosity	
Reynolds	**1880**	Transition to turbulence, Reynolds number	
G. I. Taylor	**1915-1970**	Geophysical flows, rotating flows	20世紀
Prandtle	**1904**	Boundary Layer	

Development of Thermodynamics 熱力學

19 century

雲微物理

Precipitation

First law: Energy is what makes it go and energy is conserved. 第一定律

$\triangle Q = \triangle U + WORK$

Second law: Entropy tells it where to go! 第二定律

Enthalpy

Entropy

Gibbs Free energy

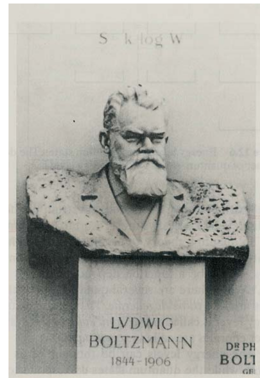
Joule, Rudolf Clausius, Lord Kelvin and others

Macro --- Micro

Classical and Statistical Thermodynamics 統計熱力學

Ludwig Boltzmann, 1844-1906, whose work led to an understanding of the macroscopic world on the basis of molecular dynamics.

$S = k \text{ Log } W$



Ideal Gas Law Equation of State 理想氣體方程

- **1662, Boyle law, PV = c when T = c.**
- **1787, Charles law, V/T = c when P = c.**
- **1803, Gay-Lussac law, P/T = c when V = c.**
- **1811, Avagardro, 1 mole gas is 22.4 l in volume.**

Universal Gas Constant

$R^* = 8314.3$ J /(deg^{-1} kmol)

$PV = n R^* T$

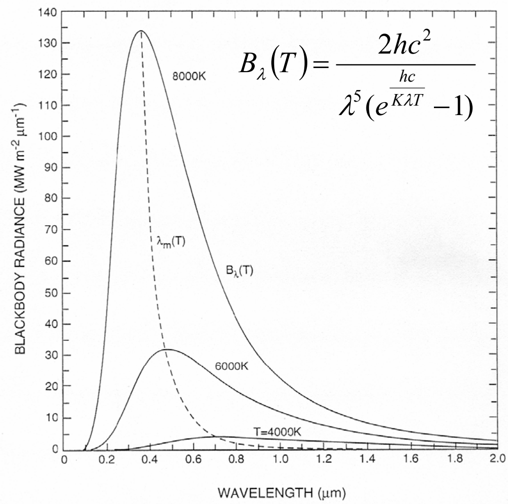
$PV = m/M \, R^* T \quad P = m/V \, R^*/M \, T$

$P = \rho R T, \; R = R^*/M$

$R_d = 287$ J/deg.kg (R^*/M_d)

$R_v = 461$ J/deg.kg (R^*/M_v)

Planck, Unwilling Revolutionary: the idea of quantization



$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (e^{\frac{hc}{K\lambda T}} - 1)}$$

Figure 8.7 Spectra of emitted intensity $B_\lambda(T)$ for blackbodies at several temperatures, with wavelength of maximum emission $\lambda_m(T)$ indicated.

1900

Hall of Fame in Science

Gravitational Law

Blackbody Radiation

E= MC^2

The Ultimate Problem in Meteorology   Bjerknes 1911

I The Present state of the atmosphere must be characterized as accurately as possible. 正確的觀測大氣現狀 [多重時空尺度]

II The intrinsic laws, according to which the subsequent states develop out of the preceding ones, must be known.
正確的大氣運作規律

Numerical Weather Prediction 數值天氣預報
[第一部電腦ENIAC, EBV model, 1950]
The Observation component 觀測
The diagnostic or analysis component 診斷分析
The prognostic component 預報

科氏力(18 19)

Momentum Conservation (18)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\nabla^2 v$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g+\nu\nabla^2 w$$

Mass conservation (18)

$$\frac{\partial \rho}{\partial t}+\frac{\partial u\rho}{\partial x}+\frac{\partial v\rho}{\partial y}+\frac{\partial w\rho}{\partial z}=0$$

Energy conservation (19)

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=Q$$

Equation of State(17,18,19)

$$p=\rho R_a T,\qquad \theta=T\left(\frac{p_0}{p}\right)^{\frac{R_a}{C_p}}$$

(19)

Radiation
大氣輻射
(19,20)
Moisture
Latent heat

雲物理
(19,20)

問蒼茫大氣，誰主浮沈？
質量、動量、能量與大氣狀態方程式

Potential vorticity in a moist atmosphere

$$\frac{DP}{Dt}=P\left(\frac{j\cdot(\nabla\times F)}{j\cdot\zeta}+\frac{k\cdot\nabla\dot{\theta}_\rho}{k\cdot\nabla\theta_\rho}+\frac{\nabla\cdot(\rho_r U)}{\rho}\right)\qquad P=\frac{1}{\rho}\zeta\cdot\nabla\theta_\rho$$

i ii iii iv

Four aspects to understanding the dynamics

(i). Advective dynamics (adiabatic manifold)
waves, turbulence, coherent vortex, and balanced 2nd circulations

(ii). Frictional source/sink (frictional manifold)
PBL, topographic friction, turbulence cascade

(iii). Diabatic source/sink (diabatic manifold)

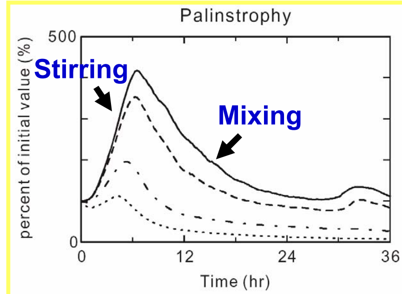
(iv). Precipitation effect

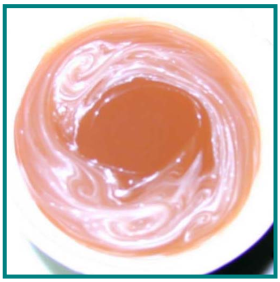
$$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \vec{V}\cdot\nabla\theta = \nu\nabla^2\theta$$

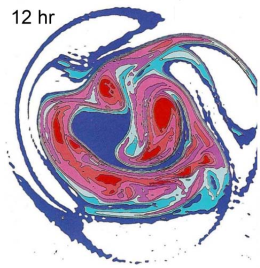
$$C = \frac{1}{2}\int \nabla\theta\cdot\nabla\theta \, dV$$

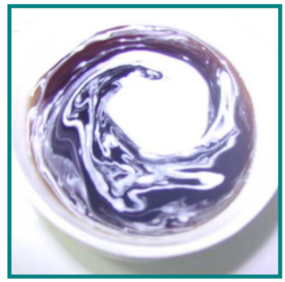
$$\frac{dC}{dt} = \int (\vec{V}\cdot\nabla\theta)\nabla^2\theta \, dV - \nu\int(\nabla^2\theta)\, dV$$

Stirring **Mixing**









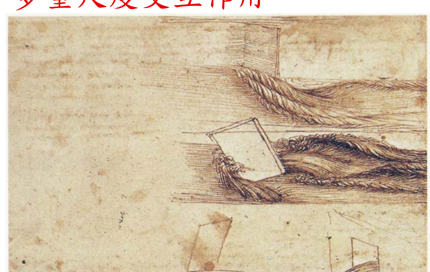
Leonardo da Vinci (1452-1519)

"Observe the movement of the surface of the water which resembles that of hair which has two motions, of which one depends on the weight of the hair and the other on the direction of the curls. Thus water forms eddying whirlpools of which one part depends on the predominant current and the other on the incidental motion and the return flow."

Multiple Scale Interactions

Steady and Turbulent Flows

多重尺度交互作用




Taylor's Experiment 1916
Strong Rotational Fluids

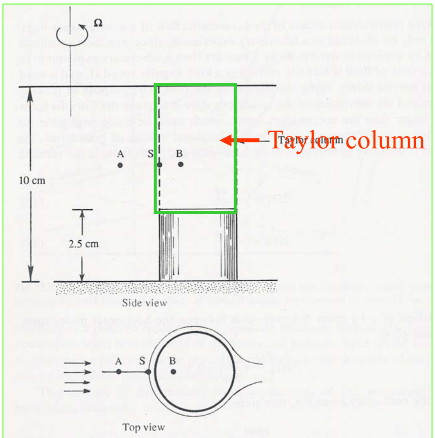


Fig. 13.6 G. I. Taylor's experiment in a strongly rotating flow of a homogeneous fluid.

Dye released from point A divided at point S, as if it had been blocked by an upward extension of the cylinder.

Dye released from a point B within the Taylor column remained there and moved with the cylinder.

The polar vortex traps the ClO rich air over Antarctica for many months. The polar vortex also prevents the lateral mixing of warm air into the pole region.

The phenomenon
is analogous to the horizontal blocking caused by a solid body in a strong stratified system.

Burger Number Rossby number
Froude number

Development of Fluid Dynamics in the 20th Century

Geophysical Fluid Dynamics (GFD)
is for those interested in doing research in the physics, chemistry, and/or biology of Earth fluid environment.

~ McWilliams

- **Vorticity Equation Helmholtz (1858)** $$\frac{\partial \vec{\zeta}}{\partial t} + \vec{V} \cdot \nabla \vec{\zeta} + \vec{\zeta}(\nabla \cdot \vec{V}) = \vec{\zeta} \cdot \nabla \vec{V} + \nabla \times \left(-\frac{1}{\rho} \nabla p \right)$$
- **Kelvin's Circulation Theorem** $$\frac{dC_a}{dt} = \frac{d}{dt} \oint U_a \cdot dl = -\oint \rho^{-1} dp$$

  Stokes' Theorem (1869) $$\oint \vec{V} \cdot d\vec{l} = \iint (\nabla \times \vec{V}) \cdot d\vec{A}$$
- **Bjerknes (1898)** $$\nabla \times \left(-\frac{1}{\rho} \nabla p \right)$$
- **Ertel/Rossby Potential Vorticity (1942)** $$PV = \frac{\vec{\zeta} \cdot \nabla \theta}{\rho}$$
- **Stommel's Model (1961)**
  - Western Boundary intensification
  - Physical oceanography
  - PV

$$\alpha \frac{d^2 \psi}{dx^2} + \beta \frac{d\psi}{dx} = \tau$$
$$\text{B.C. } \psi = 0 \text{ at } x = 0, L$$

- **Haynes and McIntyre (1987)**
  - Impermeability theorem
  - PV substance

$$\vec{\zeta}_a \cdot \nabla \theta = \zeta_\theta \frac{\partial \theta}{\partial z}$$