Balanced Atmospheric Response to a Moving Heat Source

by

Hung-Chi Kuo[1] and Wayne H. Schubert[2]

1:Department of Atmospheric Sciences
National Taiwan University Taipei, Taiwan.

2:Department of Atmospheric Sciences
Colorado State University, Fort Collins, CO, USA

Abstract

The balanced atmospheric response to a moving heat source is computed. Specifically, we consider the permanent modifications to the large-scale balanced flow rather than the transient gravity-inertia wave motion. The potential vorticity anomaly, horizontal and vertical wind shears in terms of dimensionless parameter α and swept-through distance of squall line are presented. We emphasize the importance of the α parameter so as to classify the balanced atmospheric response to the moving heat source. Observational cases from mid-latitude and subtropics are given in terms of squall line speed and α. The invertibility computations from squall lines during Taiwan Area Mesoscale EXperiment (TAMEX) are shown. The implication of the results and future research are discussed.

1. Introduction

The semigeostrophic system is a filtered set of equations providing remarkably accurate descriptions of many phenomena which lie beyond description by the quasi-geostrophic equations. Traditionally, the phenomena studied include surface and upper tropospheric fronts, jets and occluding baroclinic waves. In these studies, the potential vorticity field is of central attention. The potential vorticity view provides the simplest way to diagnose or predict balanced dynamics through the use of the Rossby-Ertel potential vorticity on isentropic or entropy surfaces (Hoskins et al., 1985). Schubert et al. (1989) studied the balanced atmospheric response to squall line in mid-latitude. They presented what is apparently the most elegant and concise version of semigeostrophic theory–the version which made simultaneous use of geostrophic and isentropic (θ) coordinates. In geostrophic and entropy coordinates, the divergent part of the circulation remains entirely implicit. In this paper, we apply the model to study the atmospheric equilibrium (geostrophy) response to squall lines during Taiwan Area Mesoscale Experiment (Kuo and Chen, 1990). We discuss the classification of squall lines in terms of the balanced atmospheric response. In Section 2 we review the invertibility principle in semigeostrophic theory. The potential vorticity anomaly, wind shears as functions of α and swept-through region by squall line in the semigeostrophic model are described in Section 3. Section 3 also contains a classification of squall lines as well as the balanced response of atmosphere to the squall lines during TAMEX. Section 4 gives the summary.

2. Semigeostrophic theory

The semigeostrophic equations in terms of "potential vorticity modeling" involves two main mathematical principles. The conservation of potential pseudo-density (the inverse of potential vorticity) and the principle of invertibility. The conservation of pseudo-density serves as the fundamental prediction equation of the model while the invertibility serves as the diagnostics to obtain the balanced wind and mass fields from the predicted potential pseudo-density. The conservation of potential pseudo-density (σ^*, the inverse of potential vorticity) on f-plane with entropy vertical coordinate ($s = c_p ln(\theta/\theta_0)$) is

$$\frac{\partial \sigma^*}{\partial T} + \frac{\partial}{\partial X}(u_g \sigma^*) + \frac{\partial}{\partial Y}(v_g \sigma^*) + \frac{\partial}{\partial S}(\dot{s}\sigma^*) = 0, \qquad (2.1)$$

where $(X, Y, S, T) = (x + v_g/f, y - u_g/f, s, t)$, $\sigma^* = f/\zeta\sigma$ and $\sigma = \partial p/\partial s$.
Defining the Bernoulli function from the Montgomery streamfunction M as

$$M^* = M + \frac{1}{2}(u_g^2 + v_g^2),$$

we have the balanced relationships of wind and mass

$$(fu_g, fv_g, T) = (-\frac{\partial M^*}{\partial Y}, \frac{\partial M^*}{\partial X}, \frac{\partial M^*}{\partial S}). \qquad (2.2)$$

The invertibility principle can be written as

$$\frac{R}{f^4}\begin{pmatrix} M^*_{XX} - f^2 & M^*_{XY} & M^*_{XS} \\ M^*_{XY} & M^*_{YY} - f^2 & M^*_{YS} \\ \rho M^*_{XS} & \rho M^*_{YS} & (\rho M^*_S)_S \end{pmatrix} + \sigma^* = 0. \qquad (2.3a)$$

The upper boundary condition is an entropy surface with entropy S_{top} and the temperature T_{top} is specified. Thus, the upper boundary condition for (2.3a) is

$$\frac{\partial M^*}{\partial S} = T_{top} \quad at \quad S = S_{top}, \qquad (2.3b)$$

where $S_{top} = c_p ln(\theta_{top}/\theta_0)$. Likewise the lower boundary is the isentropic surface with a specified potential temperature $\theta = \theta_0$ and a known surface geopotential (e.g. $\phi_{bot} = 0$ for a flat surface), then $M = c_p T + \phi_{bot}$ at $S = 0$. Written in terms of M^*, this lower boundary condition becomes

$$M^* - c_p \frac{\partial M^*}{\partial S} - \frac{1}{2f^2}(\frac{\partial M^*}{\partial X})^2 - \frac{1}{2f^2}(\frac{\partial M^*}{\partial Y})^2 = \phi_{bot} \quad at \quad S = 0. \qquad (2.3c)$$

Together with appropriate lateral boundary conditions, equations (2.1), (2.2) and (2.3) form a closed system. The computational cycle is as follows: knowing σ^*, solve (2.3) for M^*; use (2.2) to compute (u_g, v_g); use these geostrophic winds in (2.1) to predict a new σ^*.

3. Atmospheric response to a moving heat source

An ideal moving heat source with a Gaussian shape in X and a mid-tropospheric maximum in two dimension and propagating at constant speed c is considered. The source is

$$\dot{s} = S_0 exp[-\frac{(X - cT)^2}{X_0^2}] sin[\frac{\pi S}{S_{top}}], \qquad (3.1)$$

where S_0 is the magnitude which relates to a heating rate Q_0 by $S_0 = c_p Q_0/\theta_0$ and X_0 the e-folding half-width of the heat source. We define a dimensionless parameter

$$\alpha = \frac{S_{top}/(\pi S_0)}{X_0/c}, \qquad (3.2)$$

α can be interpreted as the ratio of the convective overturning time to the squall line passage time of an air parcel entering the squall line (Schubert et al. 1989). To interpret α differently, we can view α as the ratio of time rate of area in (X, S) space span by the moving squall line to the time rate of entropy change by the squall line heat source. Since the potential vorticity is the mass inside X and S, α may also be interpreted as the inverse of the potential vorticity production or reduction rate for the region swept-through by the squall line. In any event, α is small for heavily raining, wide, slowly moving squall lines while it is large for weakly raining, narrow, fast moving squall lines.

To substantiate the idea that α is the inverse of the potential vorticity production or reduction rate for the region swept-through by the squall line, we have examined the extreme PV anomaly produced by the heat source (3.1) in the semigeostrophic solution. Our calculation indicates that the extreme PV anomaly value produced by a moving heat source (squall line) is a function of α only. It is not a function of squall line speed c or time. The maximum normalized q/f as a function of α in the lower atmosphere and minimum normalized q/f as a function of α in the upper atmosphere are presented in Figure 1.

In addition to a fixed α, we have found that the balanced response of the atmosphere depends only on the swept-through distance of the heat source. The permanent modifications to the large-scale balanced flow by the moving heat source depends on the PV anomaly and the size of the anomaly region. For example, the response of $c = 5m/s$ at hour 10 is the same as the response of $c = 10m/s$ at hour 5 for a fixed α. Figures 2 and 3 give the maximum horizontal wind shear across the PV anomaly in the upper and lower atmosphere. Figure 4 presents the maximum vertical wind shear (holding X fixed) in the unit of $ms^{-1}hPa^{-1}$. The ordinate in these figures is the swept-through distance of heat source. Thus the figures give a quantitative measure in either the balanced wind shear as function of α and c at a fixed time or the time evolution of balanced wind shear for a fixed α. There is an increase in both upper and lower atmospheric horizontal wind shears as the swept-through distance and α decrease. On the other hand, the vertical wind shear increases as α decreases and swept-through distance increases. Observational squall lines from COPT81 (Roux, 1988), TAMEX (Chen and Chou, 1993) and mid-latitude Oklahoma (Schubert et al. 1989 and Hertenstein and Schubert, 1991) are given in terms of squall line speed and α in Figure 5. The TAMEX cases are different as compared to mid-latitude (OK) cases in that α and c in TAMEX are smaller. This is due to the moist environment in the TAMAX region as compared to the mid-latitude Oklahoma case. The very moist background probably leads to a stronger precipitation heating rate (thus a smaller α) and a lack of cold air behind the squall line (thus the slower moving speed). The COPT81 cases share similar α value as the TAMEX cases and possess a faster moving speed. According to Figs. 2, 3 and 4, the TAMEX squall lines tend to produce larger balanced horizontal wind shears in the upper atmosphere than the COPT81 squall lines. On the other hand, the COPT81 squall lines have twice as much balanced vertical wind shears.

Figure 6 gives the results of non-dimensionlized q (6.a), the geostrophic flow and the disturbed pressure (6.b) at $T = 16hr$ in IOP6 case in TAMEX. The θ is used in the plotting rather than the entropy S. Behind the squall line, there is a decrease in σ (stabilization) in the lower troposphere and an increase in σ (destabilization) in the upper troposphere. This is a reflection of the mutual adjustment between the wind and mass fields. This adjustment happens in such a way that $q = \zeta/\sigma$ is large in the lower part and small in the upper part. We have larger ζ and smaller σ for large $q$and the opposite for small q. It is possible that the upper destabilization provides a favorable background for the upper stratiform precipitation while the large wind shear in the lower part of the troposphere causes the instability. It is interesting to note that the induced geostrophic balanced flow covers region greater than the region swept through by the squall line. Although the solutions have similar structure as compared to the mid-latitude case studied by Schubert et al. (1989), the TAMEX cases apparently possess much stronger vertical and horizontal wind shears.

4. Summary

The balanced atmospheric response to a moving heat source is computed. We have found that the balanced atmospheric response to a moving heat source depends only on the potential vorticity anomaly intensity and anomaly size. The potential vorticity anomaly is a function of α only while the anomaly size is related to the swept-through region by the moving heat source. The potential vorticity anomaly, horizontal and vertical wind shears as functions of a dimensionless parameter α and the swept-through distance of the squall line are given. We stress the importance of the α parameter so as to classify the balanced atmospheric response to the moving heat source. Observational squall lines from COPT81, TAMEX and mid-latitude are classified in terms of their impact on the balanced atmosphere. The TAMEX cases are different as compared to mid-latitude (OK) cases in that α and c are smaller. The COPT81 cases share similar α value as the TAMEX cases and possess a faster moving speed.

Acknowledgement

The author would like to extend thanks to Prof. Scott Fulton for providing the package of multigrid routine which were so important for the completion of this research. This work was supported by the Grant NSC 82-0202-M002-073 from the National Research Council of Taiwan.

References

Chen, G. T.-J. and H.-C. Chou, 1993: On the general characteristics of squall lines observed in TAMEX. Accepted in *Mon. Wea. Rev.*.

Hertenstein, R. F. A., and W. H. Schubert, 1991: Potential vorticity anomalies associated with squall lines. *Mon. Wea. Rev.*, **119**, 1663–1672.

Hoskins, B. J., M. E. McIntyre, and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Q. J. R. Met. Soc.*, **111**, 877–946.

Kuo, Y.-H. and G. T.-J. Chen, 1990: The Taiwan Area Mesoscale Experiment (TAMEX): an overview. *Bull. Amer. Meteor. Soc.*. **71**, 488–503.

Roux, F., 1988: The west African squall line observed on 23 June 1981 during COPT 81: kinematics and thermodynamics of the convective region. *J. Atmos. Sci.*, **45**, 406–426.

Schubert, W. H., S. R. Fulton, and R. F. A. Hertenstein, 1989: Balanced atmospheric response to squall lines. *J. Atmos. Sci.*, **46**, 2478–2483.

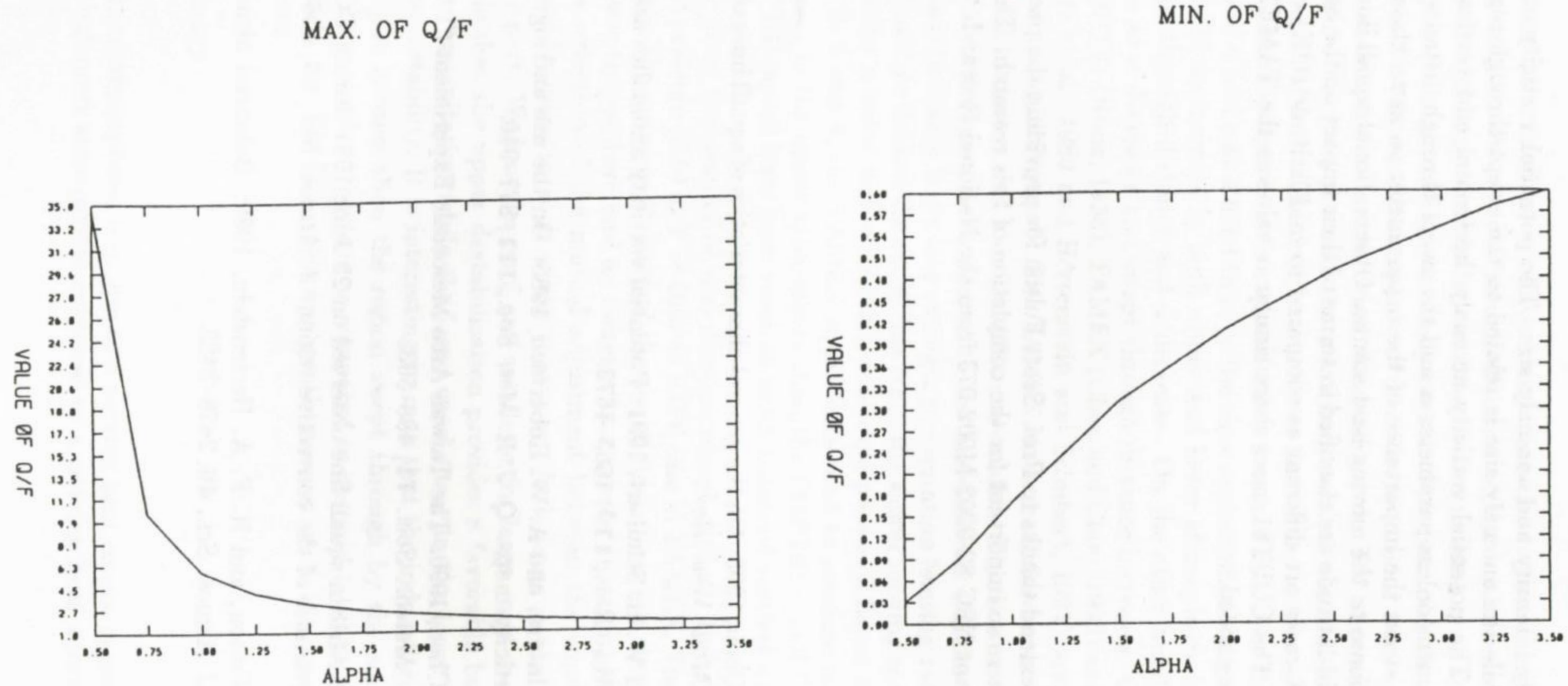

Fig. 1 The maximum normalized q/f as a function of α in the lower atmosphere and the minimum normalized q/f as a function of α in the upper atmosphere.

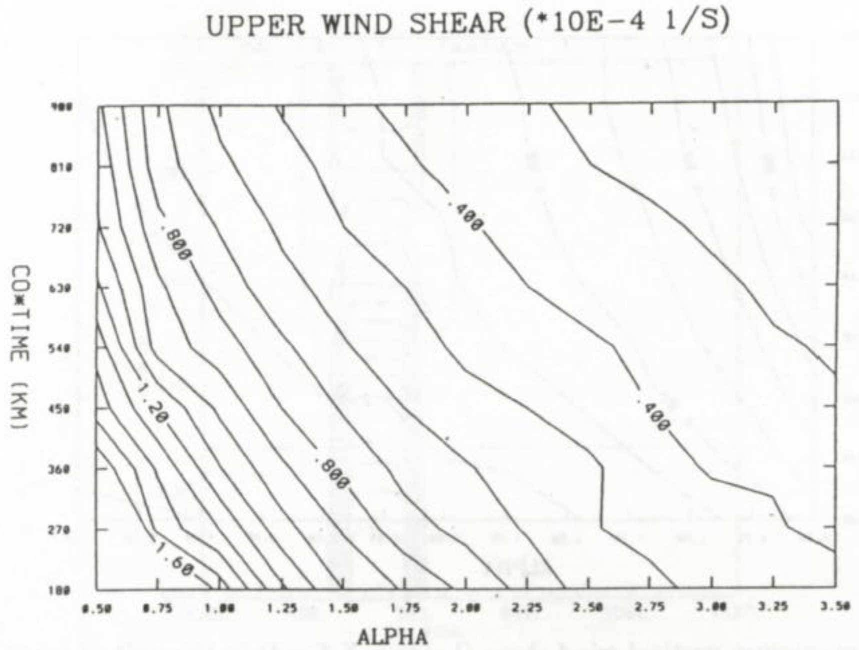

Fig. 2 The maximum horizontal wind shear across the PV anomaly in the upper atmosphere as a function of the swept-through distance of heat source (ordinate) and α (abscissa).

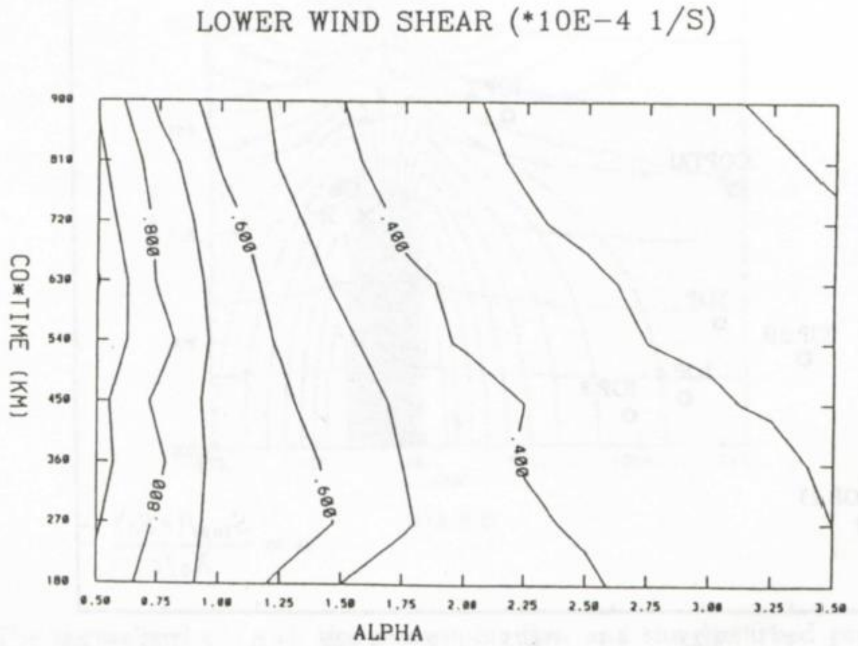

Fig. 3 The maximum horizontal wind shear across the PV anomaly in the lower atmosphere as a function of the swept-through distance of heat source (ordinate) and α (abscissa).

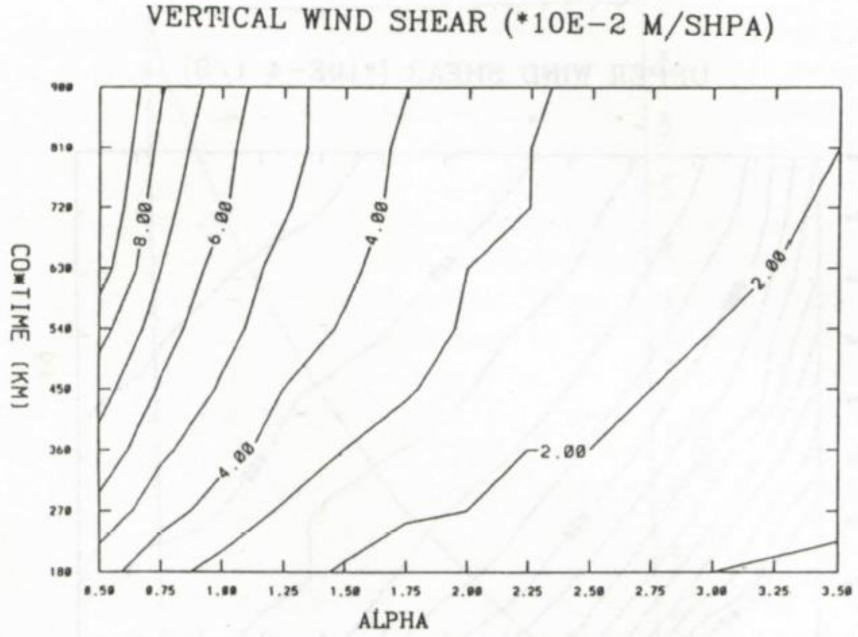

Fig. 4 The maximum vertical wind shear (holding X fixed) in the unit of $ms^{-1}hPa^{-1}$ as a function of the swept-through distance of heat source (ordinate) and α (abscissa).

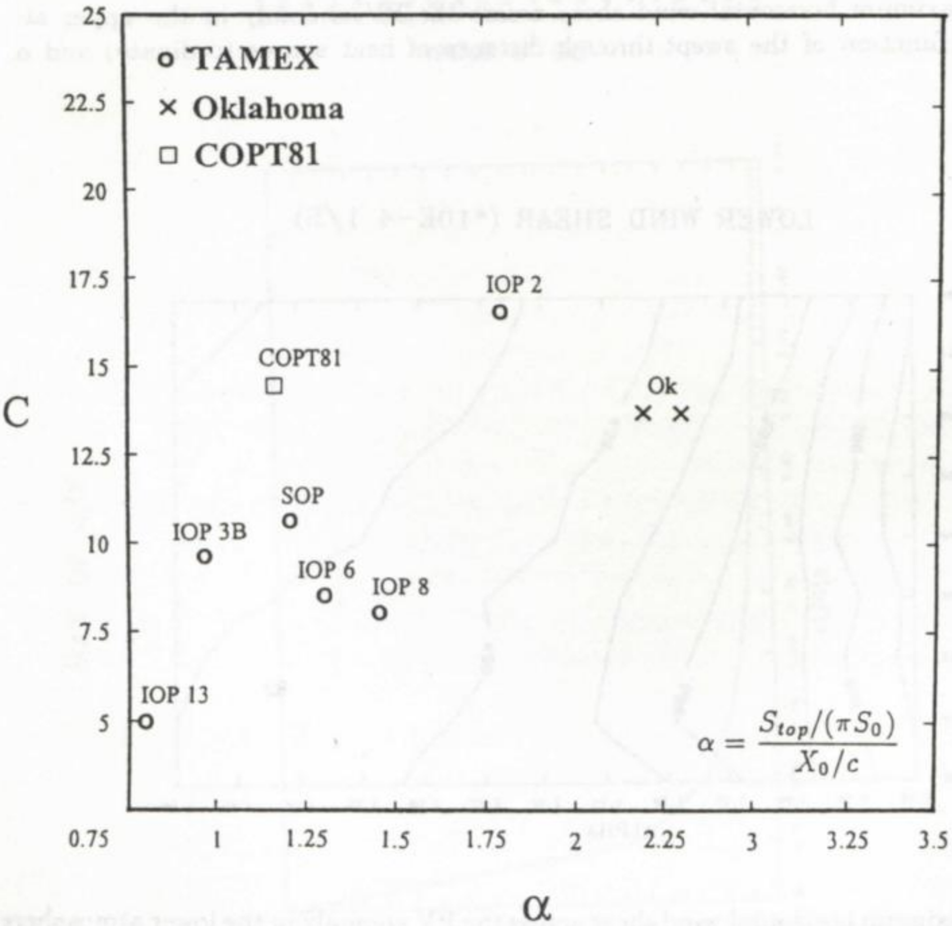

Fig. 5 Observational squall lines from COPT81 (Roux, 1988), TAMEX (Chen and Chou, 1993) and mid-latitude Oklahoma (Schubert et al. 1989 and Hertenstein and Schubert, 1991) as a function of squall line speed $C(ms^{-1})$ and parameter α.

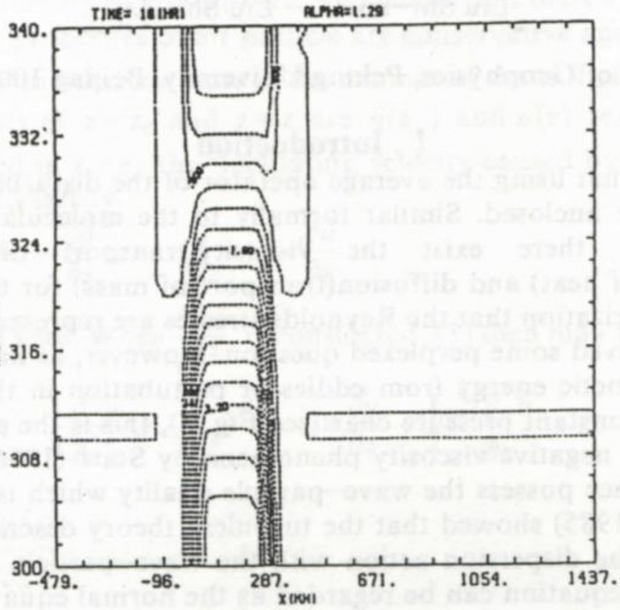

Fig.6.a

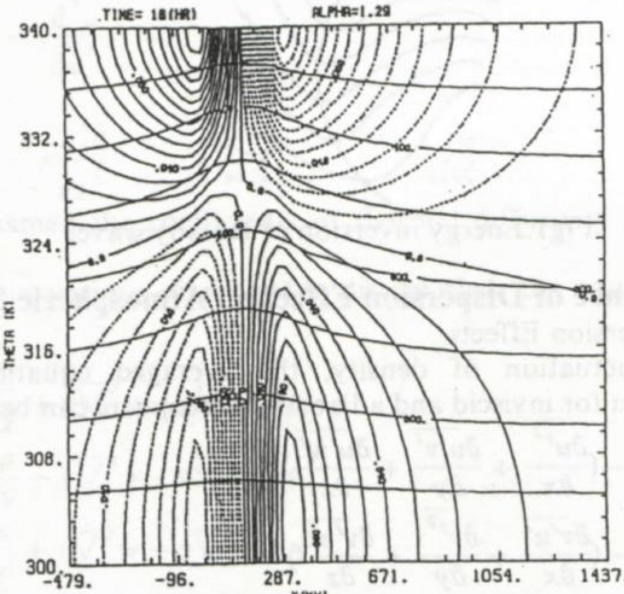

Fig.6.b

Fig. 6 The normalized q/f (a), the geostrophic flow and the disturbed pressure (b) at $T = 16hr$ in IOP6 case of TAMEX. The geostrophic wind speed is scaled by 100.0. The wind interval is $1ms^{-1}$. The dashed lines are out of page and solid lines are into the page. The disturbed pressure is in the unit of hPa.