The 1st APEC Future Scientist Conference 2008/8/20-24 Korea

Typhoon asymmetries and the rotating spring with multiple time scales

Authors : Chen-Chi Chien , Tsung-Lin Hsieh
Taipei Municipal Jianguo High School, Taipei, Taiwan
Advisor : Prof. Hung-Chi Kuo
Department of Atmospheric Sciences, National Taiwan University (NTU), Taipei, Taiwan

Abstract

We study the rotating mass held by a light spring in a horizontal plan with a mathematical model. The mass may rotate circularly in the equilibrium length. If we perturb the spring length from the equilibrium while conserving the angular momentum, the spring motion may possess a multiple-time-scale motion. The time scales are the rotating time and the spring oscillating time. Depending on the ratio of the time scales, the rotating mass may exhibit different polygonal structure. Numerical experiments are summarized with non-dimensional parameters of space and time. We use the rotating spring to interpret the typhoon asymmetrical polygonal eyewall phenomena. The corresponding pressure force oscillation in the polygonal typhoon is in analog to the elastic force variation in the spring. The results highlight the importance of pressure force variations in polygonal typhoon asymmetries.

1. Introduction

On 12 September 2003, satellite images of Atlantic Hurricane Isabel indicated in cloud observation a remarkable pentagonal pattern of small vortices in the Isabel's eye (*Fig. 1a* Kossin and Schubert 2004). Other satellite picture *Fig. 1b* indicated a trigonal structure in Typhoon Nari. The polygonal structures, commonly observed, are important in typhoon intensity changes. The patterns have been previously produced within a frame work of two-dimensional barotropic model (Kossin and Schubert 2001).

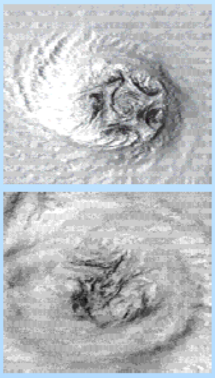
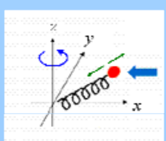


Fig. 1. (a) Visible satellite image of Hurricane Isabel at 1315 UTC on 12 Sep 2003. (Kossin and Schubert 2004) (b) The image shows a trigonal pattern in Typhoon Nari eye clouds (Kossin et al. 2002).

The swing spring is a simple mechanical device that exhibits complex dynamic of multiple time scales. Due to its simplicity and yet rich dynamics, the system can be served as a tool to understand the atmospheric motions. For example, Lynch (2003) uses the swing spring, or the elastic pendulum, to illustrate the atmospheric Rossby wave triads, resonance, and chaos. To understand the asymmetry phenomena (*Fig. 1*) in a strong rotating typhoon, our approach is the rotating spring in a horizontal plan. We, in this work, try to draw analogies between the polygonal structures of typhoon's eye and that of a rotating spring in a horizontal plan.

2. Model Equations



spring with mass.

Our system consists of a rotating heavy mass m with an angular momentum L from a fixed point in a horizontal plan (*Fig. 2*). The mass is held by a light spring (spring constant k, spring length r_0, and zero mass) which can stretch but not bend.

The governing equations in Cartesian coordinate are:

$$r=\sqrt{x^2+y^2}\,,\quad \frac{d^2x}{dt^2}=-\frac{k}{m}\frac{(r-r_0)}{r}x\,,\quad \frac{d^2y}{dt^2}=-\frac{k}{m}\frac{(r-r_0)}{r}y$$

The system is an initial value problem that can be solved with a specification initial x, y position and initial velocity dx/dt, dy/dt.

We use the 4[th] order Runge Kutta method and program the model in MATLAB to integrate the model. Note that the initial position of x_i, and y_i define the initial stretched length of the spring r_i. The initial velocity, combined with the mass and the initial stretch r_i, yield the angular momentum L of the system.

3. Model Result and Analysis

There are five variables L, r_i, r_0, k and m in our problem. According to Buckingham Pi theorem, the results can be summarized by a function only of two non-dimensional variables. The two variables turn out to be related to time and space; one is the rotation time in circular path measured in unit of the spring oscillation time and the other is the perturbed spring length from equilibrium in unit of spring original length.

The equilibrium length r^* of the spring under the rotation satisfies the relationship ,

$$k(r^*-r_0)=m\frac{v^{*2}}{r^*}.$$

Note that the mass path is circular when the initial spring length is the equilibrium length. The reference rotation time (in equilibrium circular path) "$t_{rotation}$" and the period of spring oscillation "$t_{oscillation}$" are:

$$t_{rotation}=\frac{2\pi r^*}{v^*}\,,\quad t_{oscillation}=2\pi\sqrt{\frac{m}{k}}.$$

The non-dimensional variables of time and space are:

$$K=\frac{t_{rotation}}{t_{oscillation}}=\frac{mr^{*2}}{L\sqrt{\frac{m}{k}}}\,,\quad N=\frac{(r_i-r^*)}{r_0}.$$

When the initial spring length is not the same as the equilibrium length (i.e. N≠0), polygonal path of the mass resulted. Examples of polygonal track are given in *Fig. 3*. These sharp features are produced when N is large (large initial length). We use the term wavenumber to describe the pattern, for example, hexagonal path is said to have wavenumber 6.

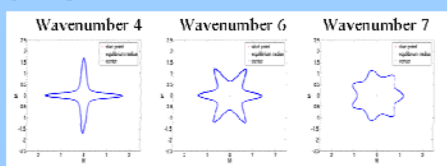


Fig. 3. The patterns with different wavenumber modeled.

Typhoon asymmetries and the rotating spring with multiple time scales

The pattern or the wavenumber can be counted by the number of the peaks along the track in a cycle. The Fourier analysis of the perturbed length time series can also be used. An example of wavenumber 7 pattern and its Fourier analysis is given in ***Fig. 4***.

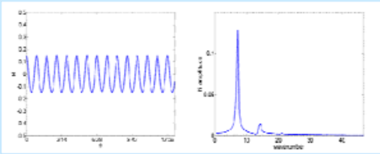


Fig. 4. The wavenumber 7 result in physical (left) and spectral (right) spaces. The spectral space calculation is by Fourier Transform.

Figure 5a shows the modeling wavenumber of the rotating spring as a function of K, N variables. ***Fig. 5b*** is the ratio of our experimental wavenumber and K as a function of K, N. When N is sufficiently small (N<0.2), the pattern of the polygonal wavenumber is precisely predicted, and highly adhere to the K value. Nevertheless, when N is large, the nonlinear effect can lead to the predicted wavenumber diverging from the linear expectation.

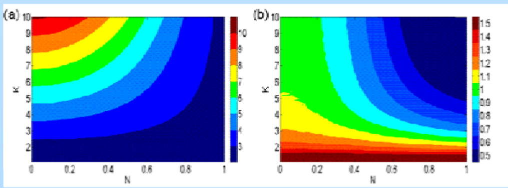


Fig. 5. (a) The wavenumber calculated by modeling. The red represents larger wavenumber, and the blue less. (b) The ratios of the wavenumber between modeling and K. The ratios of the situations in the green part are close to one.

4. Polygonal Typhoon and Rotating Spring

Kossin and Schubert (2001) argued from a simple two-dimensional barotropic model that the presence of the polygonal eyewall may be due to the presence of the small vortices at the edge of the eye. These vortices can be resulted from the wind shear instability near the edge of the eye. Satellite observations appear to support the theory. Here we reproduced the theoretical results in a double Fourier spectral model.

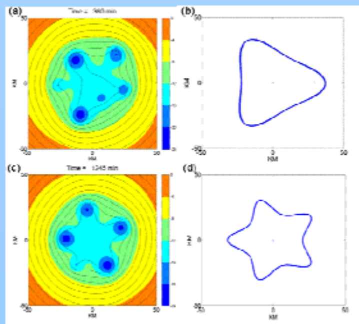


Fig. 6. (a, c) The typhoon polygonal eyewall at 360 and 1245 minutes respectively, modeled by Prof. Kuo's research group. (b, d) The rotat -ing spring track (K=2.6,N=0.31) and (K=5,N=0.23) , which possess wavenumber 3 and 5.

The left panel of ***Fig. 6*** gives the pressure field (in color) and the stream function of the typhoon at times of polygonal eye.

The stream function represents the line of instantaneous air motion and show the trigonal (pentagonal) structure with several vortices in deep blue color near the eye edge in ***Fig. 6a*** (***Fig. 6c***).

The vortices are associated with the low pressure center if they were not collapsing. The stream function near eye edge exhibits polygonal pattern; which resembles a track of rotating mass in the right panel of ***Fig. 6***.

In the polygonal eyewall, there are vortices with lower pressure at the edge. When an air particle flows near the vortex, it may be pushed out because the pressure in the center is larger than the vortex; likewise, when the particle leaves the vortex, it may be pulled in by the pressure difference. The pressure variation along the edge of the typhoon eye is similar to the elastic force variation in the rotating spring system.

5. Summary

We have studied the rotating mass held by a light spring in horizontal plan. The mass exhibits a polygonal structure track in the presence of the spring elastic force and the rotating angular momentum. The results can be summarized by two non-dimensional time and space variables. Barotropic calculations of polygonal typhoon eyewall suggest the existence of vortices and low pressure centers at the eyewall edge. The air flow around the edge of the typhoon eye will experience a variation of pressure gradient force (pull in and push out), which makes the stream function at the eye edge of typhoon appears polygonal in shape. Thus, the physical process is similar to the rotating spring we study; both systems are rotating and the alternating pressure gradient force in typhoon asymmetries is analog to that of the elastic force in our rotating spring system. The asymmetries may be important in typhoon intensity change. A dynamical equivalence with a simple mechanical system sheds light on the understanding of typhoon polygonal eye structure. It highlights the importance of the low pressure centers and vortices near the polygonal eye edge. Without these small vortices, it is difficult to vision such an inhomogeneous pressure gradient can exist in the eye edge. The origin and the stability of the vortices will be studied in the future.

Acknowledgement

The work is supported by National Science Council the Grant NSC 95-2515-S-002-003-MY2 from Taiwan. We would like to thank Hao-Tien Chiang and Tai-Ku William Kuo for useful comments.

References

Kossin, J. P. and W. H. Schubert, 2001: Mesovortices, polygonal flow patterns, and rapid pressure falls in hurricane-like vortices. *J. Atmos. Sci.*, **58**, 2196-2209

Kossin, J. P. , B.D. McNoldy, and W.H. Schubert, 2002:Vortical Swirls in Hurricane eye clouds. , *Mon. Wea. Rev.*, **130**, 3144-3149

Kossin, J. P. and W. H. Schubert, 2004: Mesovortices in Hurricane Isabel. *BAMS*, 151-153

Lynch, P., 2003: Resonant rossby wave triads and the swinging spring. *BAMS*, 605-616